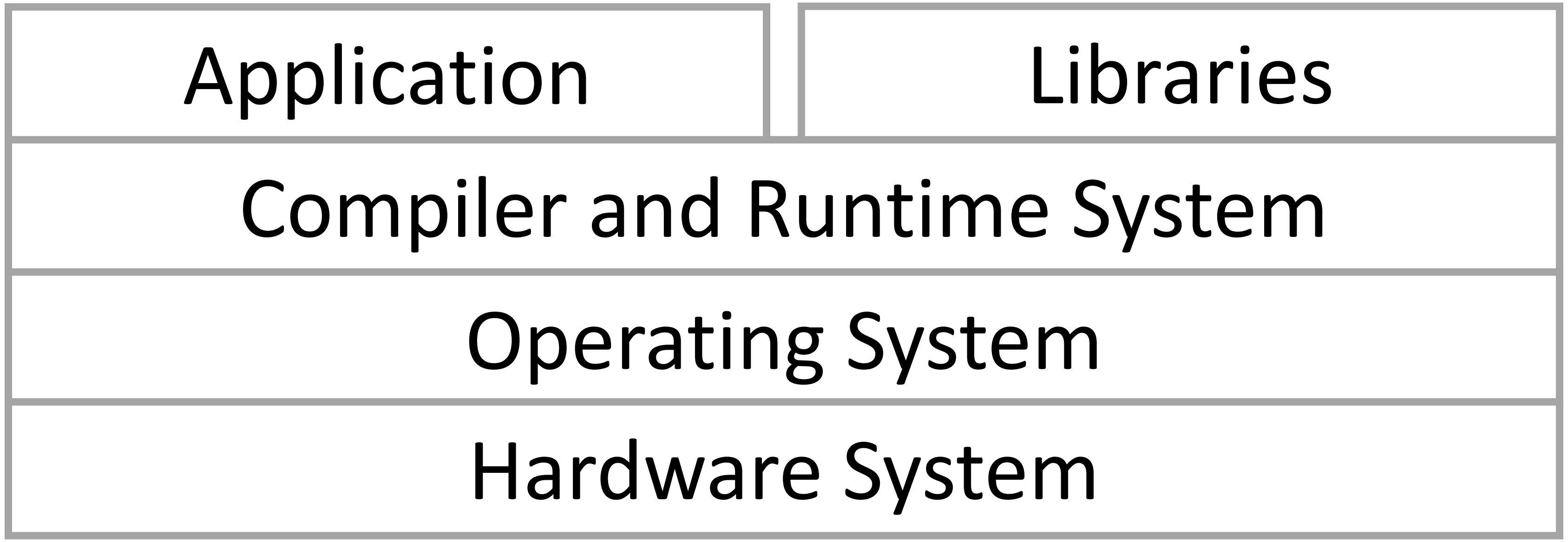
From Reliability to Resilience via Program Verification

Michael Carbin

MIT EECS & CSAIL

Software/Hardware Stack



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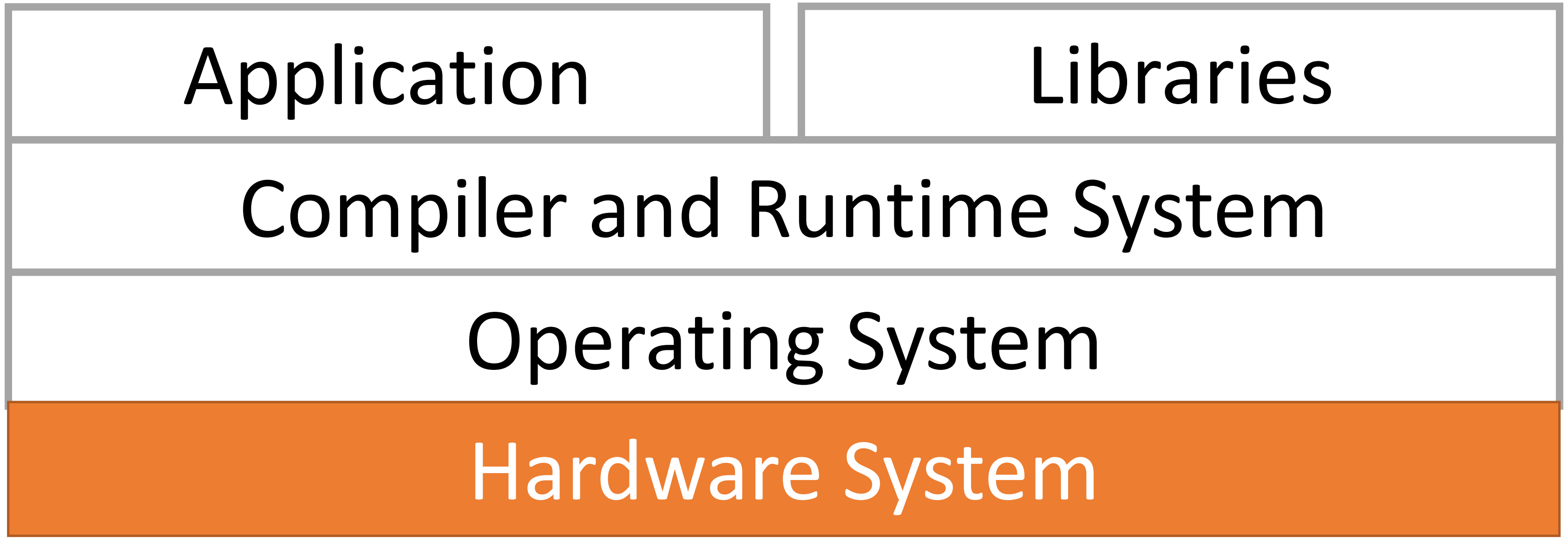


Examples

- ~107 Issues in Current Errata for Intel 6th Generation (May 2016)
- SKL057: Cache Performance Monitoring Events May be Inaccurate
- SKL082: Processor May Hang or Cause Unpredictable System Behavior
- “Under complex microarchitecture conditions, processor may **hang** with an internal timeout error ... or cause **unpredictable system behavior**.”

In Practice

- **Consumer**: “Intel Skylake bug causes PCs to freeze during complex workloads: Bug discovered while using Prime95 to find Mersenne primes.”
  - http://arstechnica.com/gadgets/2016/01/intel-skylake-bug-causes-pcs-to-freeze-during-complex-workloads/
- **Supercomputer**: “Researchers performed a study in 2010 on the then most powerful supercomputer, called Jaguar. The study found that [uncorrectable] errors occurred about once every 24 hours in Jaguar’s 360 TB of memory.”
  - http://spectrum.ieee.org/computing/hardware/how-to-kill-a-supercomputer-dirty-power-cosmic-rays-and-bad-solder

What to do?

- **Distinction**: some errors are transient (non-deterministic)
- Logic: complex workloads with multiple architectural events happening concurrently triggers corner case in design (Programming Bugs).
- Electrical: increasingly smaller transistors are more sensitive to physical variation in fabrication process (Physical Bugs)
- Environmental: cosmic-rays

What do you do?

Replication:

```
z = x + y;
```

⇨

```
do {
    z  = x + y;
    z’ = x + y;
} while (z != z’)
```

Checkable Computations:

```
x = newton_method(f, guess);
```

⇨

```
do {
    x = newton_method(f, guess);
} while (abs(f(x)) > eps);
```

What if we just let errors *happen*?

Faster and consumes less energy!

May give ~~the wrong~~ result.

a ***different***

Different Results

- Produce an inaccurate result

  5 + 5 = 8

- Produce correct results too infrequently

  $\Pr(5 + 5 = 10)$ too low

- Produce an invalid result

  5 + 5 = "hello"

- Crash or do something nefarious

  5 + 5 = exec "/bin/launch_missiles"

Approaches

- Self-Stabilizing Algorithms
  - Algorithm-based Fault Tolerance for Matrix Operations (Huang and Abraham, 1984)
  - Fault-Tolerant GMRES (Sao et al., 2011)
  - Self-Stabilizing Conjugate Gradient (Sao et al., 2013)
  - Self-Correct Connected Components (Sao et al., 2016)
- Non-interference + Empirical Guarantees
  - Enerj (Sampson et al. 2011), Truffle (Esmaeilzadeh et al., 2012)
  - ExpAx (Park et al., 2014), FlexJava (Park et al., 2015)
- Traditional Verification
  - Faulty Logic (Meola and Walker, 2010)
  - Relaxed Programs (Carbin et al. 2012)
- Probabilistic Analysis
  - Rely (Carbin et al. 2013), Chisel (Misailovic et al. 2014)
  - Uncertainty Quantification

In Progress Systems

- Leto: Verifying Fault Tolerance with First-Class Execution Models
  - Programmer, platform designer specifies an stateful execution model that prescribes a semantics for the platform
  - Verification system weaves in model and enables check fault tolerance properties such as memory safety,, non-interference, and accuracy
  - Student: Brett Boston

- Shuffle: Typesafe Handcoded Probabilistic Inference
  - NIPS – Machine Learning Systems Dec, 2016
  - Students: Eric Atkinson and Cambridge Yang

Noise Model

$$f(x) = h(g(x))$$

$$\hat{f}(x) = h(g(x) + e_1) + e_2 \qquad\qquad e_1 \sim N(\mu_1, 1) \quad e_2 \sim N(\mu_2, 1)$$

- Distribution of Error: $P\left(f(x) - \hat{f}(x)\right)$
- Expected Error : $E[f(x) - \hat{f}(x)]$
- Variance of Error $\mathrm{Var}[f(x) - \hat{f}(x)]$
- Estimation: $P(\mu_1, \mu_2 | \ obs)$ where obs are outputs from the noisy computation

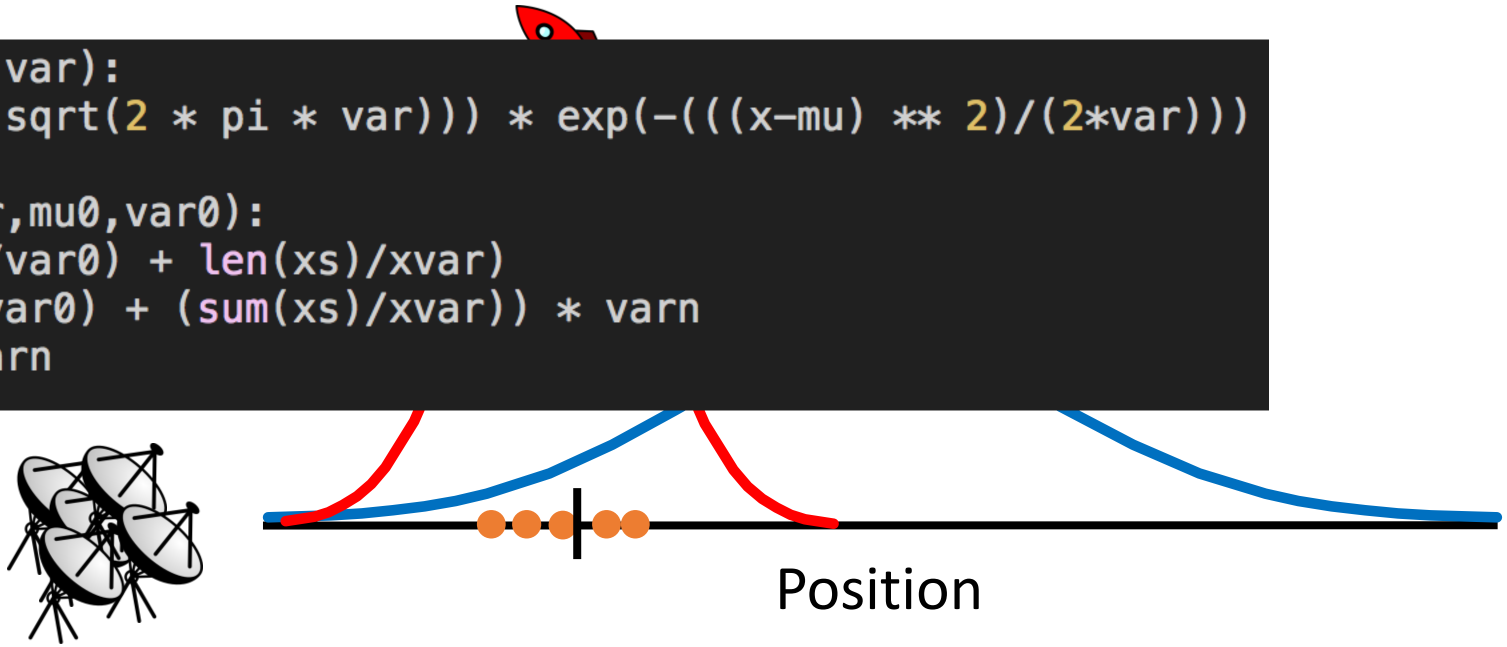
Probabilistic Inference

$\Pr(\mu)$

```
def normald(x,mu,var):
    return (1 / (sqrt(2 * pi * var))) * exp(-(((x-mu) ** 2)/(2*var)))

def infer(xs,xvar,mu0,var0):
    varn = 1/((1/var0) + len(xs)/xvar)
    mun = ((mu0/var0) + (sum(xs)/xvar)) * varn
    return mun,varn
```

$\Pr(x_i|\mu)$



$\Pr(\mu|x)$

Probabilistic Inference

$$\Pr(\mu_j) = N(\mu_j, 0{,}10)$$

$$\Pr(z_i) = U(z_i)$$

$$\Pr(x_i|\mu_j, z_i = j) = N(x$$

$$\Pr(\mu|x) = \cdots$$

```
def normald(x,mu,var):
    return (1 / (sqrt(2 * pi * var))) * exp(-(((x-mu) ** 2)/(2*var)))

def conjugate(xs,xvar,mu0,var0):
    varn = 1/((1/var0) + len(xs)/xvar)
    mun = ((mu0/var0) + (sum(xs)/xvar)) * varn
    return mun,varn

def obscond(obsi,i,obsprev,z,xvar,var0,mu0):
    relobs = []
    for ii in range(0,i):
        if z[ii] == z[i]:
            relobs += [obs[ii]]

    muc, varc = conjugate(relobs,xvar,mu0,var0)
    return normald(obsi,muc,varc + xvar)

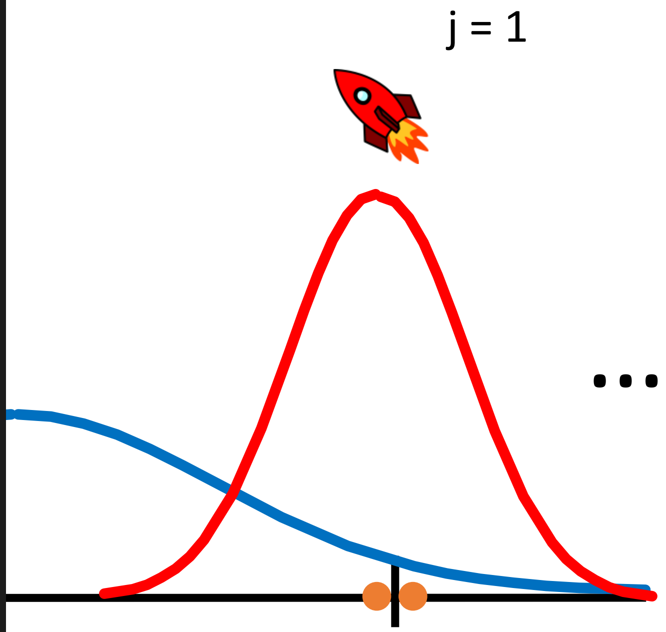
def obsdens(obs,z,xvar,var0,mu0):
    p = 1
    obsprev = []
    for i in range(len(obs)):
        p *= obscond(obs[i],i,obsprev,z,xvar,var0,mu0)
        obsprev += [obs[i]]
    return p

def zdens(z,obs,xvar,var0,mu0):
    s = 0
    for z0 in range(0,2):
        for z1 in range(0,2):
            for z2 in range(0,2):
                for z3 in range(0,2):
                    for z4 in range(0,2):
                        for z5 in range(0,2):
                            zp = [z0,z1,z2,z3,z4,z5]
                            s += obsdens(obs,zp,xvar,var0,mu0)
    return obsdens(obs,z,xvar,var0,mu0) / s

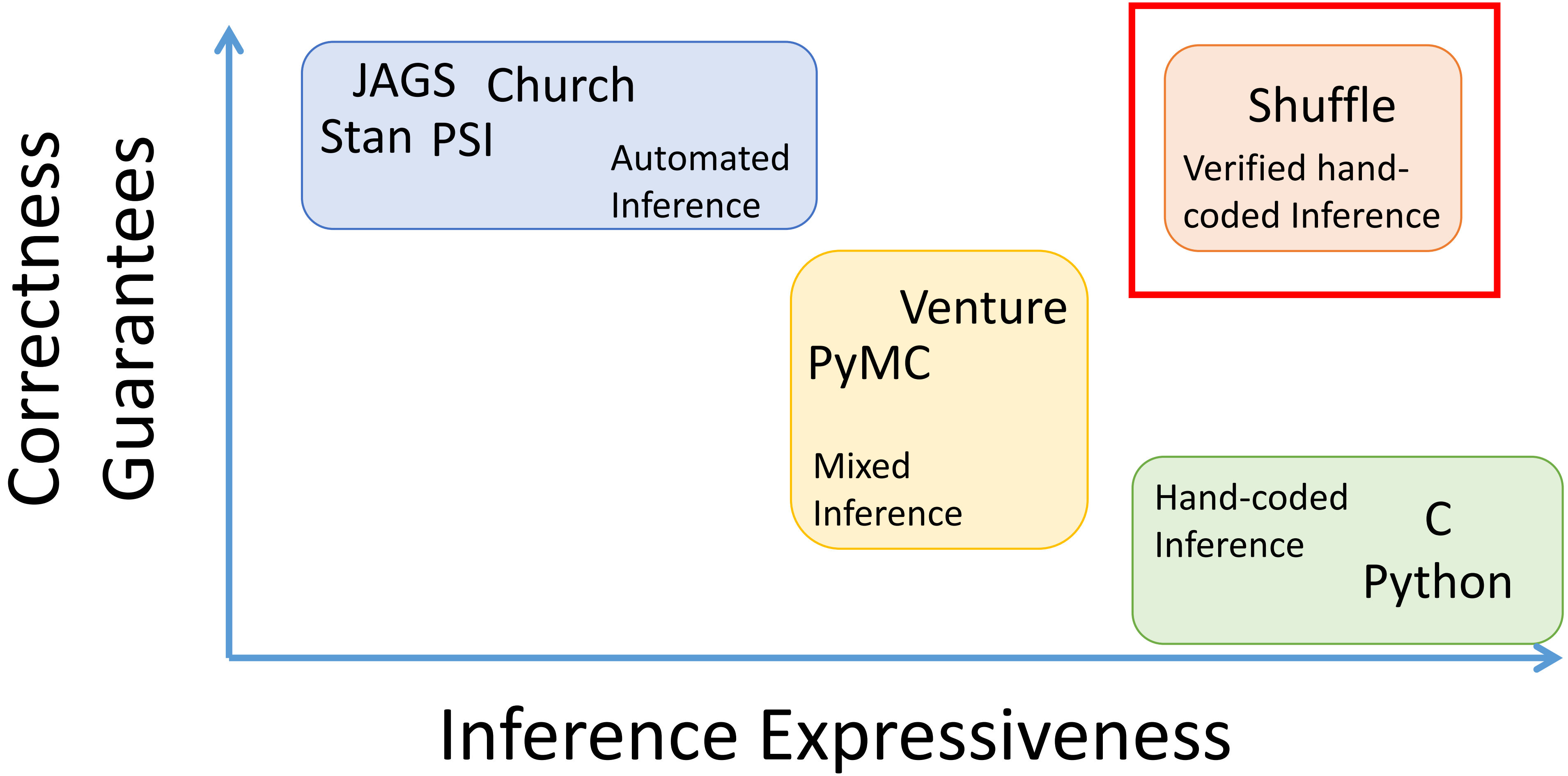
def mupostf(mu,j,z,obs,xvar,var0,mu0):
    relobs = []
    for i in range(len(obs)):
        if z[i] == j:
            relobs += [obs[i]]

    return conjugate(relobs,xvar,mu0,var0)

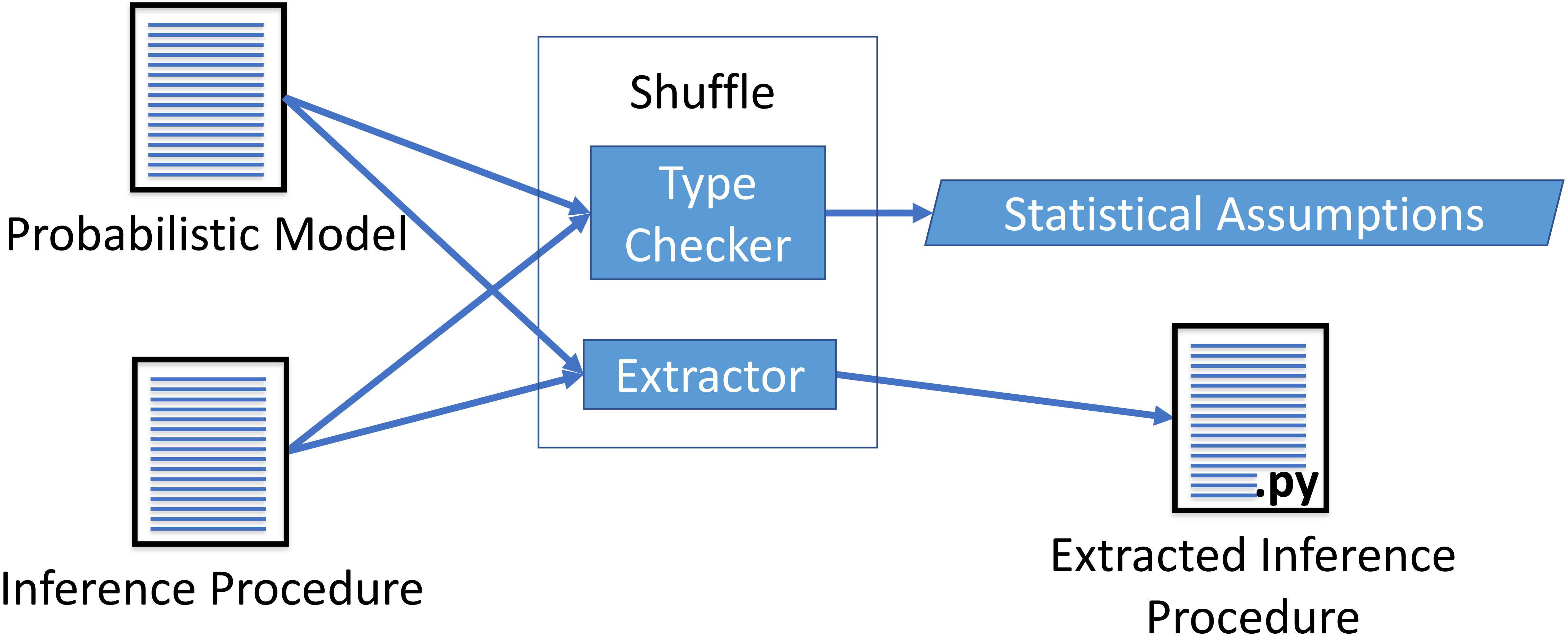
def finalpost(mu,j,obs,xvar,var0,mu0):
    s = 0
    for z1 in range(0,2):
        for z2 in range(0,2):
            for z3 in range(0,2):
                for z4 in range(0,2):
                    for z5 in range(0,2):
                        z = [0,z1,z2,z3,z4,z5]
                        mup,varp = (mupostf(mu,j,z,obs,xvar,var0,mu0))
                        s += 2 * (zdens(z,obs,xvar,var0,mu0)) * normald(mu,mup,varp)
    return s
```



Existing Approaches

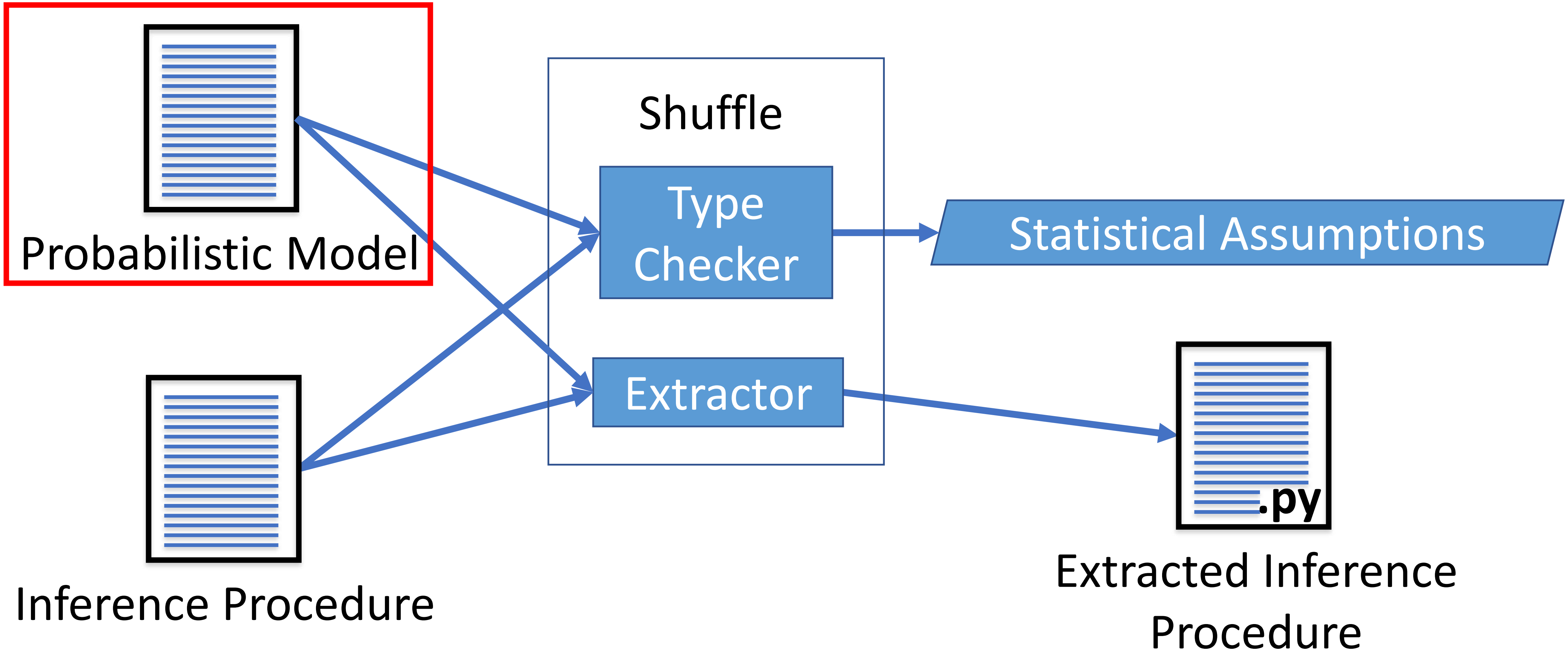


Shuffle



Probabilistic Model
Inference Procedure
Shuffle
Type Checker
Extractor
Statistical Assumptions
.py
Extracted Inference Procedure

Shuffle



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Model Specification

```
model GMM {
   domain Samples, Mus;
   variable R[Samples] obs;
   variable R[Mus] mu;
   variable Mus[Samples] z;

   def muiPrior(i in Mus) : density(mu[i]) =
      normal(mu[i], 0, 100);

   def ziPrior(i in Samples) : density(z[i]) =
       uniform(Mus, z[i]);

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Random Variable Specification

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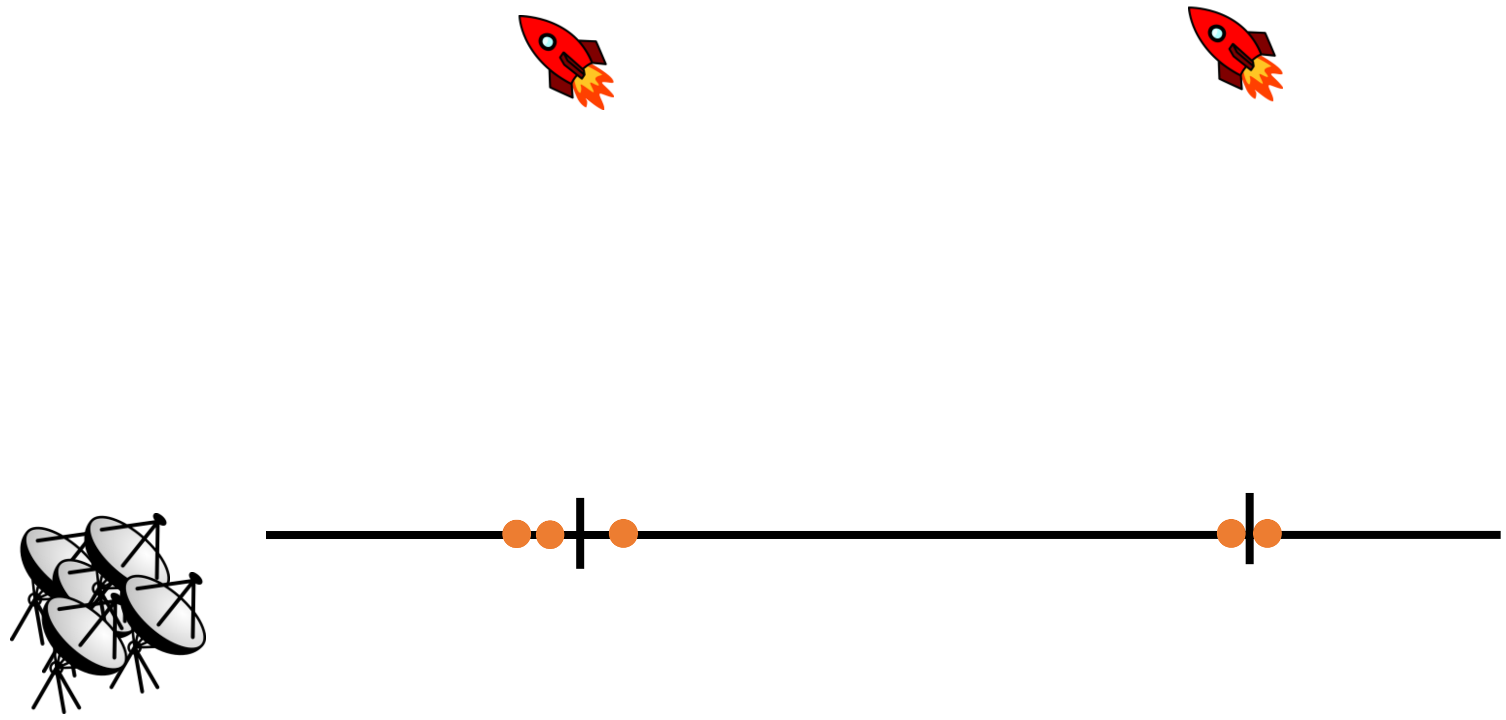

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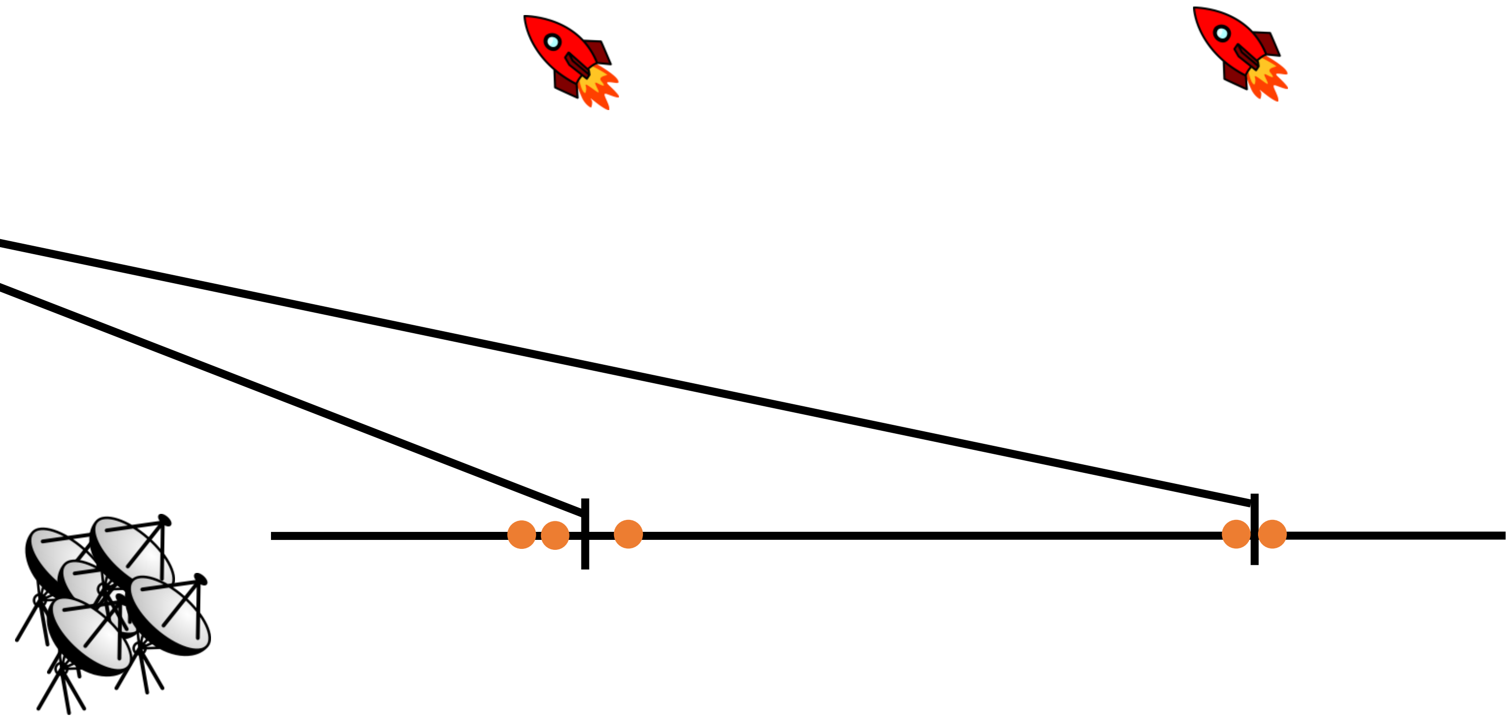

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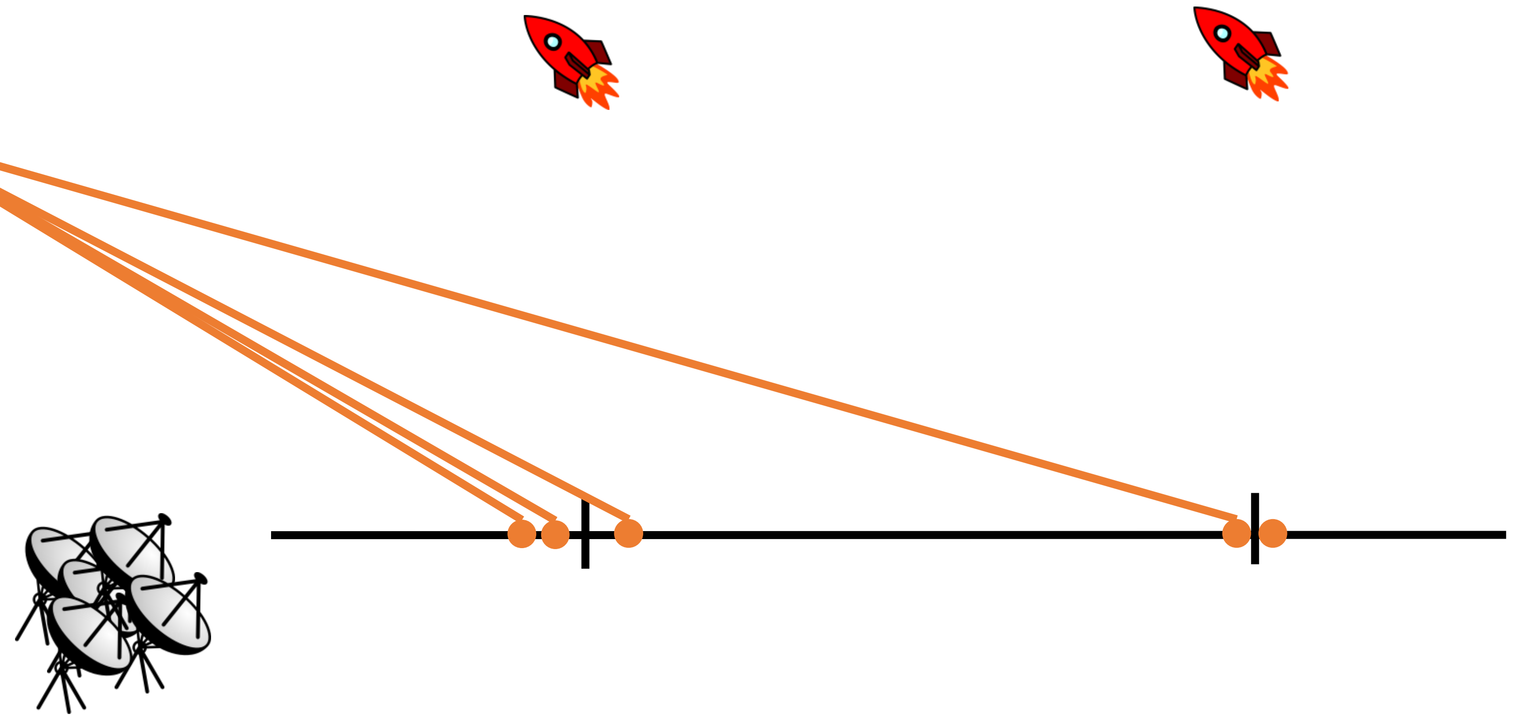

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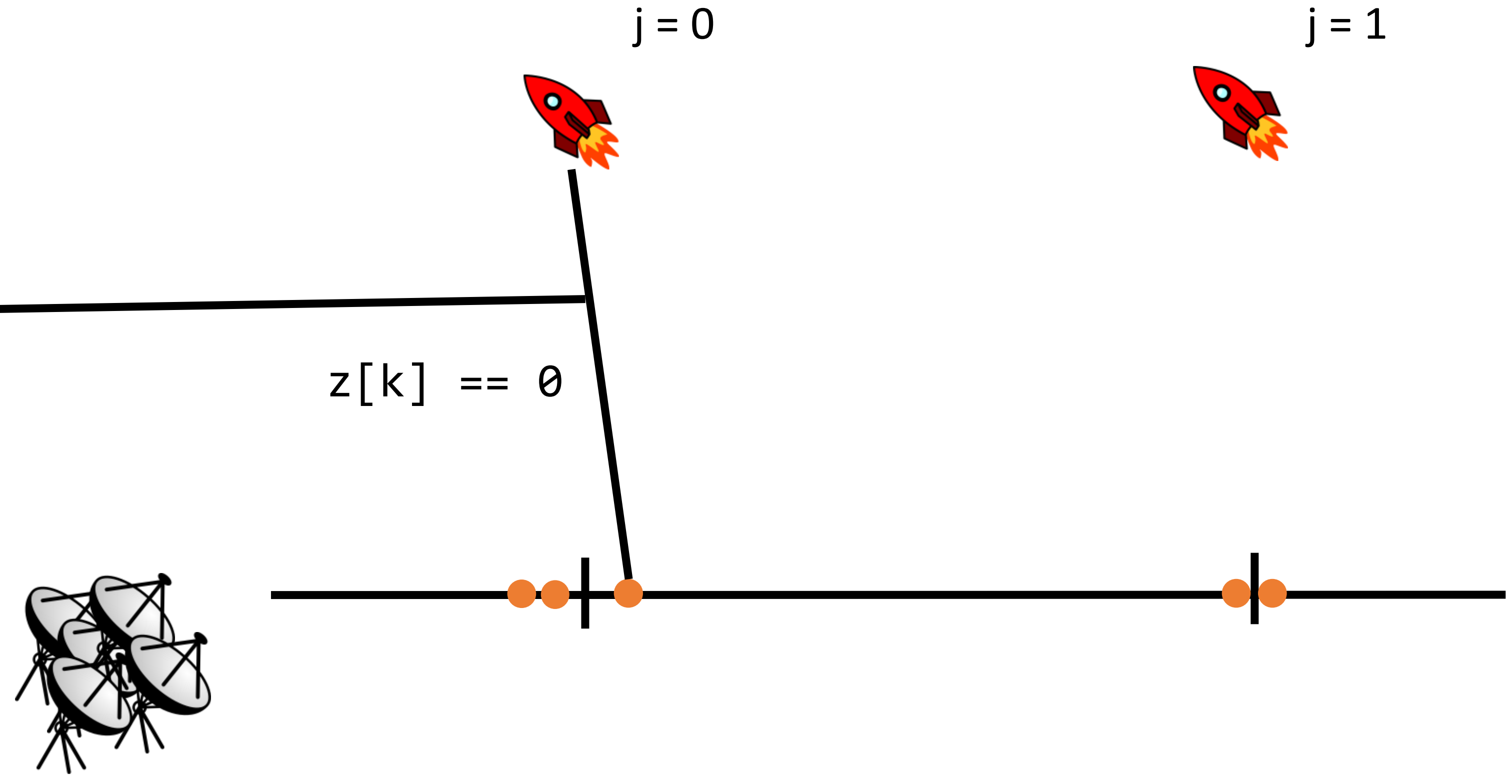

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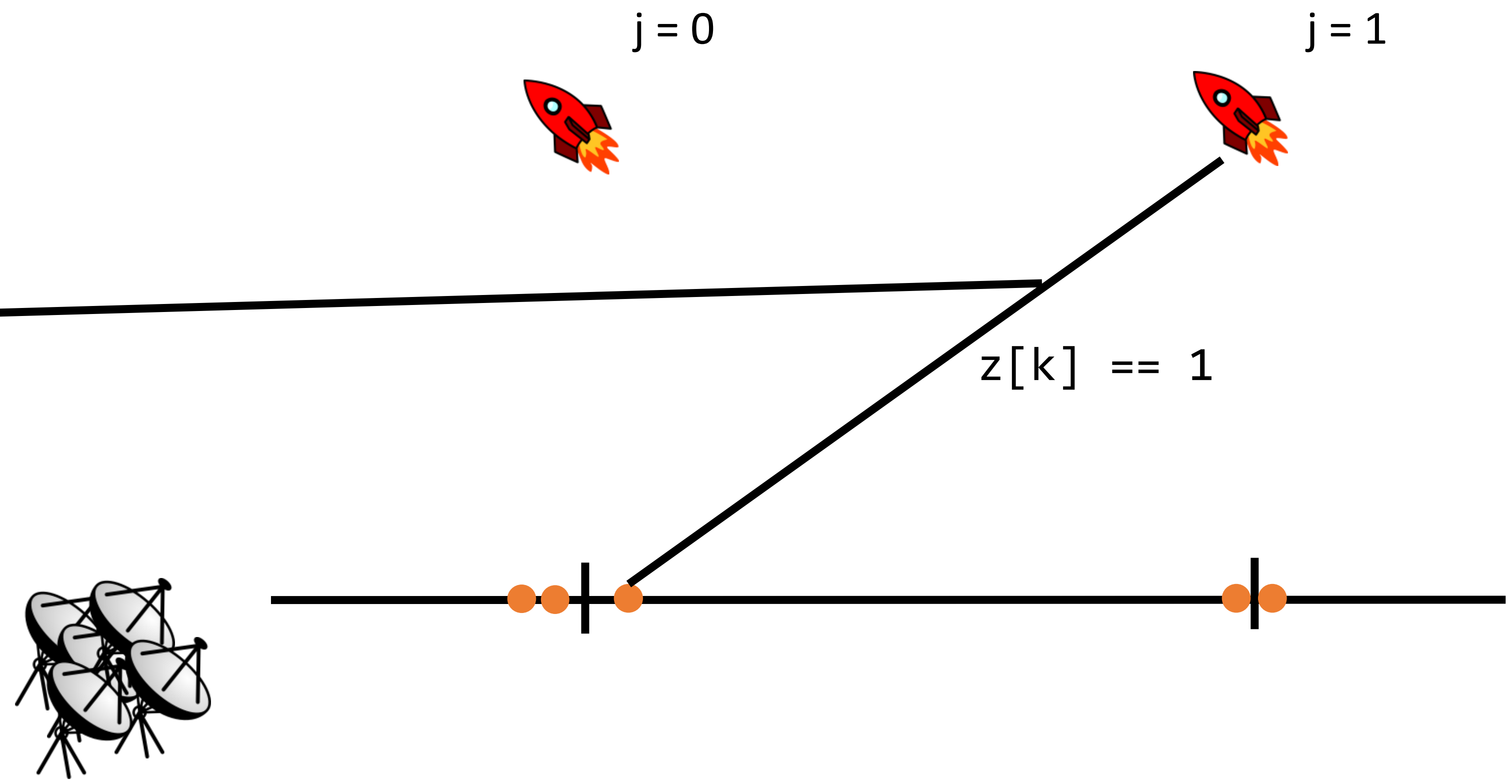

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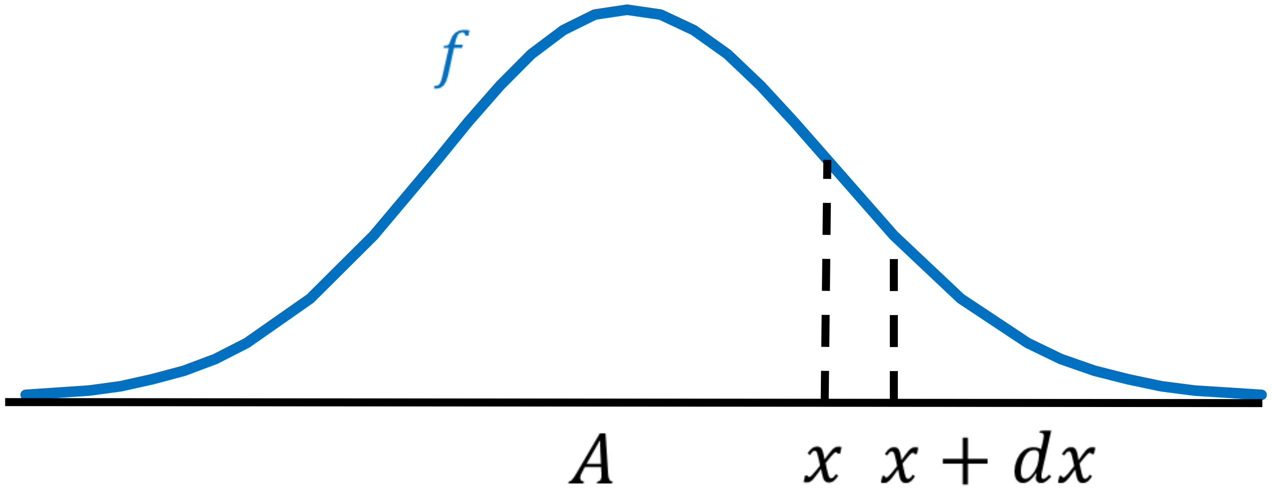
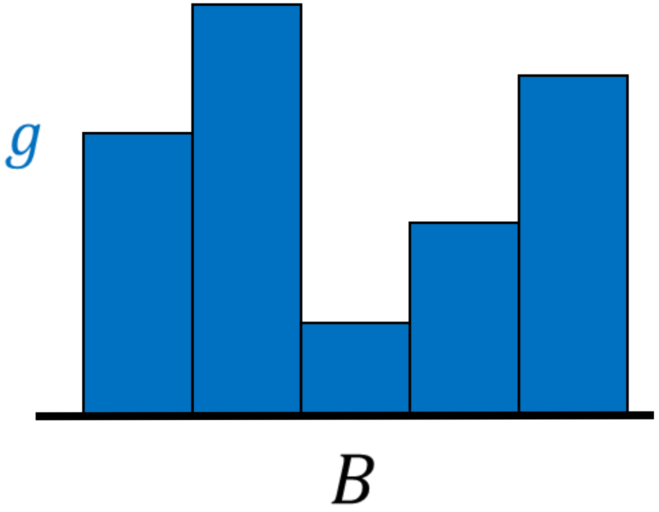
Probability Densities



$v : Rand(A)$

$f : A \to \mathbb{R}$

$f(x) = \Pr(v \in [x, (x + dx)])$



$w : Rand(B)$

$g : B \to \mathbb{R}$

$g(x) = \Pr(w = x)$

Density Semantics

```
def muiPrior(i in Mus) =
      normal(mu[i], 0, 100);
```

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Density Primitives

Density Semantics

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Density Primitives

Random Variables

Density Semantics

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Density Primitives   Random Variables   Quantified Variables

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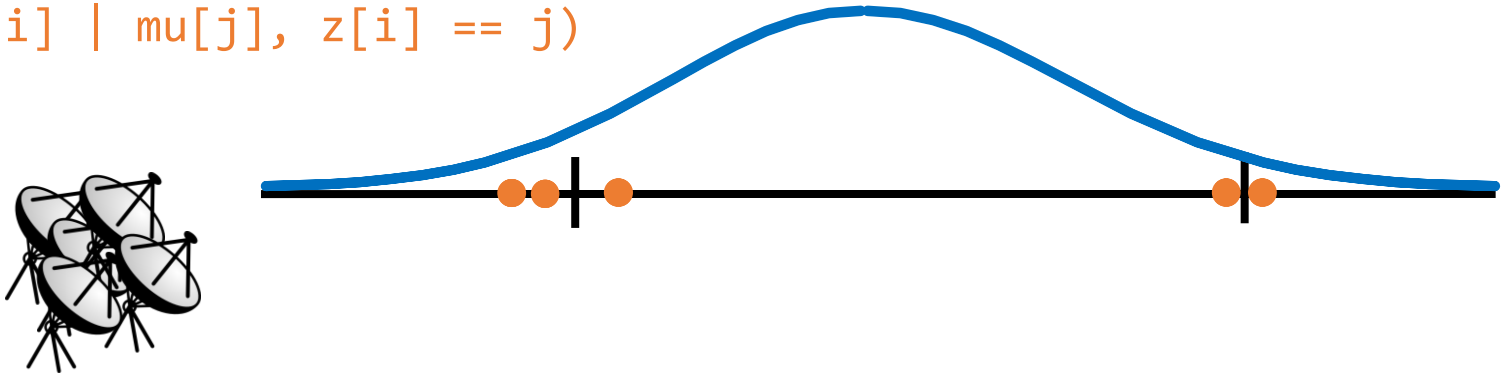
```

Types of Probability Densities

```
muiPrior : i. density(mu[i])

 ziPrior : i. density(z[i])

obsiDensity : i,j. density(obs[i] | mu[j], z[i] == j)
```



Type Structure

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density(A | B, ϕ)
```

- Compare to Pr(A | B)
- A and B are disjoint sets of random variables
- Constrained: supports dynamic affine dependencies

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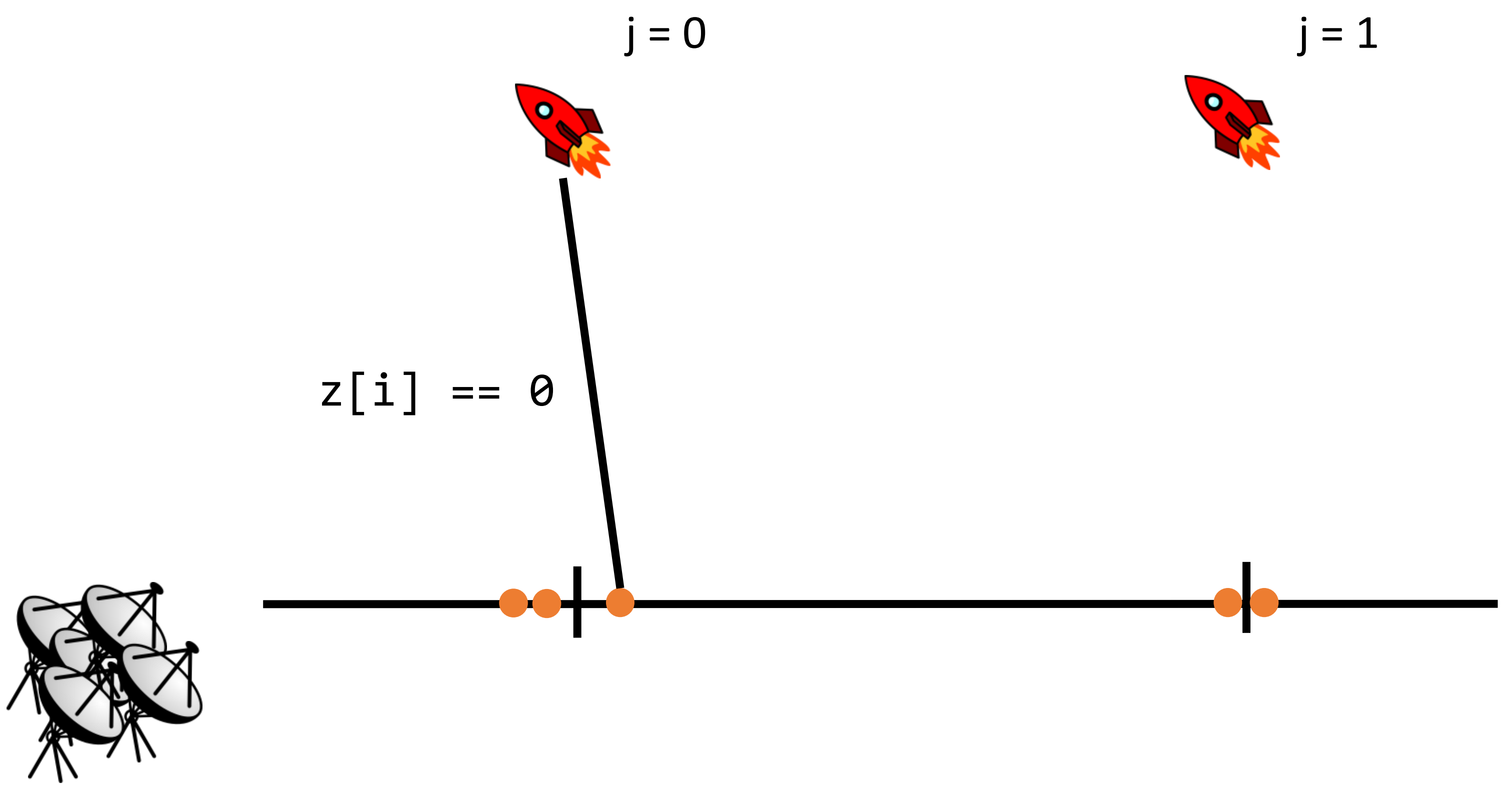

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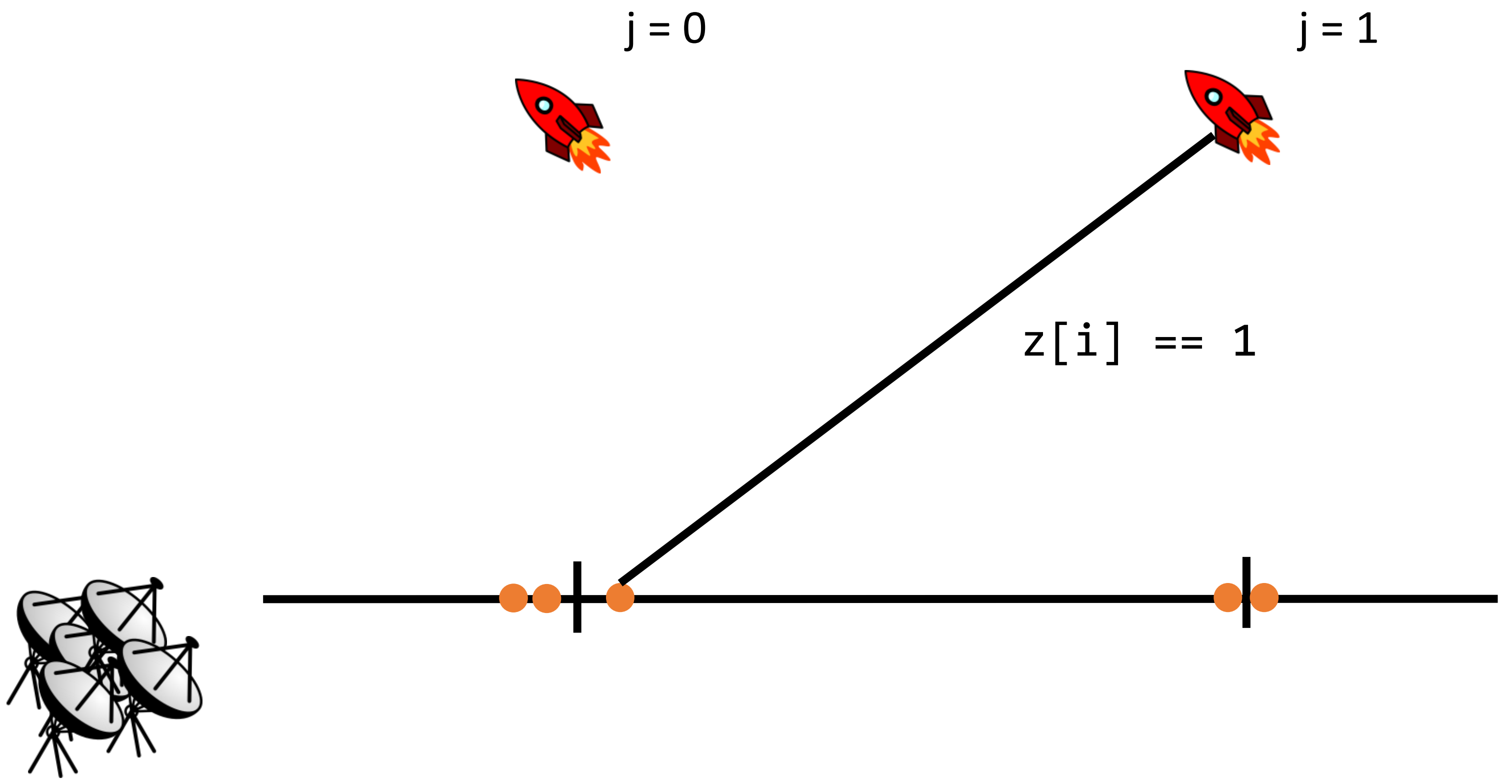

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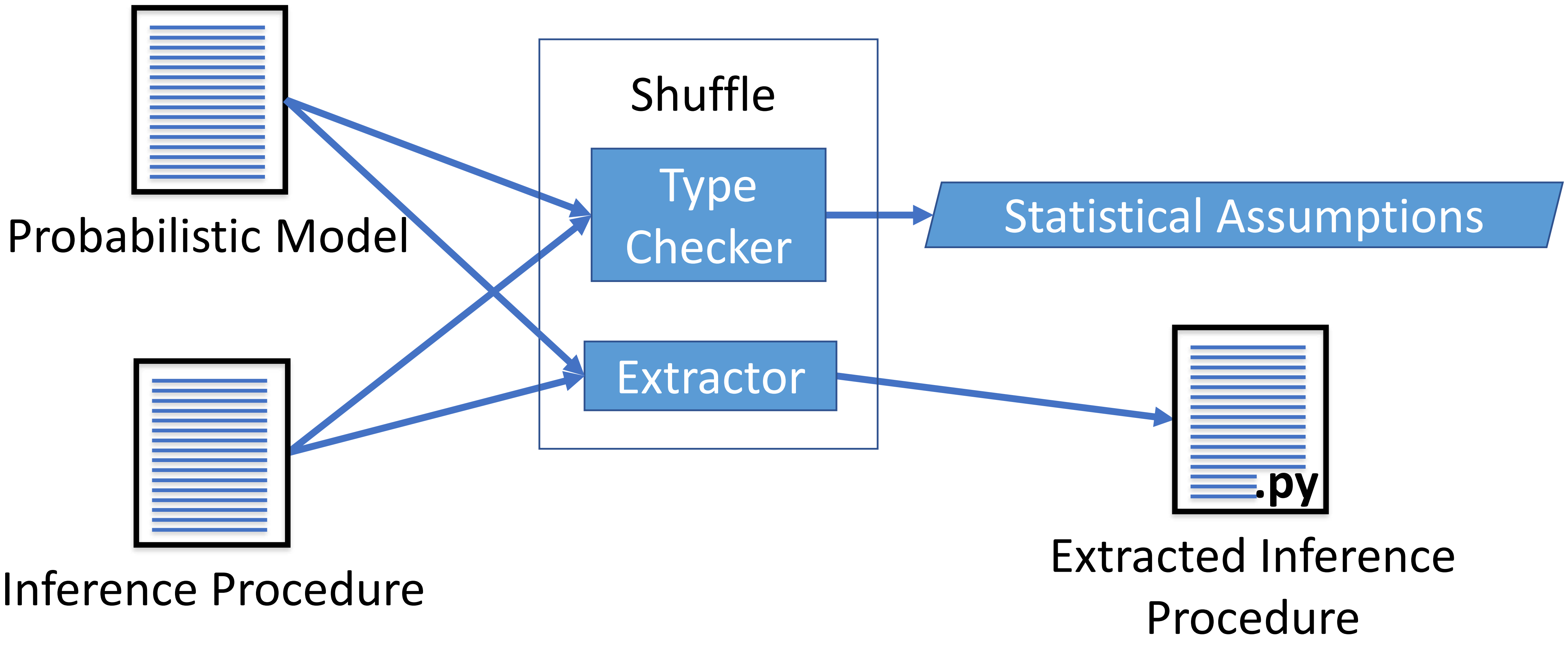
```

$$\Pr(\texttt{obs},\texttt{mu},\texttt{z}) = \prod_{j}^{\texttt{Mus}} \texttt{muiPrior}(j) \prod_{i}^{\texttt{Samples}} \texttt{ziPrior}(i) \cdot \begin{cases} \texttt{obsiDensity}(i,j), & \texttt{z[i] == j} \\ 1, & \texttt{else} \end{cases}$$

Shuffle


Probabilistic Model
Inference Procedure
Shuffle
Type
Checker
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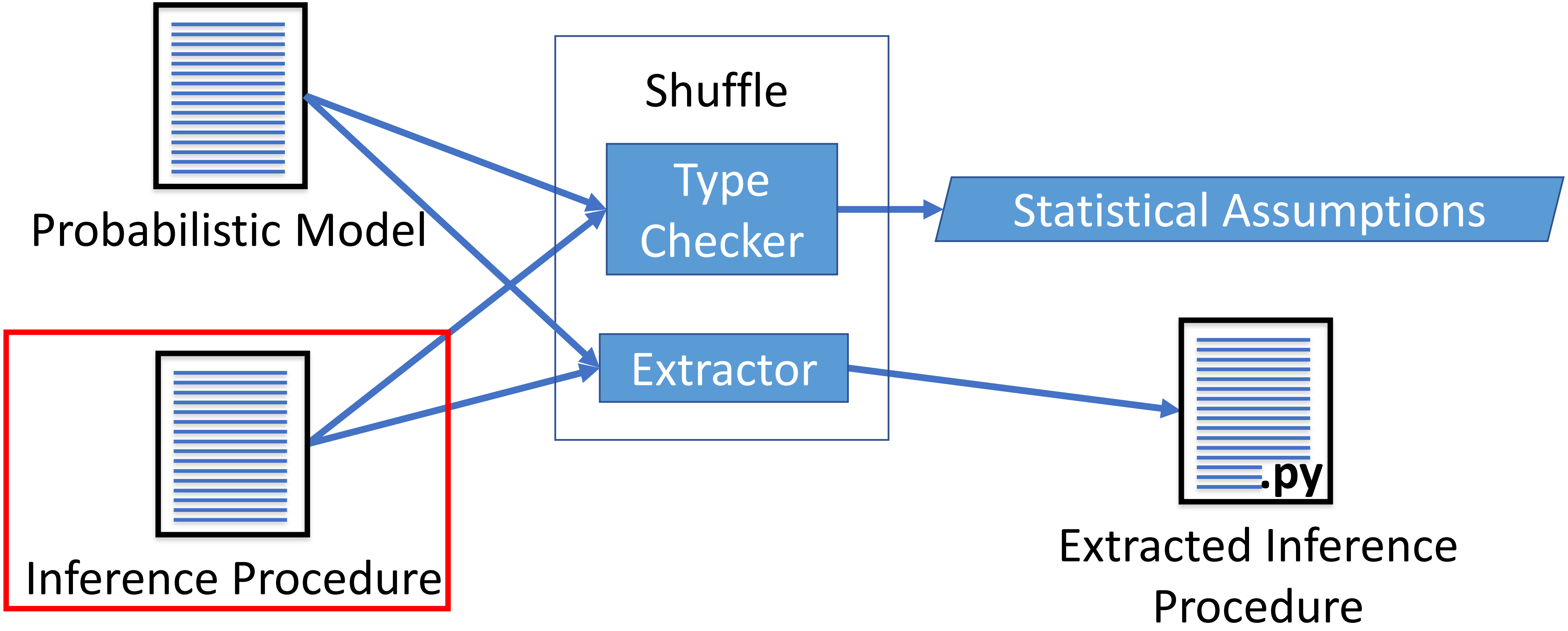
.py
Extracted Inference
Procedure

Shuffle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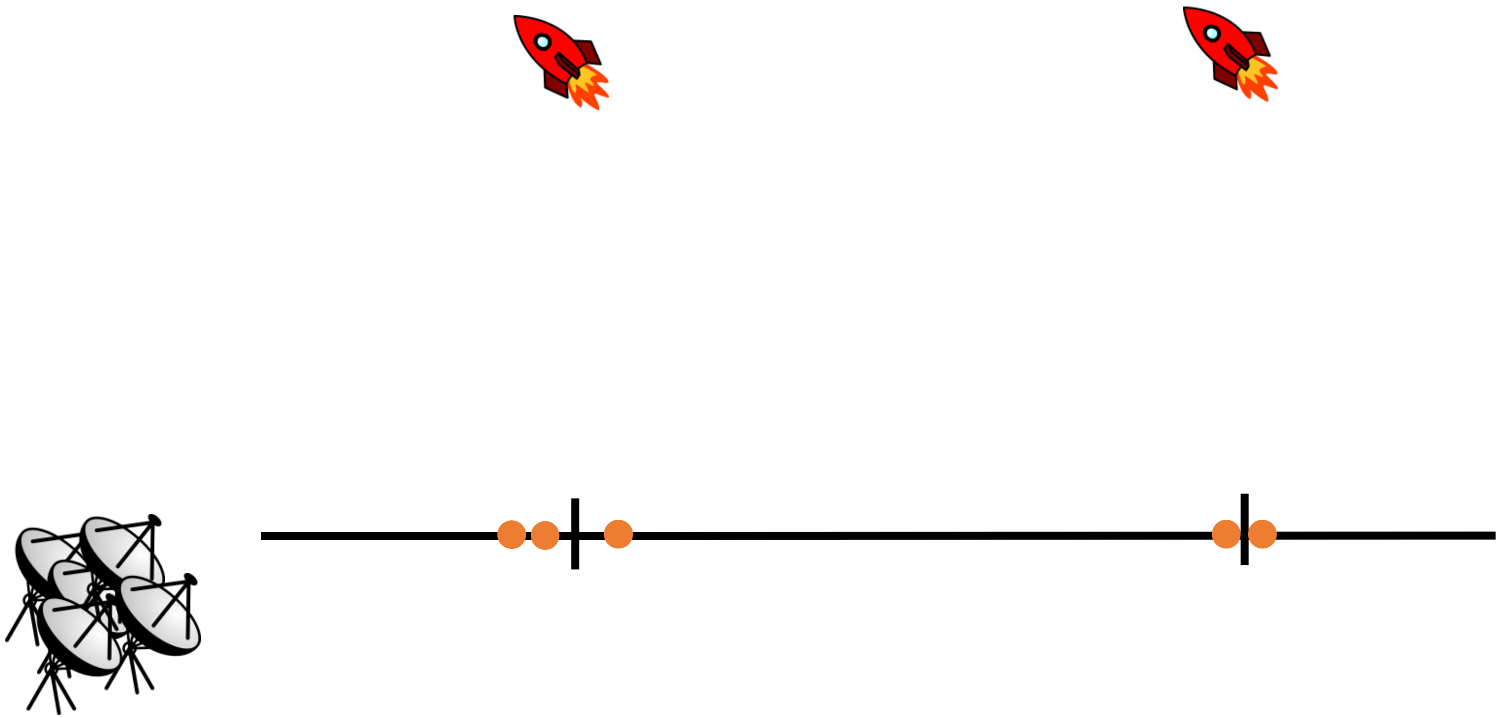
Extracted Inference Procedure

Inference Procedures

Goal Distribution:

`density(mu|obs)`

`density(z|obs)`



Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Inference Procedures

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```

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

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def zPrior : density(z) = …;

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def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Compute *prior probability* of entire z vector

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

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def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Computes likelihood of data given an assignment

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Computes joint probability of data and assignments

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Computes marginal likelihood of data

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Apply Bayes' Rule

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Computes joint probability of data and assignments

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Multiplication Operator

```
def obsZJoint : density (obs, z) = obsLikelihood * zPrior
```

Python

```
#input state assigns values to all random variables
#returns a real number
def obsZJoint(state):
   return obsLikelihood(state) * zPrior(state)
```

Typing

$$\frac{\Gamma \vdash \texttt{obsLikelihood} : \texttt{density(obs | z)} \qquad \Gamma \vdash \texttt{zPrior} : \texttt{density(z)}}{\Gamma \vdash \texttt{obsLikelihood * zPrior} : \texttt{density(obs, z)}}$$

$$\frac{\Gamma \vdash \texttt{e1} : \texttt{density(A | B)} \qquad \Gamma \vdash \texttt{e2} : \texttt{density(B)}}{\Gamma \vdash \texttt{e1 * e2} : \texttt{density(A, B)}}$$

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Computes marginal likelihood of data

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Integration (Summation)

```
def obsMarginal : density (obs) = int obsZJoint z;
```

Python

```
def obsMarginal(state):
   ret = 0
   for z in exprange(Samples,Mus):
      state' = state.clone()
      state'.z = z
      ret += obsZJoint(state')
   return ret
```

Computes the set $\mathrm{Mus}^{\mathtt{Samples}}$

Typing

$$\frac{\Gamma \vdash \mathtt{obsZJoint} : \mathtt{density\ (obs,\ z)}}{\Gamma \vdash \mathtt{int\ obsZJoint\ z} : \mathtt{density(obs)}} \qquad \frac{\Gamma \vdash \mathtt{e} : \mathtt{density(A,\ B)}}{\Gamma \vdash \mathtt{int\ e\ B} : \mathtt{density(A)}}$$

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Apply Bayes' Rule

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Division (Bayes' Rule)

```
def zPost : density (z | obs) = obsZJoint / obsMarginal;
```

Python

```
def zPost(state):
   return obsZJoint(state) / obsMarginal(state)
```

Typing

$$\frac{\Gamma \vdash \texttt{obsZJoint} : \texttt{density(obs, z)} \qquad \Gamma \vdash \texttt{obsMarginal} : \texttt{density(obs)}}{\Gamma \vdash \texttt{zJoint / obsPrior} : \texttt{density(z | obs)}}$$

$$\frac{\Gamma \vdash \texttt{e1} : \texttt{density(A, B)} \qquad \Gamma \vdash \texttt{e2} : \texttt{density(B)}}{\Gamma \vdash \texttt{e1 / e2} : \texttt{density(A | B)}}$$

Inference Pro

- Arithmetic operat
- Integrals
- Definition
- Invocation
- Independence dent
- Primitive recursion
- Conditionals } else { d2 }

```
def independent obsDens1 (i in dataPoints, j in Mus) :
    density(obs[i] | mu[j], z; z[i] == j) =
    obsDens(i,j) in

def obsDens2 (i in dataPoints, j in Mus) :
    density(obs[i] | mu[j] , z ; z[i] == j) =
    obsDens1(i,j) in

def independent obsDens3 (i in dataPoints, j in Mus) :
    density(
        obs[i] |
        obs{i0 in dataPoints: i0 < i && z[i0] == j}, mu[j], z ;
        z[i] ==j
    ) =
    obsDens2(i,j) in

def independent muDens1(j in Mus) : density(mu[j] | z, none; true)
    = muPrior(j) in

def muDens2(j in Mus) :
    density(mu[j] | z; true) = muDens1(j) in

def rec obsProdHelper(i in dataPoints, j in Mus) :
    density(obs{i0 in dataPoints: i0 <= i && z[i0] == j}
        | mu[j], z; true) =
    if (z[i] == j) {
        obsDens3(i,j) * obsProdHelper(i-1,j)
    } else {
        obsProdHelper(i-1,j)
    } in

def obsProd(j in Mus) :
    density(mu[j], obs{i0 in dataPoints: z[i0] == j} | z; true) =
    obsProdHelper(max(dataPoints),j) * muDens2(j) in

def muPost (j in Mus) :
    density(mu[j] | obs{i0 in dataPoints: z[i0] == j}, z; true)
    = obsProd(j) / int obsProd(j) by mu[j] in

def independent obsMarg(j in Mus) :
    density(obs{i0 in dataPoints: z[i0] == j} | obs{i0 in dataPoints: z[i0] < j},
        z; true) =
    obsProd(j) / muPost(j) in

def rec obsMargAllHelper(j in Mus) :
    density(obs{i0 in dataPoints: z[i0] <= j} | z; true) =
    obsMarg(j) * obsMargAllHelper(j - 1) in

def obsLikelihood() : density(obs | z; true) =
    obsMargAllHelper(max(Mus)) in

def independent ziPriorI(i in dataPoints) :
    density(z[i] | z{i0 in dataPoints: i0 < i}; true) =
    zPrior(i) in

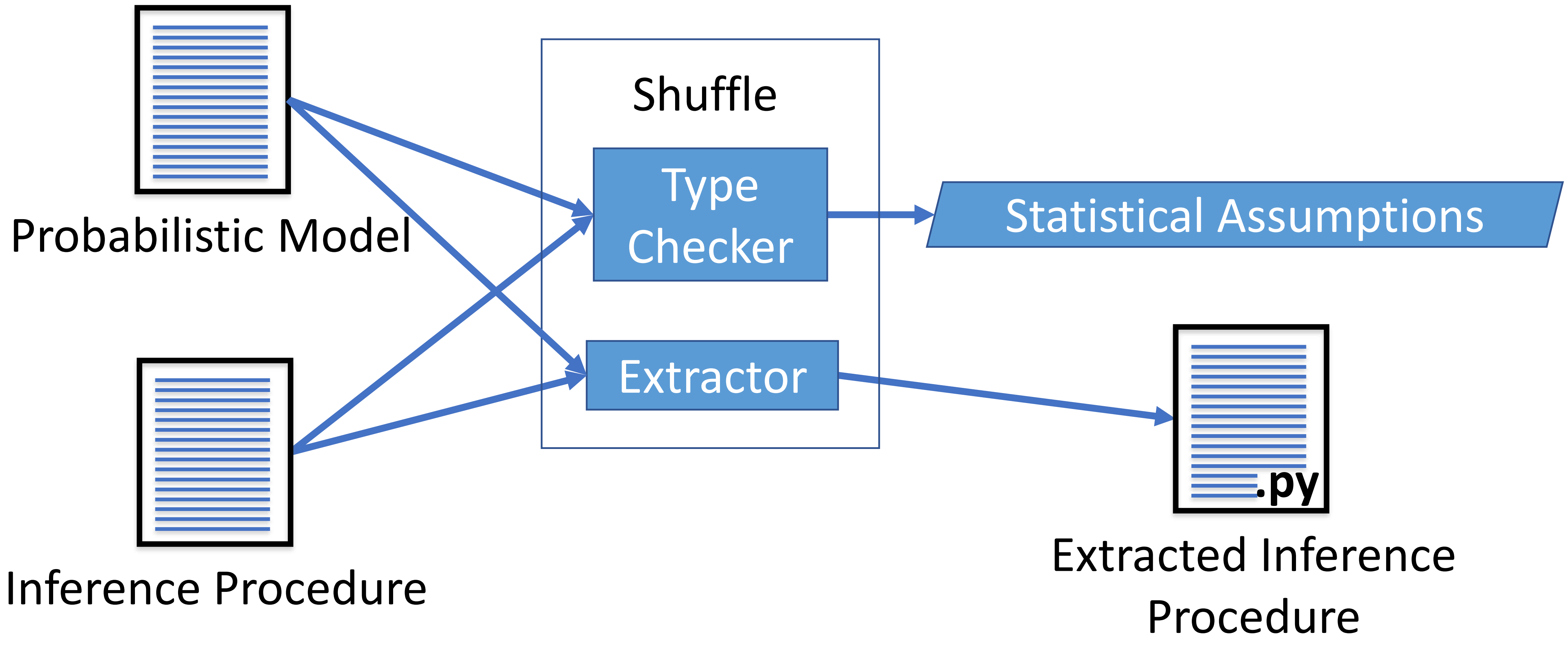
def rec zPriorHelper(i in dataPoints) :
    density(z{i0 in datAPoints: i0 <= i} | none; true) =
    ziPriorI(i) * zPriorHelper(i - 1) in

def zPriorAll() : density(z | none; true) =
    zPriorHelper(max(dataPoints)) in

def zPost() : density(z | obs; true) =
    obsLikelihood() * zPriorAll() / int (obsLikelihood() * zPriorAll()) by z in

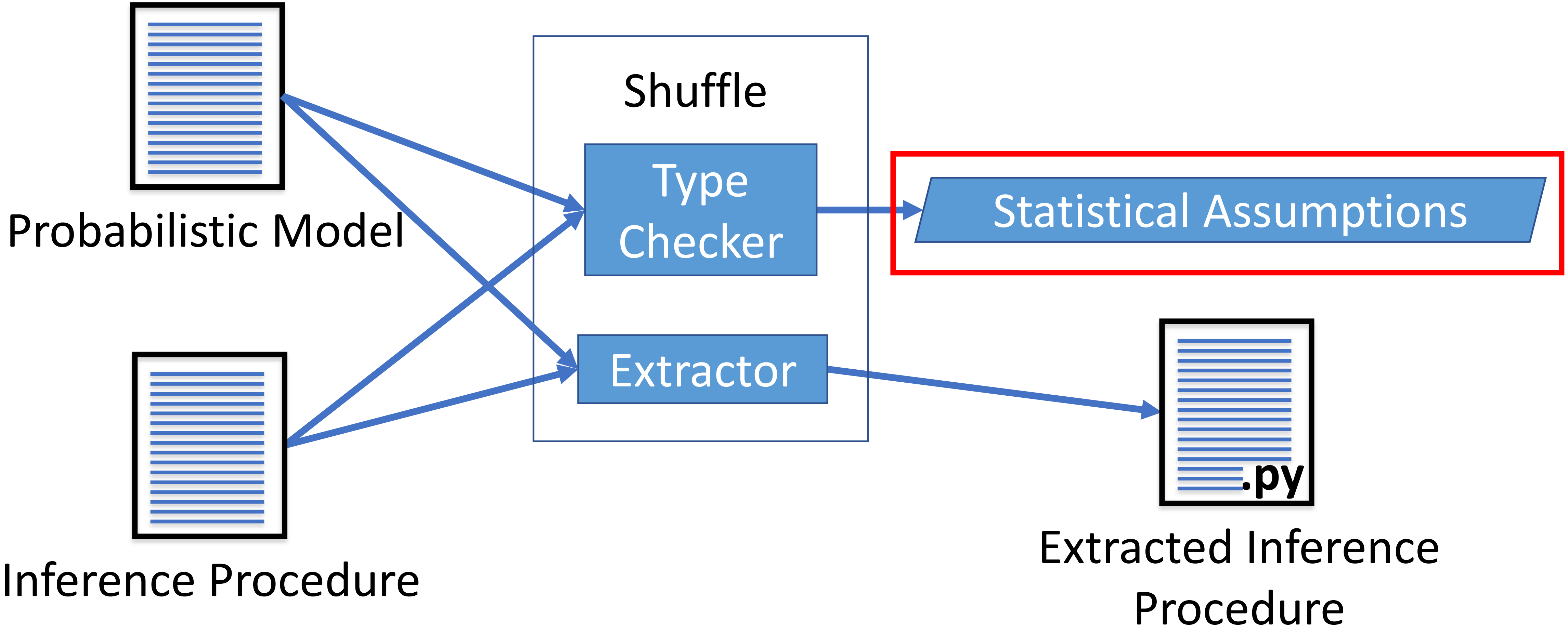
def export zPost1() : density(z | obs; true) =
    zPost()
```

Shuffle



Probabilistic Model
Inference Procedure
Shuffle
Type Checker
Extractor
Statistical Assumptions
.py
Extracted Inference Procedure

Shuffle



Probabilistic Model
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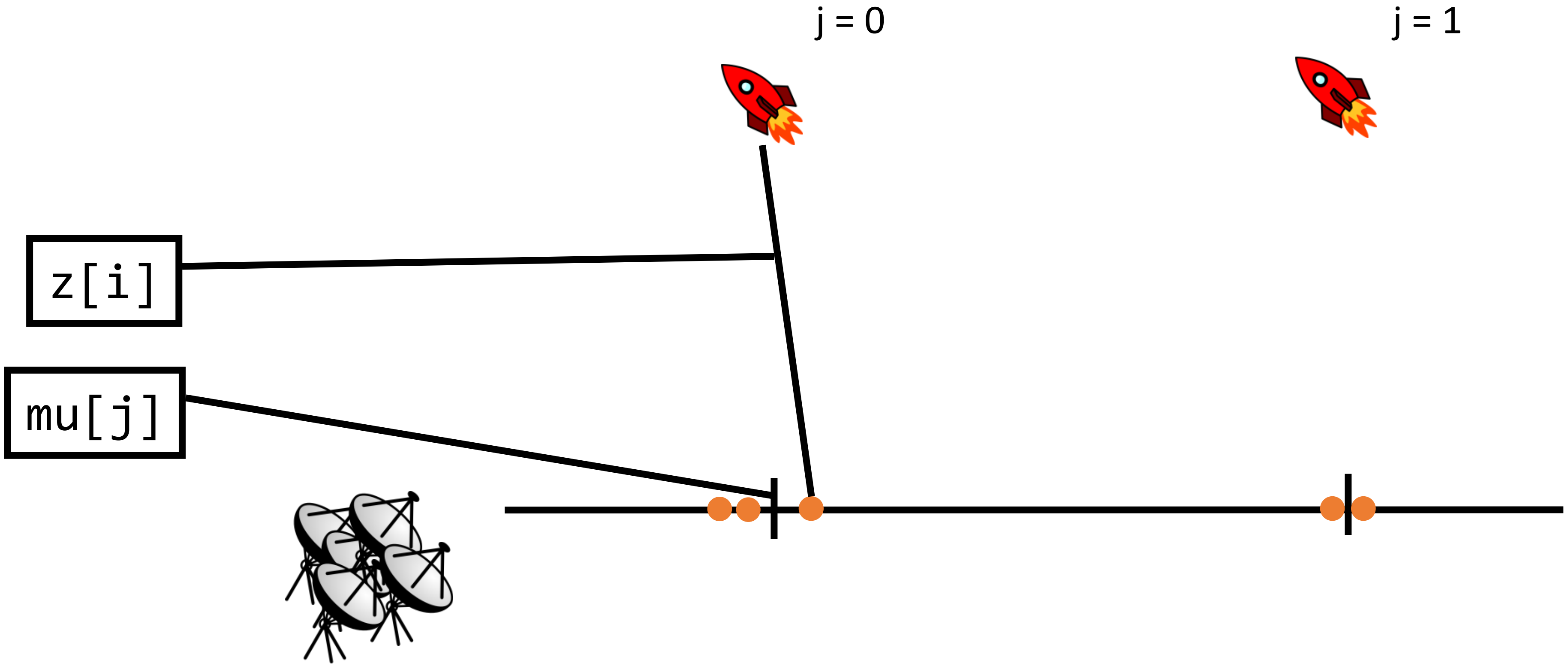
Extracted Inference Procedure

Independence



z[i] _||_ mu[j]

Assumptions

- Independence

```
// Assuming ziPrior : i. density(z[i])

def independent ziPriorI(i in dataPoints):
   density(z[i] | mu[j]) =
   ziPrior(i);
```

Assumptions

- Independence

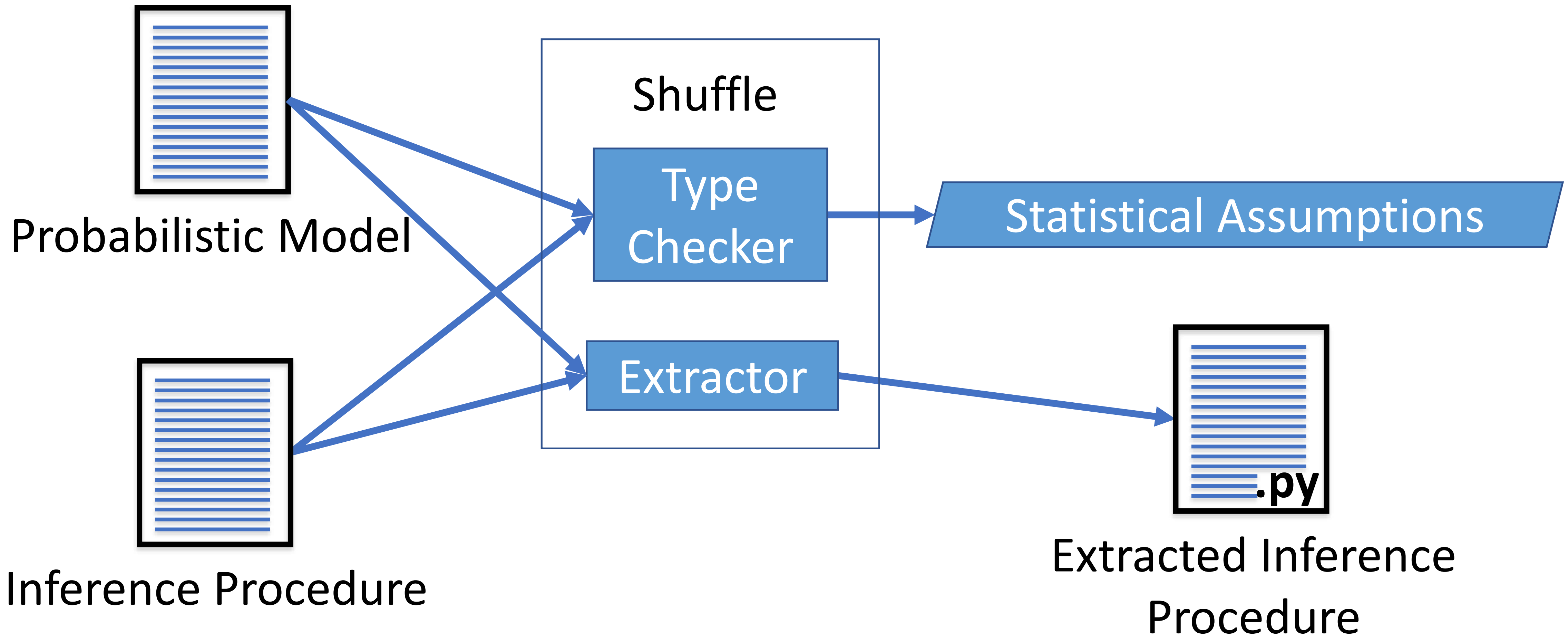
```
// Assuming ziPrior : i. density(z[i])

def independent ziPriorI(i in dataPoints):
   density(z[i] | mu[j]) =
   ziPrior(i);
```

- Saturation

- Annotated by developer, but recorded and reported in a log by Shuffle.

Shuffle


Probabilistic Model
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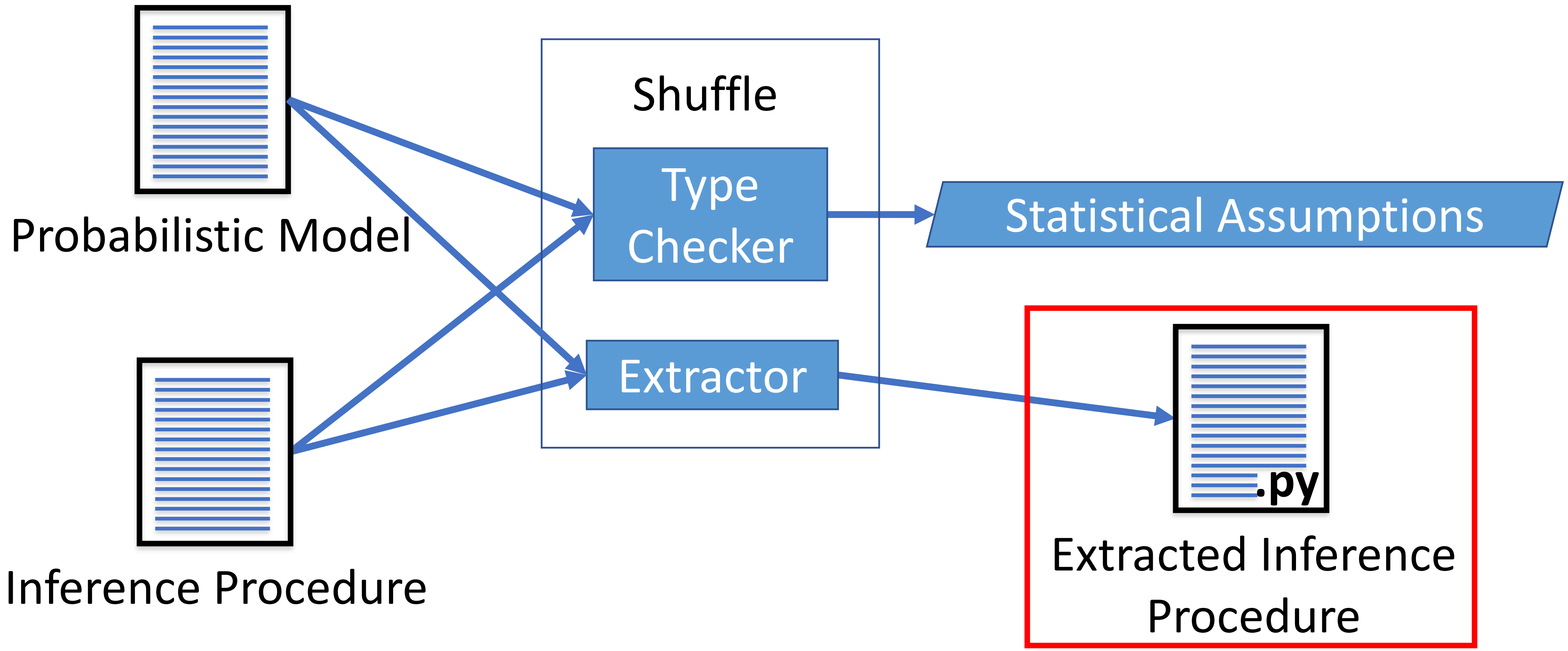
Shuffle



Probabilistic Model
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.py
Extracted Inference Procedure

Extraction

```
def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

```
def zPosterior(state) :
   return obsZJoint(state) / obsMarginal(state);
```

Integrals

- Simplify known opportunities for closed forms: For example, the conjugate prior in the posterior distribution:

Prior

$$\mathrm{P}(mu[j] \mid obs[i]) = \frac{P(obs\ [i] \mid mu[j]) \cdot P(mu[j])}{\int P(obs[i] \mid mu_1) \cdot P(mu_1) dmu_1}$$

Likelihood

- If normal, $\Pr(\mathrm{mu}[j] \mid obs[i]) = \dfrac{\frac{\mu_0}{\sigma_0^2} + \frac{obs[i]}{\sigma^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}}$

Pattern Matching

```
(normal(obs[i],mu[j],1) * normal(mu[j],0,10)) /
int normal(obs[i],mu[j],1) * normal(mu[j],0,10) by mu[j]
```

Likelihood

Prior

```
normal(mu[j],(obs[i]/1)/(1/10 + 1/1), 1/(1/10 + 1/1))
```

Automatic Incrementalization

```
accs = []
for i in range(N) :
    acc = 0

    for j in in range(i)
        acc += obs[j]

    accs += [acc]
```

```
accs = []

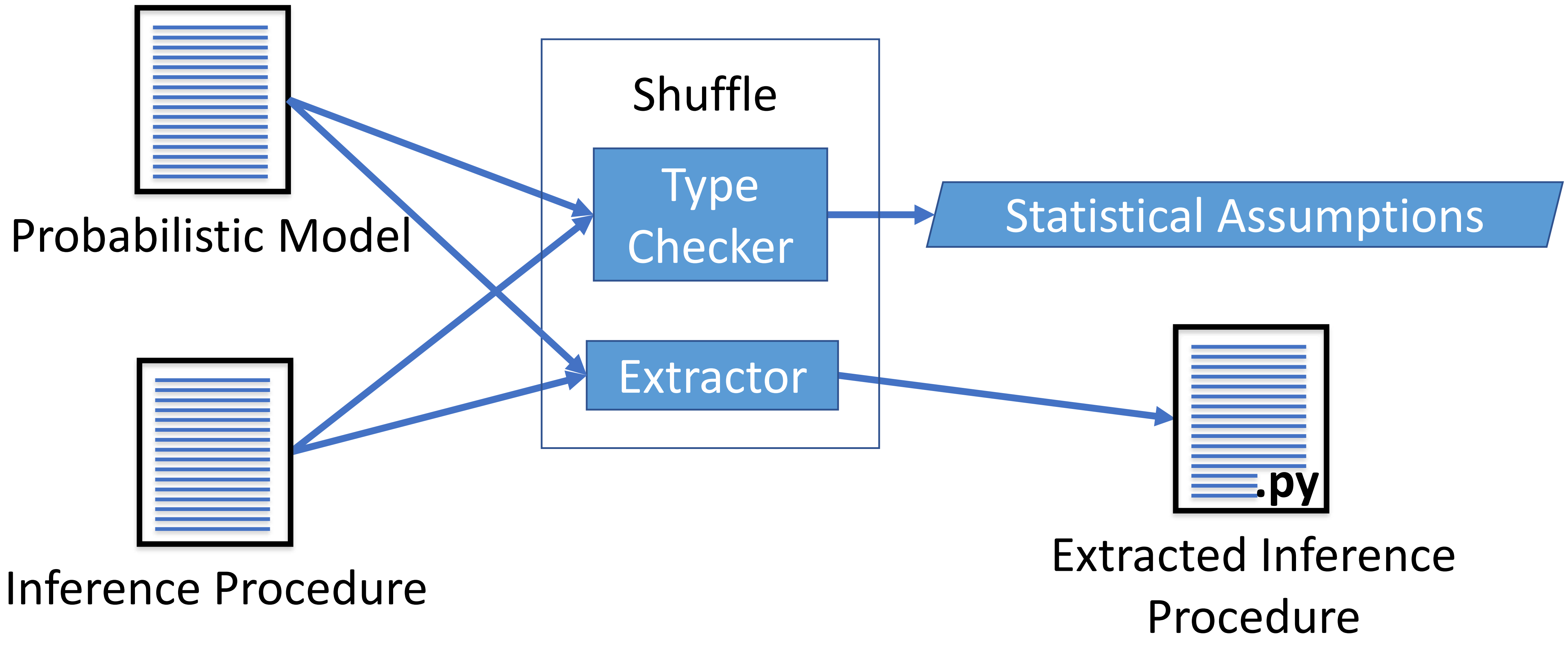
acc = 0

for i in range(N) :
    acc += obs[i]

    accs += [acc]
```

- Commutative and associative reductions with overlapping ranges
- Shuffle’s language is such that determining if two iteration ranges overlap is computable

Shuffle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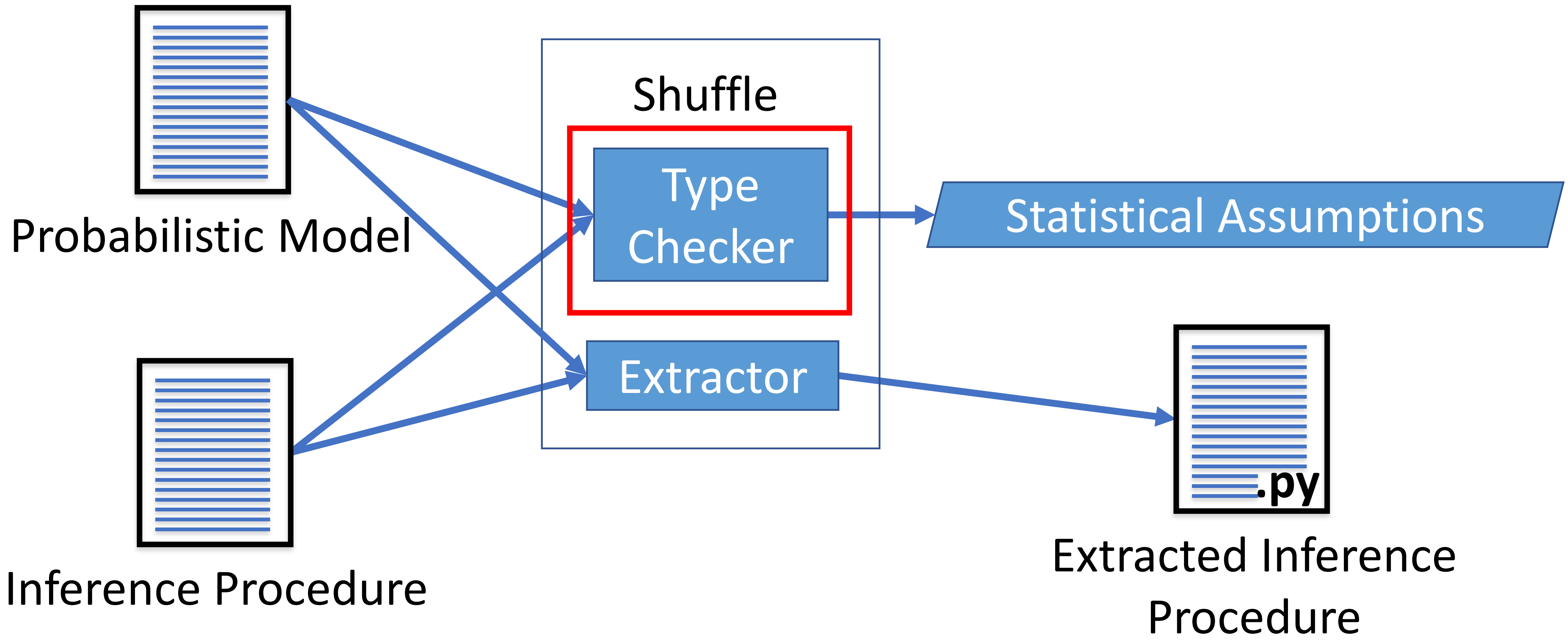
Shuffle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Extracted Inference Procedure

Type Checking

- Preservation:

$$\Gamma \vdash d\colon \texttt{density}(A \mid B) \Rightarrow \forall \sigma.\, d(\sigma) = \Pr(A \mid B)$$

- Progress:
  - Well-typed terms evaluate successfully

Type Checking

`density(A | B, ϕ)`

- Must verify that $A \cap B = \emptyset$ and `FRV(ϕ)` $\subseteq \mathrm{B}$

`density(A | B, ϕ)` → `density(A' | B', ϕ')`

- Must verify $A \equiv A'$, $B \equiv B'$ and $\phi' \Rightarrow \phi$

- Formalize with QF theory of arrays and BitVectors (Solve with Z3)

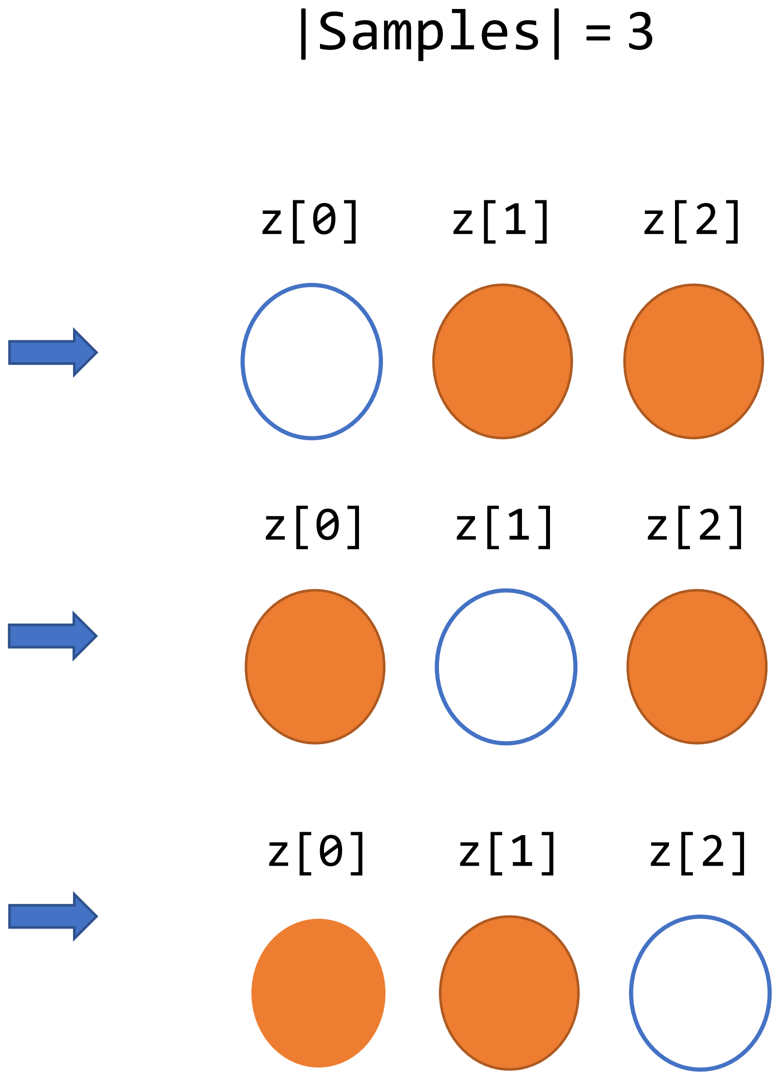
Variable Sets

```
density(z[i] | z{k : k != i}, obs)

density(z[0] | z{k : k != 0}, obs)

density(z[1] | z{k : k != 1}, obs)

density(z[2] | z{k : k != 2}, obs)
```



Benchmarks

Benchmark Name	Shuffle Code (Model/Inference LoC)	Generated Code (LoC)
Burglary	28/30	36
Context-Specific Inference	26/30	24
Dirichlet-Categorical	11/20	20
Healthiness	76/101	118
Hurricane	29/29	12
Naïve Bayes	29/229	318
Normal-Normal	11/20	27
Weather	12/20	24

Benchmarks

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Healthiness	76/101	118
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Naïve Bayes	29/229	318
Normal-Normal	11/20	27
Weather	12/20	24
Gaussian Mixture Model	19/93	55
Latent Dirichlet Allocation	21/160	293
Simultaneous Localization and Mapping	40/68	38

Inference Procedures

```
def zPrior : density(z) = …;

def obsLikelihood : density(obs | z) = … ;

def obsZJoint : density(obs, z) = obsLikelihood * zPrior;

def obsMarginal : density(obs) = int obsZJoint z;

def zPosterior : density(z | obs) = obsZJoint / obsMarginal;
```

Computes marginal likelihood of data

$$\mathrm{P}(\mathrm{z} \mid \mathrm{obs}) = \frac{P(\mathrm{obs} \mid \mathrm{z}) * P(\mathrm{z})}{\int P(\mathrm{obs} \mid z_1) * P(z_1) dz_1} = \frac{P(\mathrm{obs},\ \mathrm{z})}{\int P(\mathrm{obs},\ z_1) dz_1}$$

Integration (Summation)

```
def obsMarginal : density (obs) = int obsZJoint z;
```

Python

```
def obsMarginal(state):
    ret = 0
    for z in exprange(Samples,Mus):
        state' = state.clone()
        state'.z = z
        ret += obsZJoint(state')
    return ret
```

Computes the set $\text{Mus}^{\texttt{Samples}}$

Approximate Inference

- Alternative: build a *sampler* for P(z | obs)

- For example, given a boolean predicate pred : Mus[Samples] -> bool

```
def muApprox (state, count, pred) :
    sum = 0
    total = 0

    for state in [zSample(state) for x in range(count)] :
       sum = sum + (1 if pred(state.z) else 0)
       total = total + weight

    return sum / total
```

```
Consider pred =
   lambda x : (z[0] != z[1])
```

Approximate Inference

$$\Gamma \vdash d : \texttt{density}(A \mid B) \Rightarrow \forall \sigma.\, d(\sigma) = \Pr(A \mid B)$$

$$\Gamma \vdash s : \texttt{sampler}(A \mid B) \Rightarrow \forall \sigma. \int_{sr} f\big(s(\sigma, sr)\big) = \int_{A} f(x) * P(x|B)$$

$$\Gamma \vdash k : \texttt{kernel}(A \mid B) \Rightarrow \forall \sigma. \int_{sr_1} \int_{sr_2} f(k(s(\sigma, sr_1), sr_2)) = \int_{A} f(x) * P(x|B)$$

MCMC

$$\Gamma \vdash k : \texttt{estimator}(A \mid B) \Rightarrow \forall \sigma. \int_{sr} \frac{fst\big(e(\sigma, sr)\big) * f\big(snd\big(e(\sigma, sr)\big)\big)}{\int_{sr} fst(e(\sigma, sr))} \int_{A} f(x) * P(x|B)$$

Importance Sampling

Approximate Infe

- Sampling
- Composition
- Lifting
- Factor
- Definition
- Invocation
- Independence
- Primitive recursion
- Conditionals

```
def independent obsDens1 (i in dataPoints, j in Mus) :
    density(obs[i] | mu[j], z; z[i] == j) =
    obsDens(i,j) in

def obsDens2 (i in dataPoints, j in Mus, k in dataPoints) :
    density(obs[i] | mu[j] , z ; z[i] == j && i != k) =
    obsDens1(i,j) in

def independent obsDens3 (i in dataPoints, j in Mus, k in dataPoints) :
    density(
        obs[i] |
        obs{i0 in dataPoints: i0 < i && (z[i0] == j && i0 != k)}, mu[j], z ;
        z[i] ==j && i != k
    ) =
    obsDens2(i,j,k) in

def independent muDens1(j in Mus) : density(mu[j] | z, none; true)
    = muPrior(j) in

def muDens2(j in Mus, k in dataPoints) :
    density(mu[j] | z; true) = muDens1(j) in

def rec obsProdHelper(i in dataPoints, j in Mus, k in dataPoints) :
    density(obs{i0 in dataPoints: i0 <= i && z[i0] == j && i0 != k}
        | mu[j], z; true) =
    if (z[i] == j && i != k) {
        obsDens3(i,j,k) * obsProdHelper(i-1,j,k)
    } else {
        obsProdHelper(i-1,j,k)
    } in

def obsProd(j in Mus, k in dataPoints) :
    density(mu[j], obs{i0 in dataPoints: z[i0] == j && i0 != k} | z; true) =
    obsProdHelper(max(dataPoints),j,k) * muDens2(j,k) in

def muPost (j in Mus, k in dataPoints) :
    density(mu[j] | obs{i0 in dataPoints: z[i0] == j && i0 != k}, z; z[k] == j)
    = obsProd(j,k) / int obsProd(j,k) by mu[j] in

def independent obsDensNew (j in Mus, k in dataPoints) :
    density(obs[k] | mu[j], z; z[k] == j) =
    obsDens(k,j) in

def independent obsDensNew2 (j in Mus, k in dataPoints) :
    density(
        obs[k] |
        obs{i0 in dataPoints: z[i0] == j && i0 != k}, mu[j] , z ; z[k] == j
    ) = obsDensNew(j,k) in

def muJoint (k in dataPoints, j in Mus) :
    density(
        mu[j], obs[k] |
        obs{i0 in dataPoints: z[i0] == j && i0 != k}, z; z[k] == j
    ) = obsDensNew2(j,k) * muPost(j,k) in

def independent obsPred(k in dataPoints, j in Mus) :
    density(
        obs[k] | obs{i0 in dataPoints: i0 != k}, z; z[k] == j) =
    int muJoint(k,j) by mu[j] in

def obsPred1(k in dataPoints) :
    density(
        obs[k] | z[k],
        z{i0 in dataPoints : i0 != k},
        obs{i0 in dataPoints: i0 != k}; true
    ) =
    obsPred(k, z[k]) in

def independent zDensNew(i in dataPoints) :
    density(z[i] |
        z{i0 in dataPoints: i0 != i}, obs{i0 in dataPoints: i0 != i}; true) =
    zPrior(i) in

def zJoint(k in dataPoints) :
    density(
        z[k], obs[k] |
        z{i0 in dataPoints: i0 != k}, obs{i0 in dataPoints: i0 != k};
        true
    ) = obsPred1(k) * zDensNew(k) in

def zPost(k in dataPoints) :
    density(z[k] | z{i0 in dataPoints: i0 != k}, obs; true) =
    zJoint(k) / int zJoint(k) by z[k] in

def rec zPostHelper(k in dataPoints) :
    kernel(z{i0 in dataPoints: i0 <= k} |
        z{i0 in dataPoints: k < i0}, obs; true) =
    zPostHelper(k-1); lift z[k] := sample zPost(k) in

def export zPostFinal() : kernel(z | obs; true) =
    zPostHelper(max(dataPoints))
```

sample d

ifte

e by d

ependent

{ d1 } else { d2 }

Small Scale Performance

Benchmark	Algorithm	Assumptions		Speedup vs Venture
		Ind.	Reach.	
Gaussian Mixture Model	Collapsed Gibbs Sampler	7	1	6x
Latent Dirichlet Allocation (LDA)	Collapsed Gibbs Sampler	5	1	.01x
Simultaneous Mapping and Location (SLAM)	Rao-Blackwellized Particle Filter	13	0	20x

Automatic Incrementalization

```
accs = []
for i in range(N) :
    acc = 0

    for j in in range(i)
        acc += obs[j]

    accs += [acc]
```

```
accs = []

acc = 0

for i in range(N) :
    acc += obs[i]

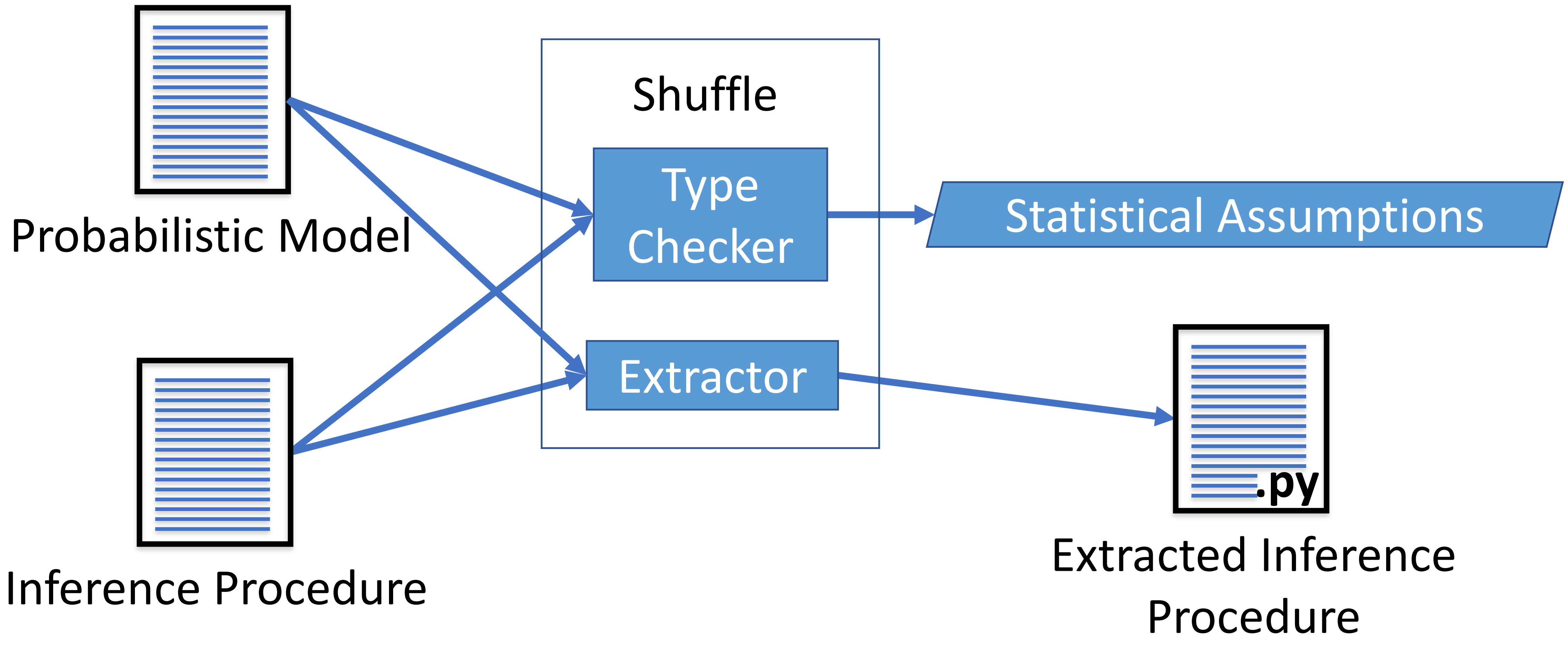
    accs += [acc]
```

- Commutative and associative reductions with overlapping ranges
- Shuffle’s language is such that determining if two iteration ranges overlap is computable

Performance at Scale

Benchmark	Algorithm	Assumptions		Speedup vs Venture
		Ind.	Reach.	
Gaussian Mixture Model	Collapsed Gibbs Sampler	7	1	34x
Latent Dirichlet Allocation (LDA)	Collapsed Gibbs Sampler	5	1	50x
Simultaneous Mapping and Location (SLAM)	Rao-Blackwellized Particle Filter	13	0	1.3x

Shuffle


Probabilistic Model
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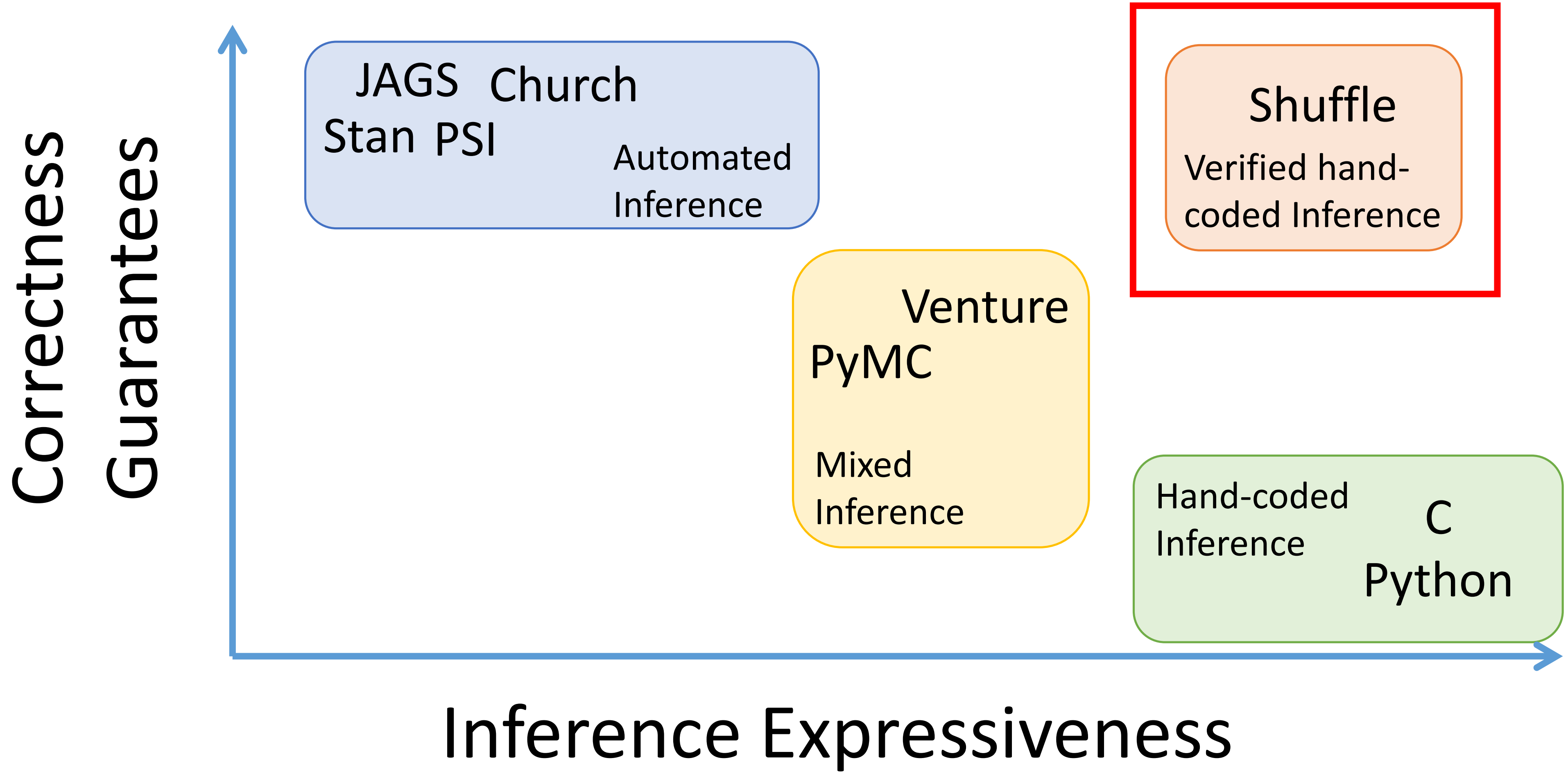
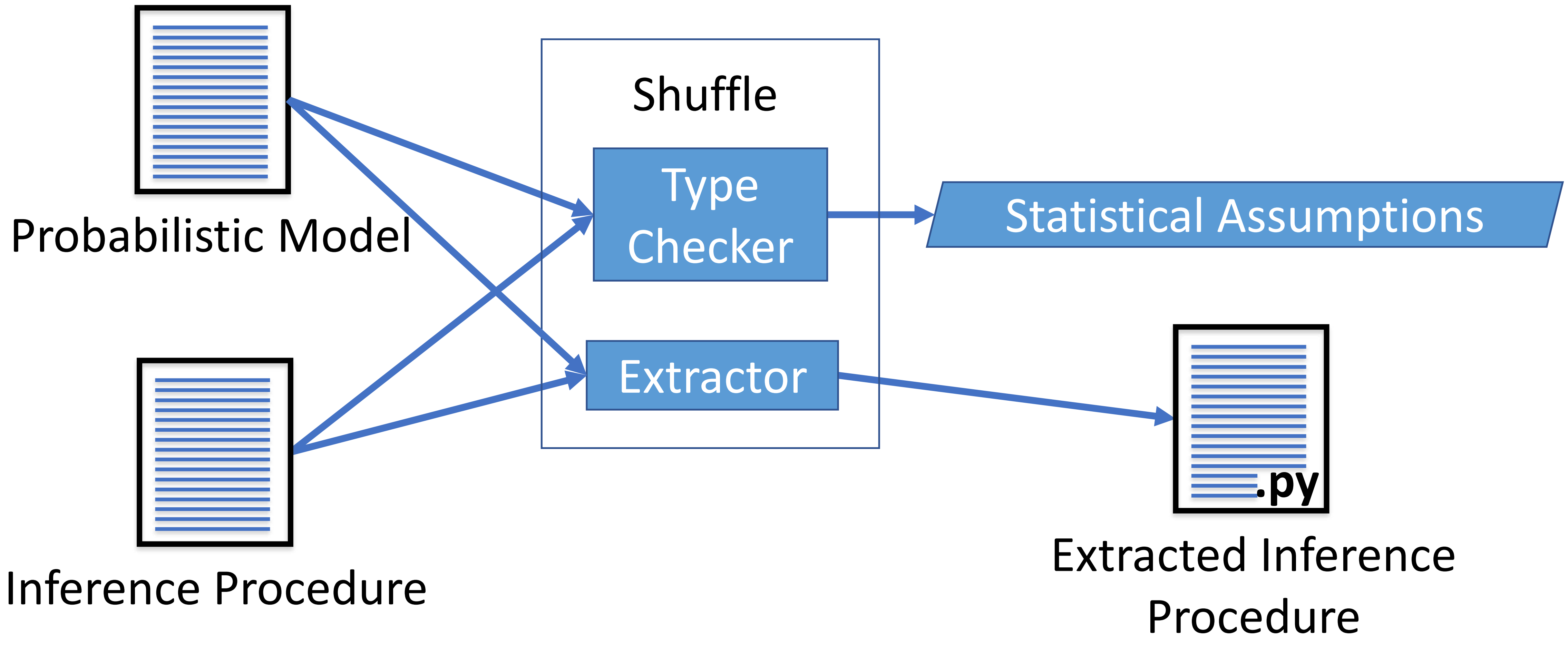
Existing Approaches



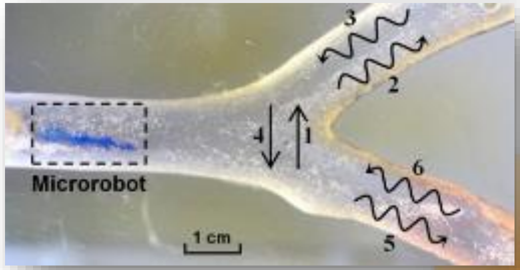
Shuffle

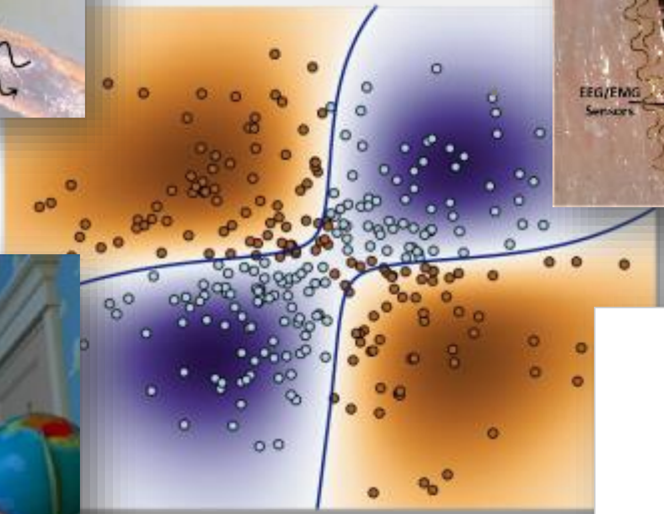

Probabilistic Model
Inference Procedure
Shuffle
Type Checker
Extractor
Statistical Assumptions
.py
Extracted Inference Procedure

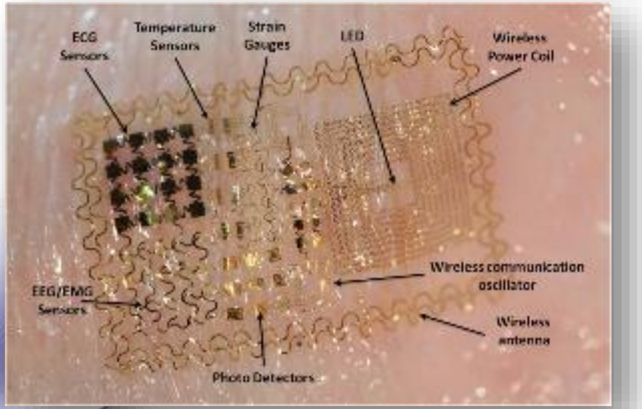
Conclusion

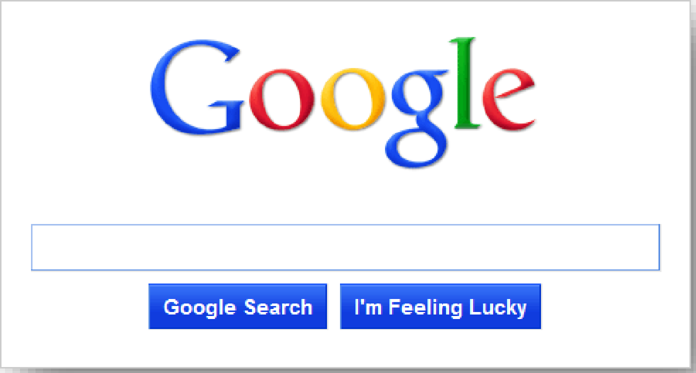
- Many opportunities for resilience
  - Mechanism for dealing with inherently unreliable hardware
  - Mechanism for increased performance (up to 7x)
  - It’s also possible to verify the resulting applications

Takeaway: Methodology for Programming General Uncertain Computations









First-Class Execution Models (SEU)

```
model
{
  bool upset = false;

  operator +(x1, x2) :
    ensures result == (x1 + x2)

  operator +(x1, x2) :
    when !upset,
    ensures upset,
    ensures |result - (x1 + x2)| <= e

  ...
}
```

Model State

Postcondition

Guard

Verifying Fault Tolerance with First-class Execution Models.
Brett Boston and Michael Carbin. In Submission

Leto: Verifying Application-Specific Fault Tolerance with First-Class Execution Models

First-Class Execution Models (Jacobi)

```
spec bool last_upset = model.upset;
while(...) {
    for (int i = 0; i < x.length; ++i) {
        float sigma = 0;
        for(int j = 0; j < x.length; ++j) {
            if (j != i) {
                float delta = A[i][j] *. last_x[j];
                sigma = sigma +. delta;
            }
        }
        float num = b[i] -. sigma;
        x[i] = num /. A[i][i];
    }
    assert (!last_upset && model.upset) ->
           (norm2(x<o> - x<r>) < eps))
    last_x = x;
    last_upset = model.upset;
}
```

Reflect on fault model state

Unreliable Computation

Relational Assertion: bound difference in solution vector

Logics for Verifying Properties

1. Safety – properties required to produce a valid result

Relational Program Logic

```
assert (x != null) ∧ x<o> == x<r> ⊨ x<r> != null
```

2. Accuracy – worst-case difference in program result

Relational Program Logic

```
assert_r |res<o> - res<r>| <= .02 * res
```

Result

- Model: First class fault model specification
- Semantics: system weaves fault model into program semantics
- Verification: automatically generate *relational* weakest preconditions and discharge using SMT
- Broad Motivation: model adversarial environments such as hardware faults (unreliable compute, memory (RowHammer)) and system attackers

First-Class Execution Model

- Rowhammer
- Approximate Multiplication
- Chaos