Equality Saturation & egg

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2022-10-08 @ ETH Workshop on Dependable and Secure Software Systems


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$$(a * 2) / 2 \Rightarrow a$$

$$(a * 2) / 2 \Rightarrow a$$

REWRITE!

```
(a * 2) / 2 ⇒ a
```

REWRITE!

Useful

```
(x * y) / z = x * (y / z)

      x / x = 1

      x * 1 = x
```

$$(a * 2) / 2 \Rightarrow a$$

REWRITE!

Useful

```
(x * y) / z = x * (y / z)
x / x = 1
x * 1 = x
```

Less Useful

```
x * 2 = x << 1
x * y = y * x
x = x * 1
```

```
(a * 2) / 2
```

"happy path"

```
(x * y) / z = x * (y / z)

x / x = 1

x * 1 = x
```

$$(a * 2) / 2 \Rightarrow a * (2 / 2)$$

"happy path"

$$(x * y) / z = x * (y / z)$$

$$x / x = 1$$

$$x * 1 = x$$

```
(a * 2) / 2 ⇒ a * (2 / 2) ⇒ a * 1
```

"happy path"

```
(x * y) / z = x * (y / z)

x / x = 1

x * 1 = x
```

$$(a * 2) / 2 \Rightarrow a * (2 / 2) \Rightarrow a * 1 \Rightarrow a$$

“happy path”

$$(x * y) / z = x * (y / z)$$

$$x / x = 1$$

$$x * 1 = x$$

```
(a * 2) / 2 ⇒ a * (2 / 2) ⇒ a * 1 ⇒ a
```

“happy path”

```
(x * y) / z = x * (y / z)

       x / x = 1

       x * 1 = x
```

✅

```
(a * 2) / 2
```

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
```

```
(a * 2) / 2 ⇒ (a << 1) / 2
```

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
```

`(a * 2) / 2 ⇒ (a << 1) / 2` ❌ order

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
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```
(a * 2) / 2 ⇒ (a << 1) / 2
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(a * 2) / 2
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Pitfalls

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x * 2 = x << 1
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(a * 2) / 2 ⇒ (a << 1) / 2 ❌ order

(a * 2) / 2 ⇒ (2 * a) / 2

Pitfalls

x * 2 = x << 1

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(a * 2) / 2 ⇒ (a << 1) / 2
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(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2
```

Pitfalls

```
x * 2 = x << 1
x * y = y * x
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```
(a * 2) / 2 ⇒ (a << 1) / 2
```
❌ order

```
(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2
```
❌

diverge

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
```

```
(a * 2) / 2 ⇒ (a << 1) / 2
```
❌ order

```
(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2
```
❌

diverge

```
a
```

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
```

```
(a * 2) / 2 ⇒ (a << 1) / 2
```
❌ order

```
(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2
```
❌

diverge

```
a ⇒ a * 1
```

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
```

```
(a * 2) / 2 ⇒ (a << 1) / 2
```
❌ order

```
(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2
```
❌

diverge

```
a ⇒ a * 1 ⇒ a * 1 * 1
```

Pitfalls

```
x * 2 = x << 1
x * y = y * x
    x = x * 1
```

(a * 2) / 2 ⇒ (a << 1) / 2 ❌ order

(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2 ❌

diverge

a ⇒ a * 1 ⇒ a * 1 * 1 ⇒ ... ❌ infinite size

Pitfalls

x * 2 = x << 1

x * y = y * x

x = x * 1

`(a * 2) / 2 ⇒ (a << 1) / 2` ❌ order

`(a * 2) / 2 ⇒ (2 * a) / 2 ⇒ (a * 2) / 2` ❌

diverge

`a ⇒ a * 1 ⇒ a * 1 * 1 ⇒ ...` ❌ infinite size

Critical for other inputs!

Pitfalls

`x * 2 = x << 1`

`x * y = y * x`

`x = x * 1`

`(a * 2) / 2 ⇒ a`

Which rewrite? When?

Useful	Less Useful
`(x * y) / z = x * (y / z)`	`x * 2 = x << 1`
`x / x = 1`	`x * y = y * x`
`x * 1 = x`	`x = x * 1`

`(a * 2) / 2 ⇒ a`

Which rewrite? When?

Equality Saturation

Try applying all the rules in every order!

`(a * 2) / 2` ⇒ `a`

Which rewrite? When?

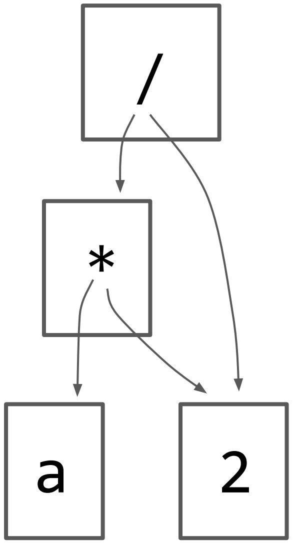
Equality Saturation

Try applying **all** the rules in **every** order?!

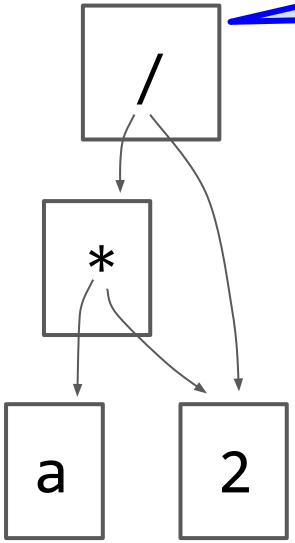
E-graphs

- Data structure from Greg Nelson's PhD thesis (1980)
- Used for congruence closure (Downey, Sethi, Tarjan 1980)
  - Intuition: union-find (Tarjan 1975) but function-aware
- Key for equality and uninterpreted funcs (EUF) theory in SMT
  - Intuition: the "glue" that connects other theories to SAT
- Historically: "baked in" to SMT solvers, no general libraries 😐

E-graphs

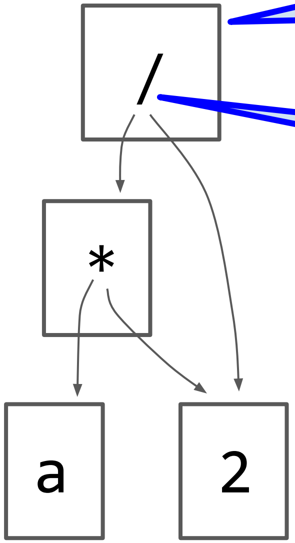


E-graphs



e-classes contain e-nodes (ops)

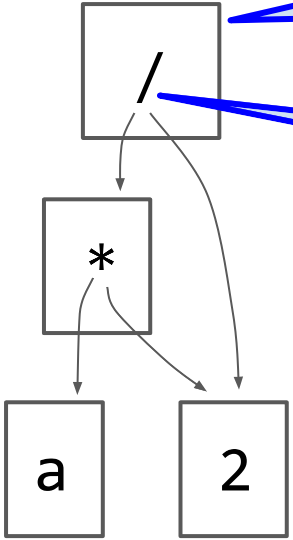
E-graphs



e-classes contain e-nodes (ops)

e-nodes' arguments are e-classes!

E-graphs

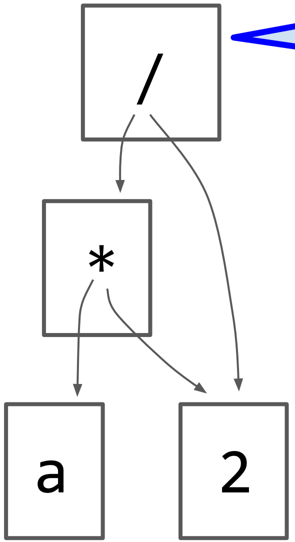


e-classes contain e-nodes (ops)

e-nodes' arguments are e-classes!

e-graphs maximize sharing
(no copies of same e-node)

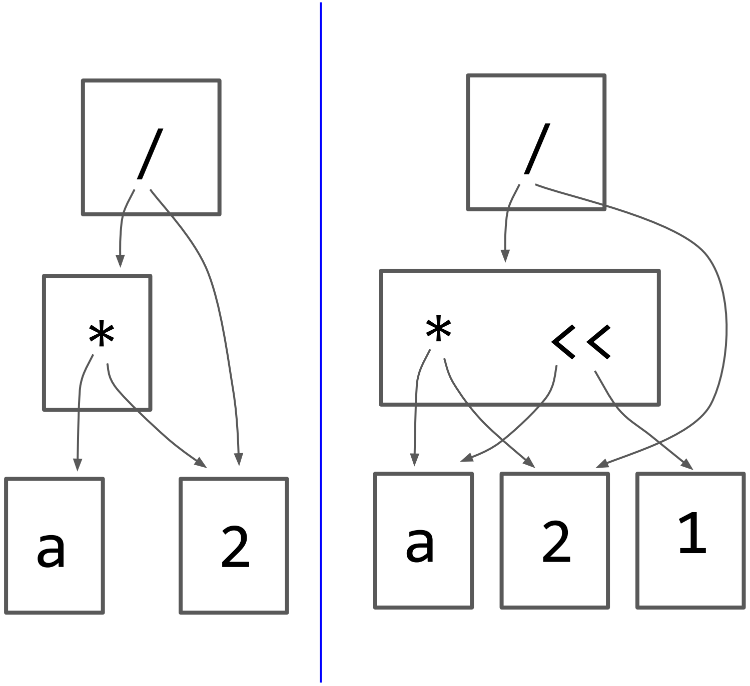
E-graphs



This e-classes *represents*

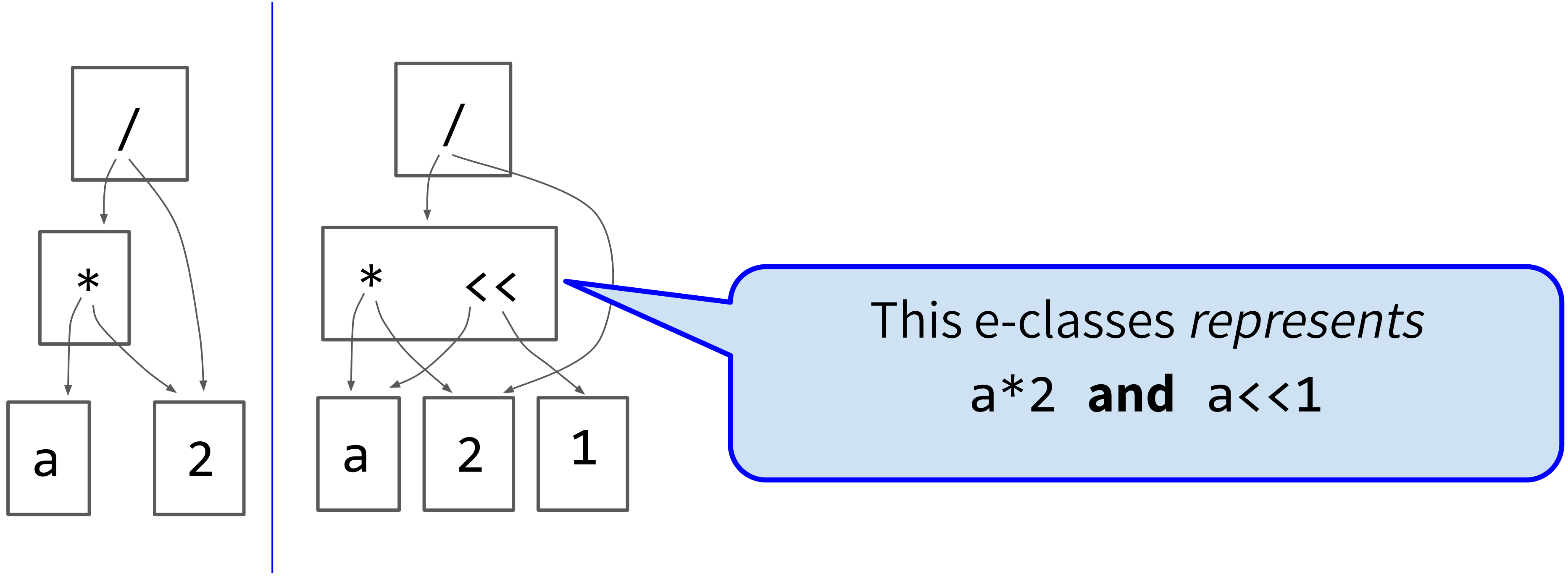
`(a * 2) / 2`

E-graphs: applying rewrite rules



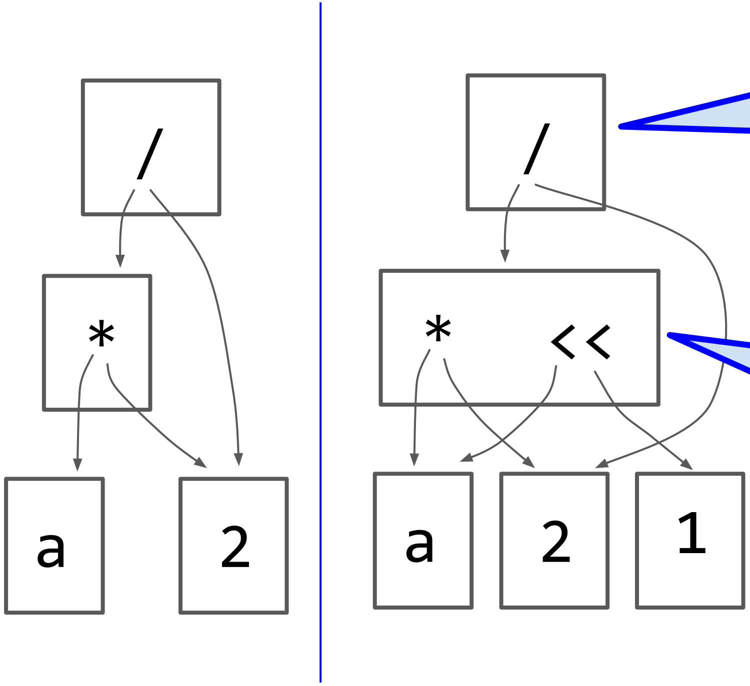
`x * 2 → x << 1`

E-graphs: applying rewrite rules



x * 2 → x << 1

E-graphs: applying rewrite rules

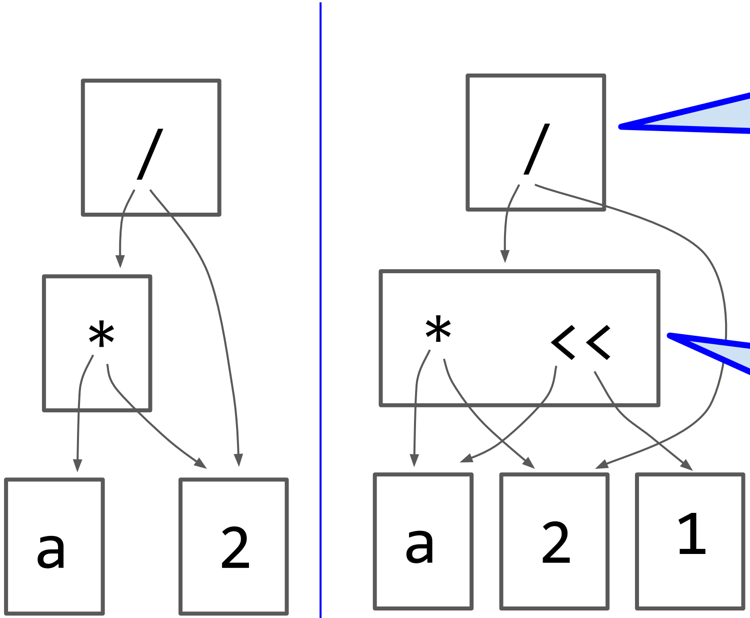


This e-classes *represents*
`(a*2)/2` **and** `(a<<1)/2`

This e-classes *represents*
`a*2` **and** `a<<1`

`x * 2 → x << 1`

E-graphs: applying rewrite rules

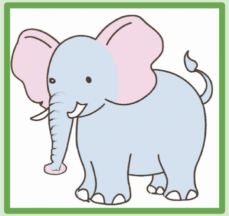


x * 2 → x << 1

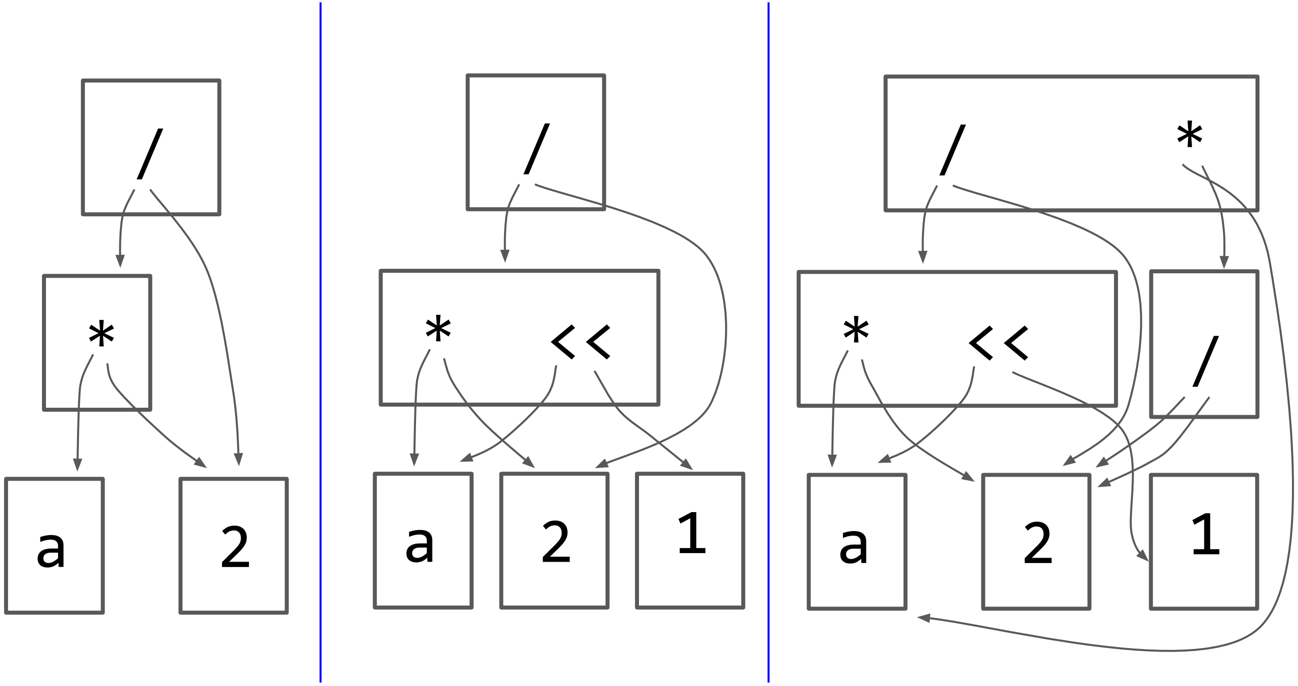
This e-classes *represents*
`(a*2)/2` **and** `(a<<1)/2`

This e-classes *represents*
`a*2` **and** `a<<1`

E-graphs never forget.
Rewrites don't lose info!

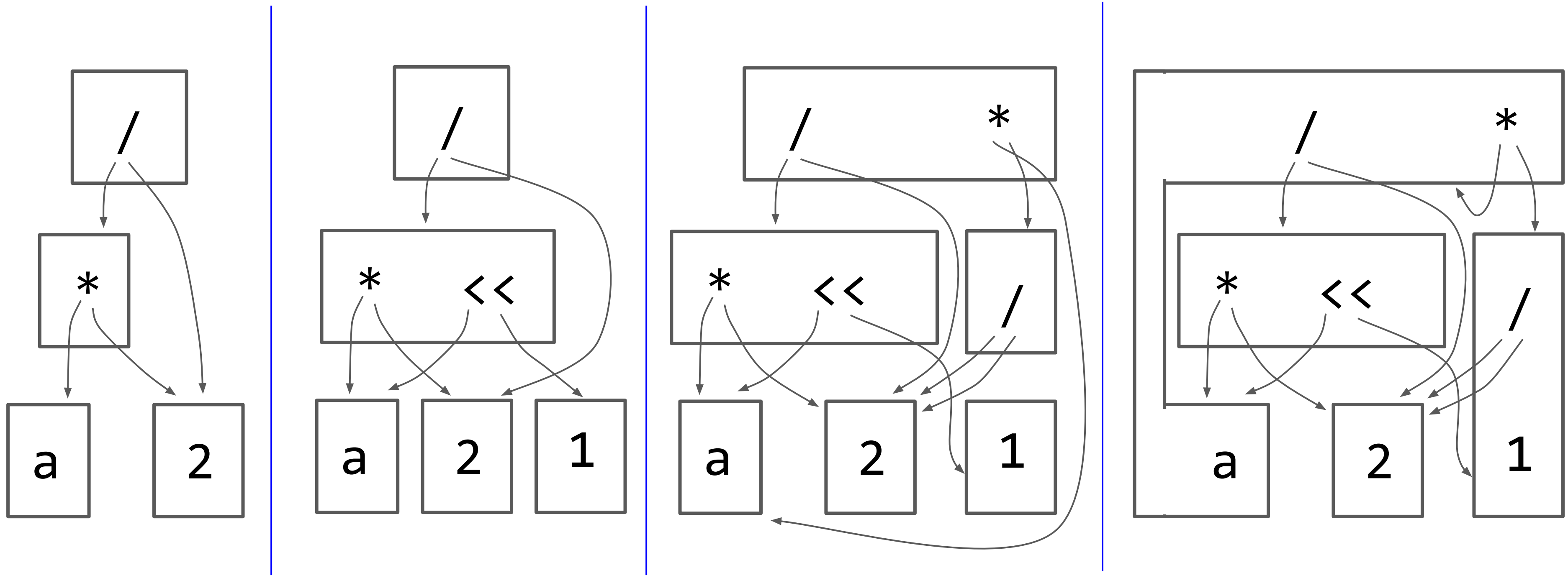


E-graphs: applying rewrite rules



x * 2 → x << 1   (x*y)/z → x*(y/z)

E-graphs: applying rewrite rules



x * 2 → x << 1

(x*y)/z → x*(y/z)

x / x → 1

x * 1 → x

E-graphs: compact representation

Rewrites can **shrink** e-graphs!

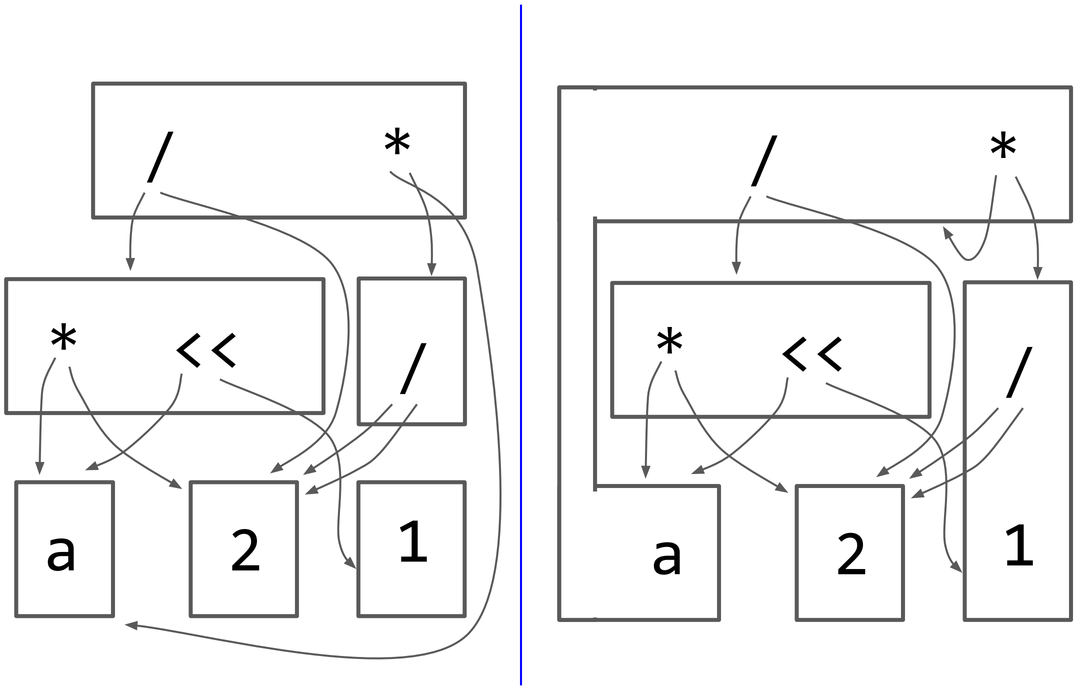
- 6 → 5 eclasses

E-graphs can represent ∞ terms

- `a, a * 1, a * 1 * 1, …`

E-graphs can "*saturate*"

- learn all derivable eqs ✅



`x / x → 1`

`x * 1 → x`

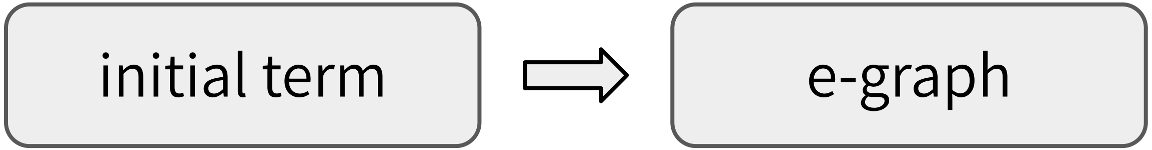
Equality Saturation

- Technique first used in Denali (Joshi, Nelson, Randall 2002)
  - Optimizing straight-line assembly kernels for Alpha
- Extended to loops in Peggy [POPL 2009]
  - Coined term “Equality Saturation”
  - Coinductive stream operators for algebraic loop rewrites
  - Used Rete algo from expert sys for incremental e-matching

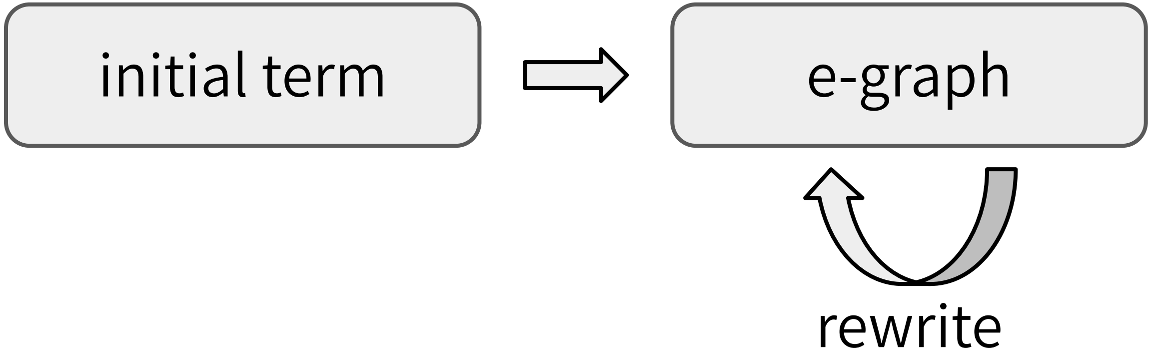
Equality Saturation

initial term

Equality Saturation



Equality Saturation



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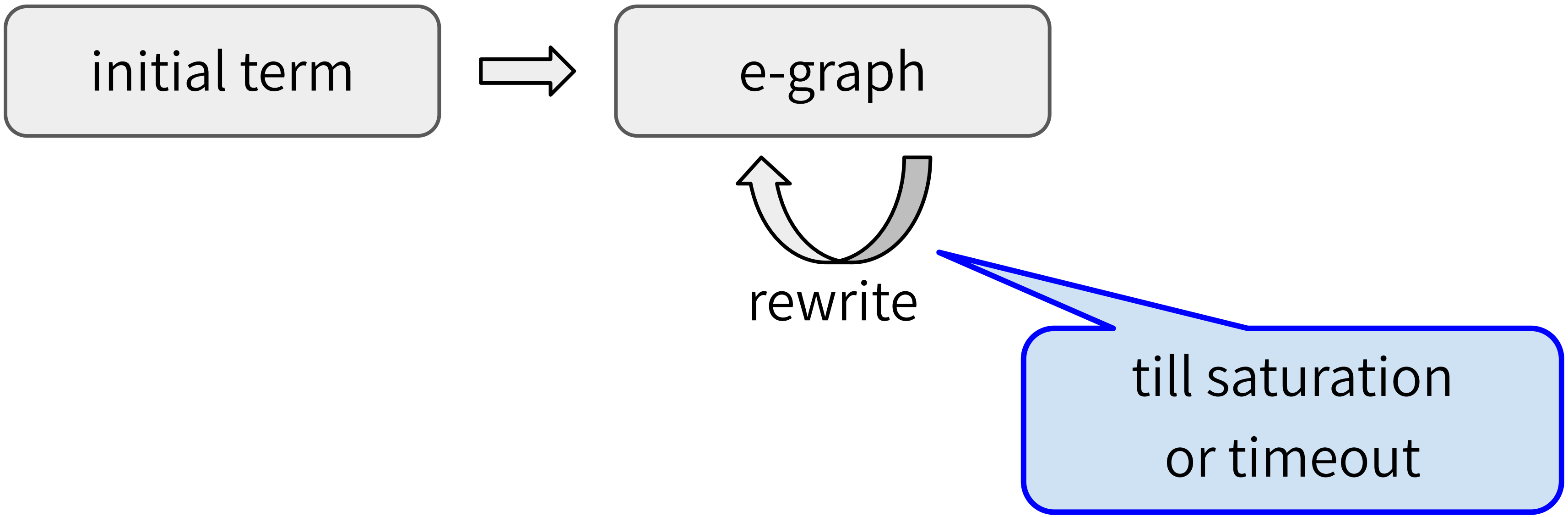

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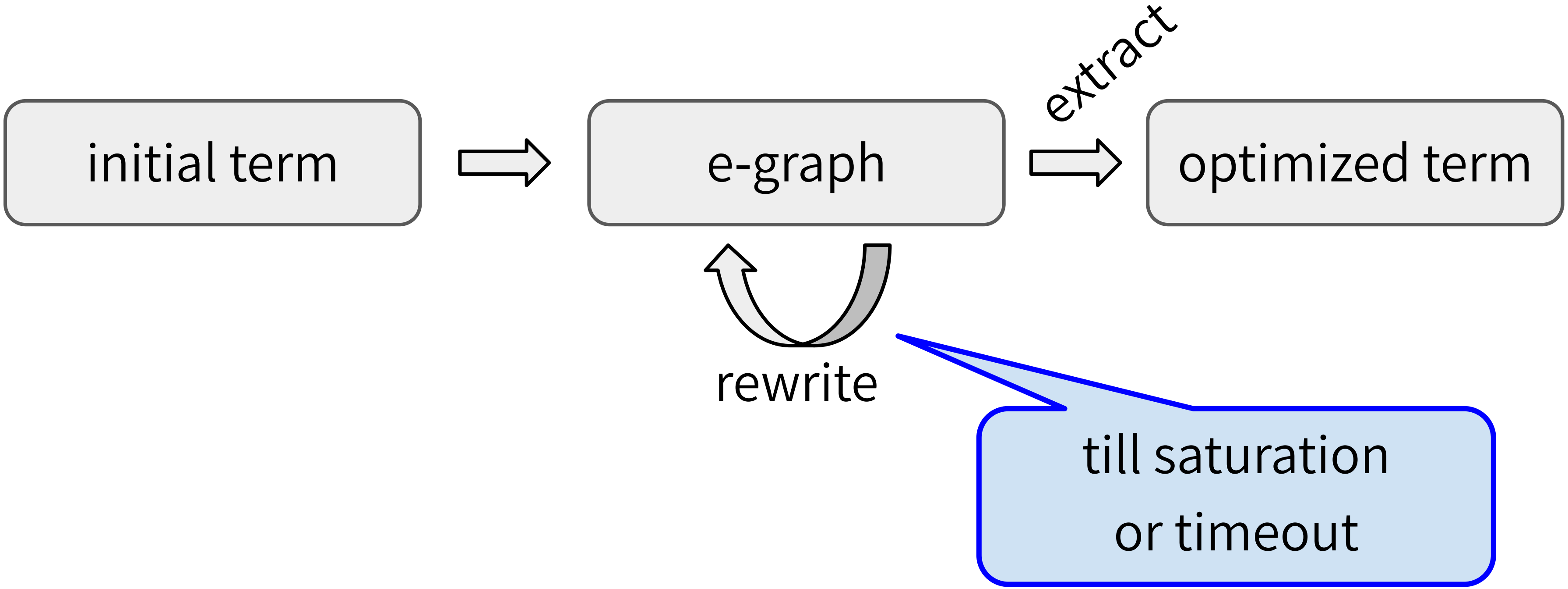

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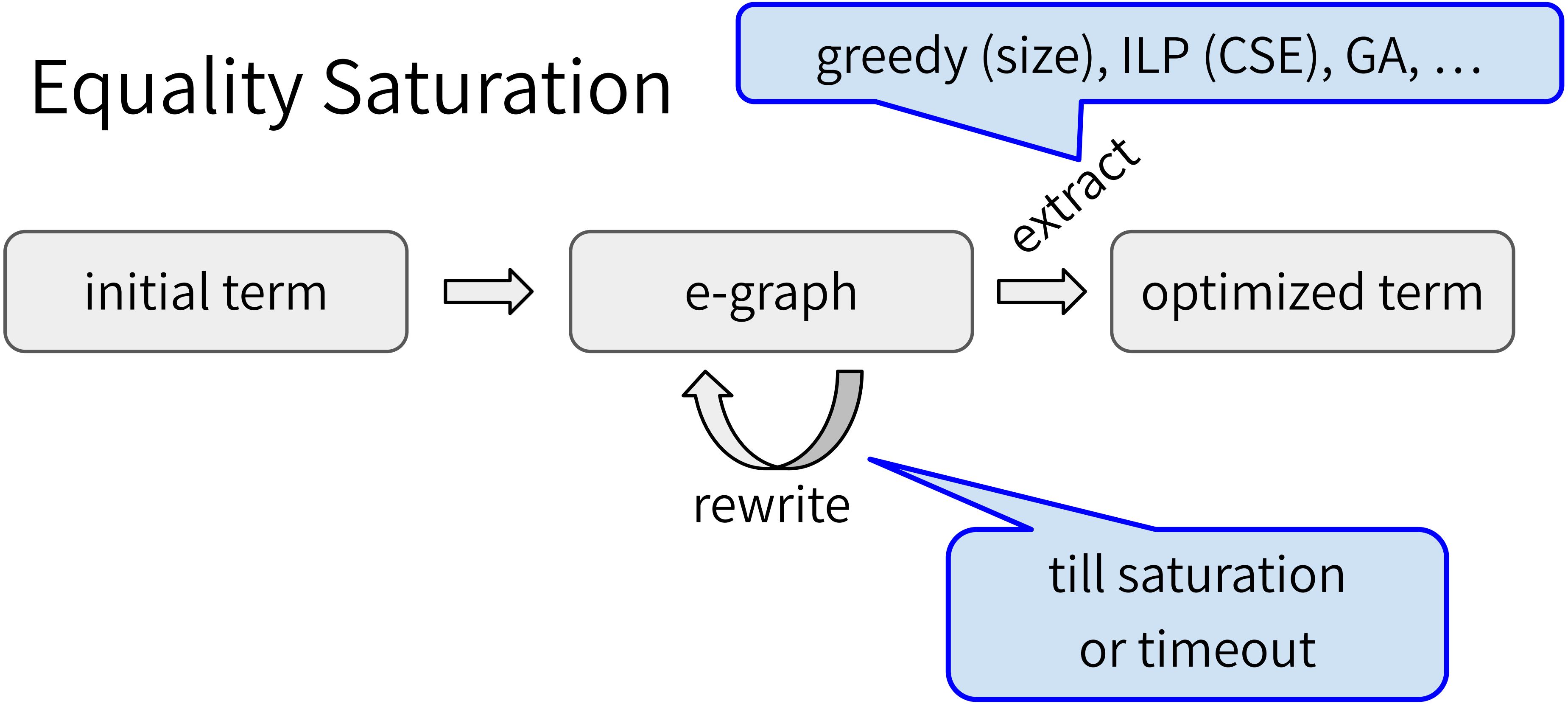

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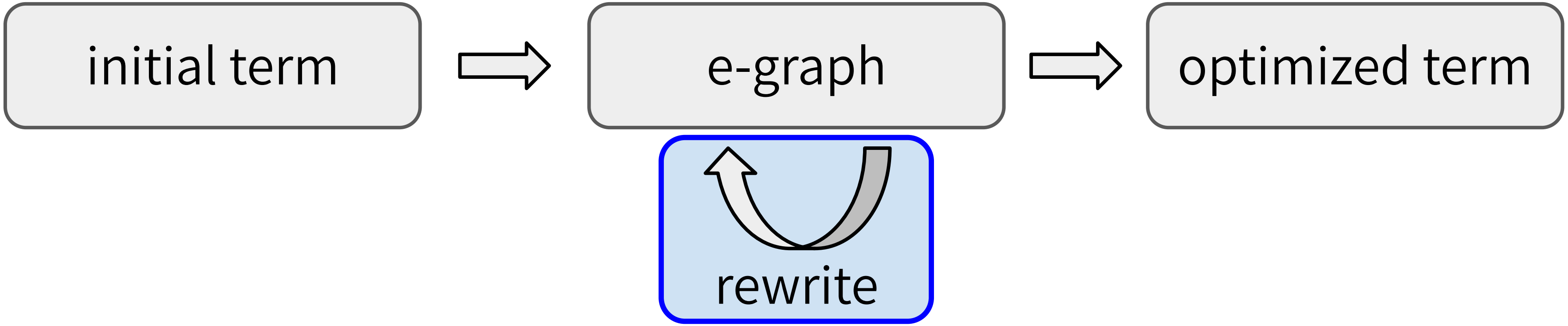

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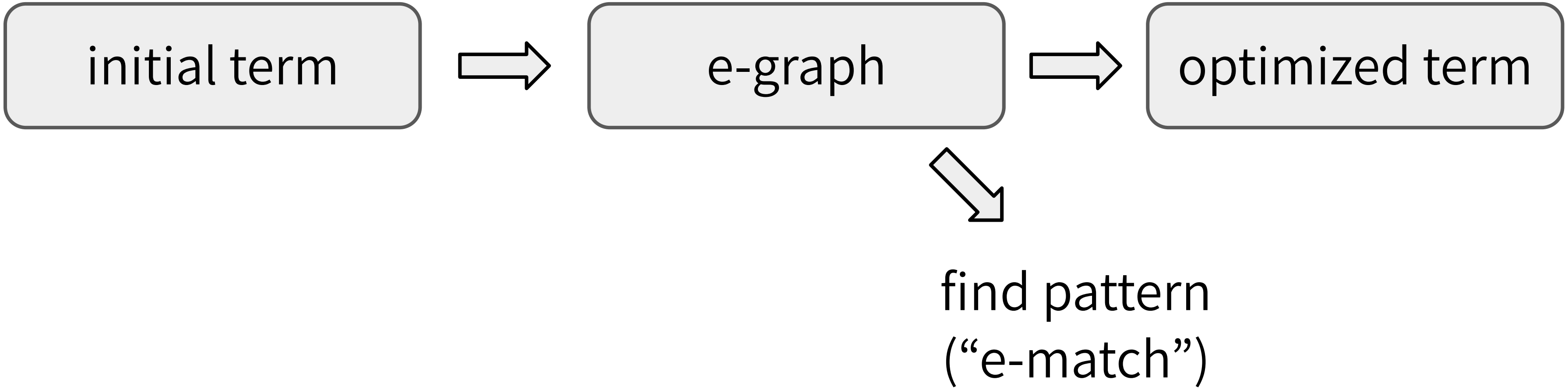

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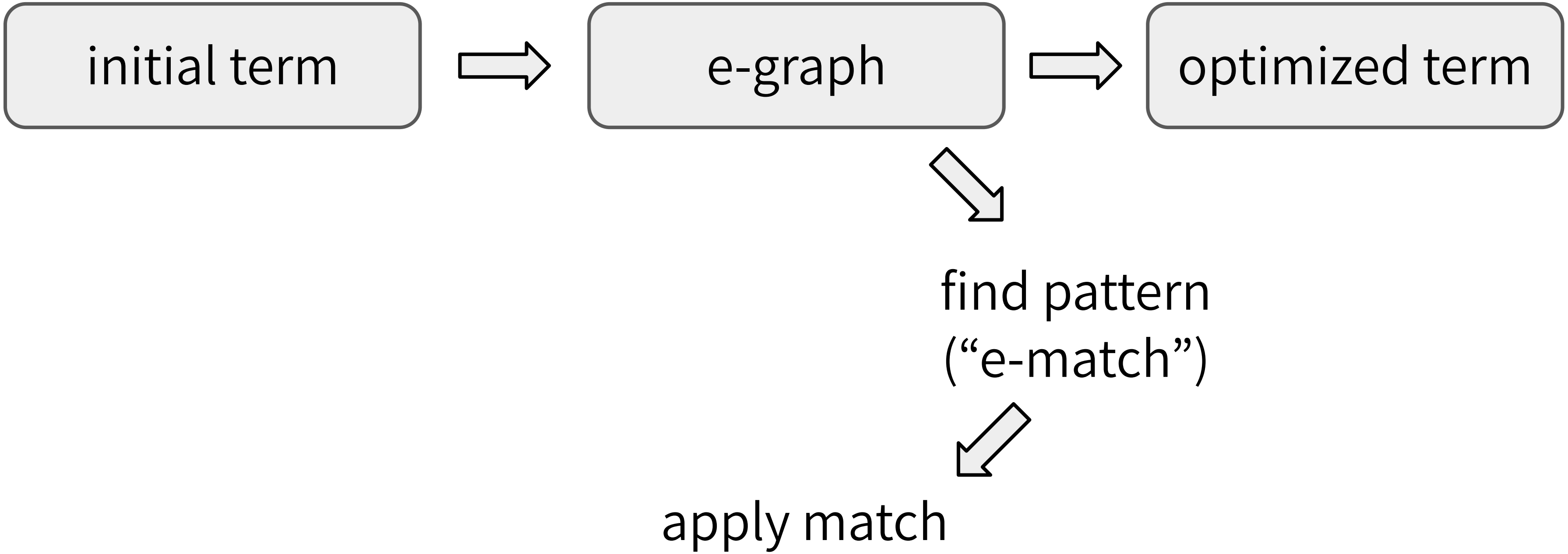

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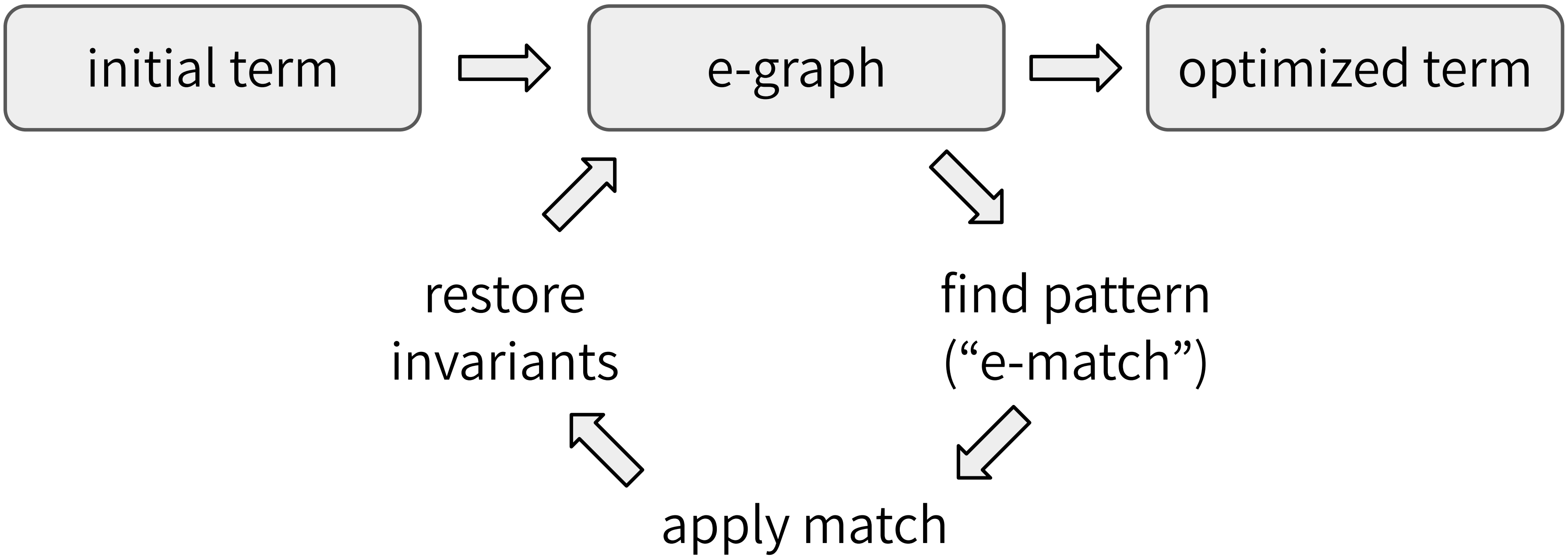

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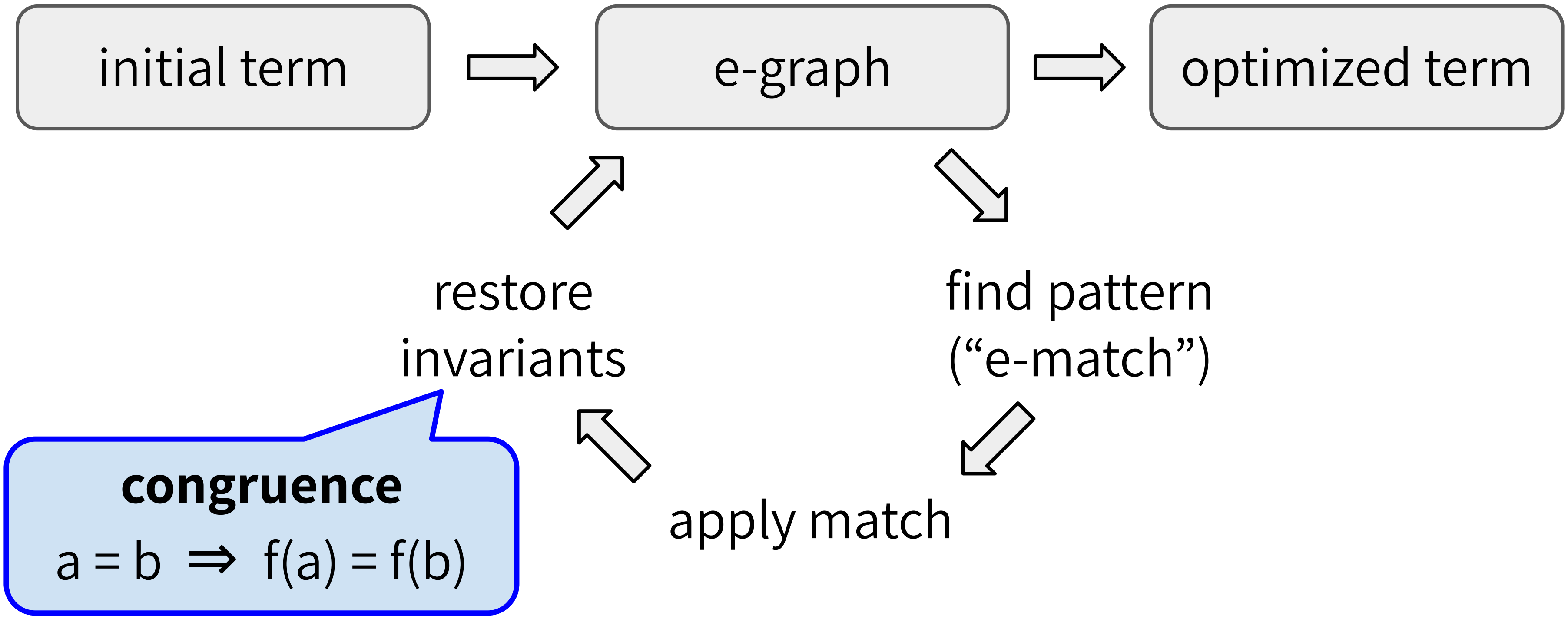

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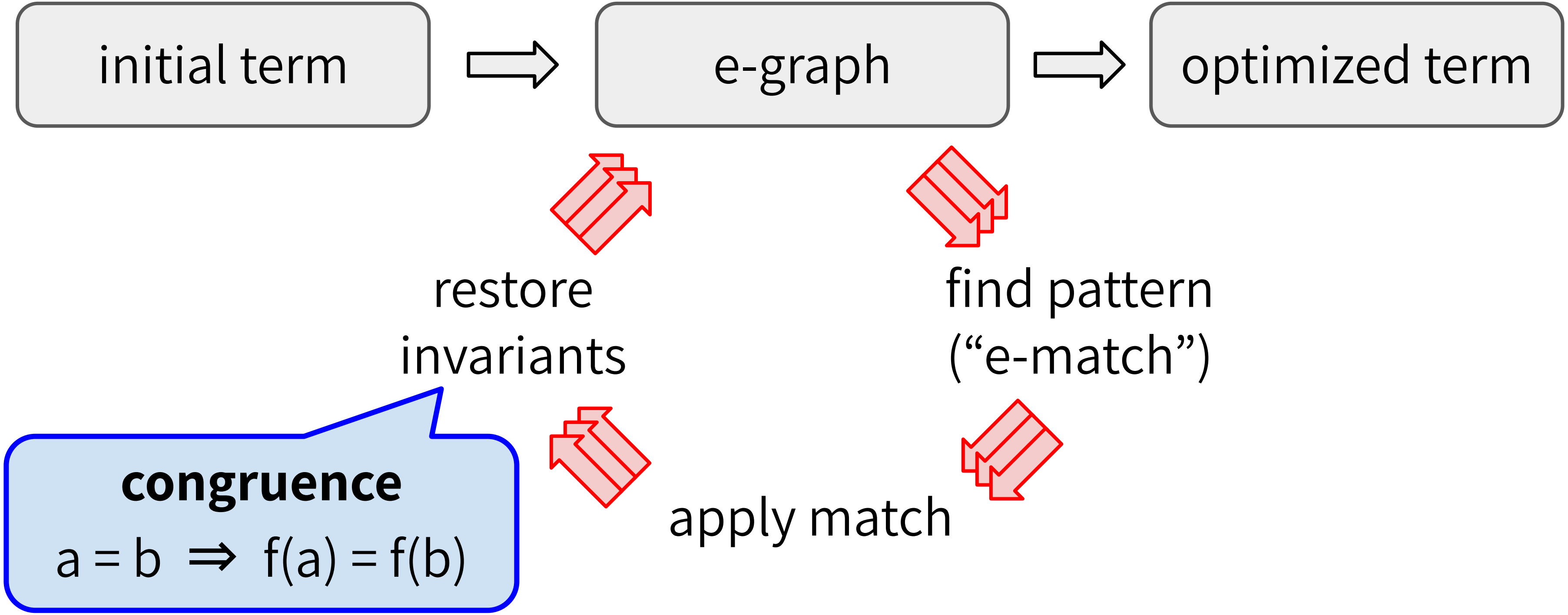

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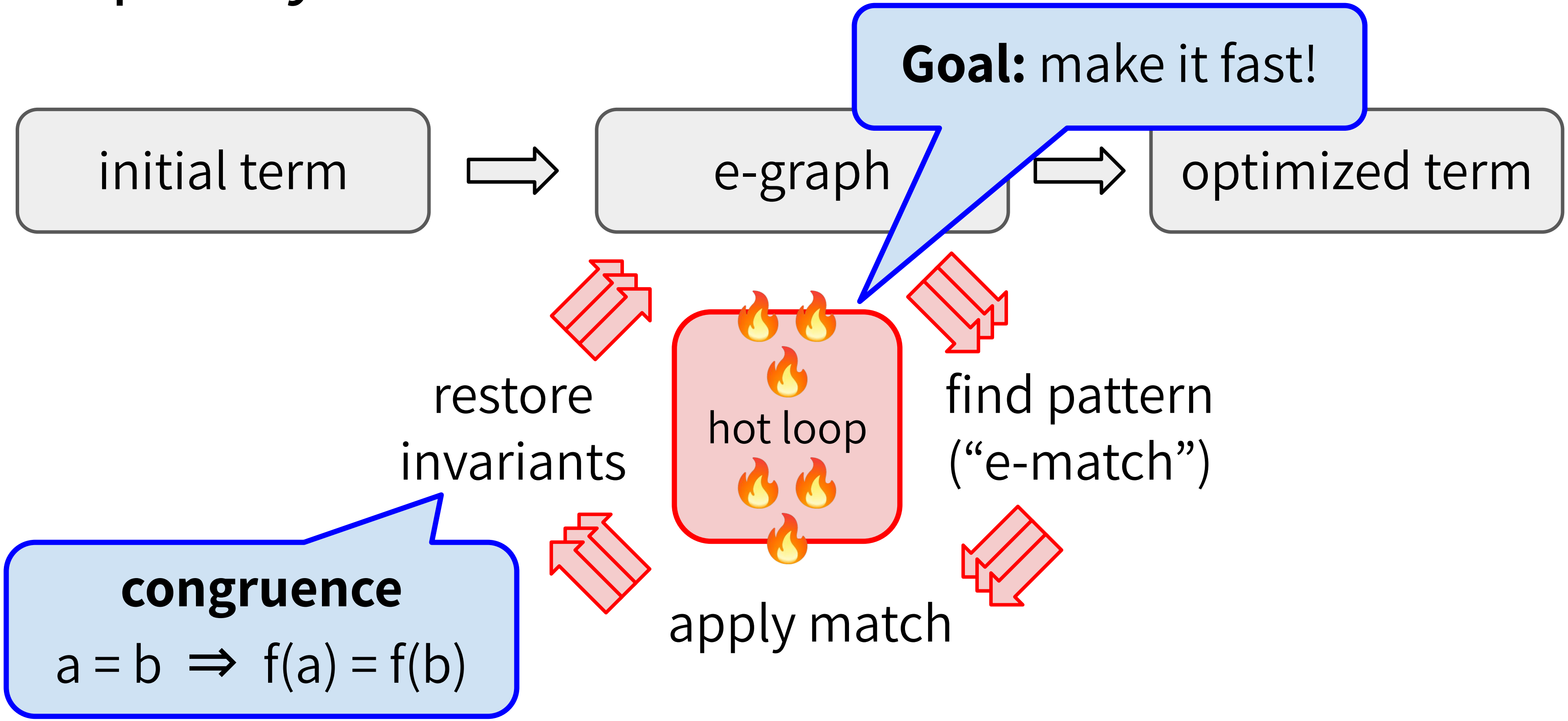

egg EqSat Toolkit [POPL 2021, Distinguished Paper]

- ❑ Deferred invariant maintenance & batching
- ❑ Relational e-matching [POPL 2022]
- ❑ E-class analyses
- ❑ Rewrite rule synthesis with Ruler [OOPSLA 2021, Distinguished Paper]
- ❑ Applications
  - ❑ 3D CAD in Szalinski, FP Accuracy in Herbie, Lib Learning in Babble, …
  - ❑ EVM simplify @ Certora, wasm JIT @ Fastly, datapath optimize @ Intel, …

Equality Saturation

```
def equality_saturation(expr, rewrites):
  egraph = initial_egraph(expr)

  while not egraph.is_saturated_or_timeout():
    for rw in rewrites:
     for (subst, ec) in egraph.ematch(rw.lhs):
       ec2 = egraph.add(rw.rhs.subst(subst))
       egraph.merge(ec, ec2)



  return egraph.extract_best()
```

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```

read

Equality Saturation

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read

write

Equality Saturation

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read

write

restore invariant

Equality Saturation

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- rewrites are ordered
- read/write interleaved
  - more invariant maint
- invariants baked-in

Deferred Invariant Maintenance in egg

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       matches.append((rw, subst, ec))
    for (rw, subst, ec) in matches:
     ec2 = egraph.add(rw.rhs.subst(subst))
     egraph.merge(ec, ec2)
    egraph.rebuild()

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```

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      egraph.merge(ec, ec2)
    egraph.rebuild()

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```

Deferred Invariant Maintenance in egg

```
def equality_saturation(expr, rewrites):
  egraph = initial_egraph(expr)

  while not egraph.is_sat
    for rw in rewrites:
      for (subst, ec) i
        ec2 = egraph.a
        egraph.merge(e

  return egraph.extr
```

batch reads

batch writes
(invariants broken)

invariants restored
once per iteration

```
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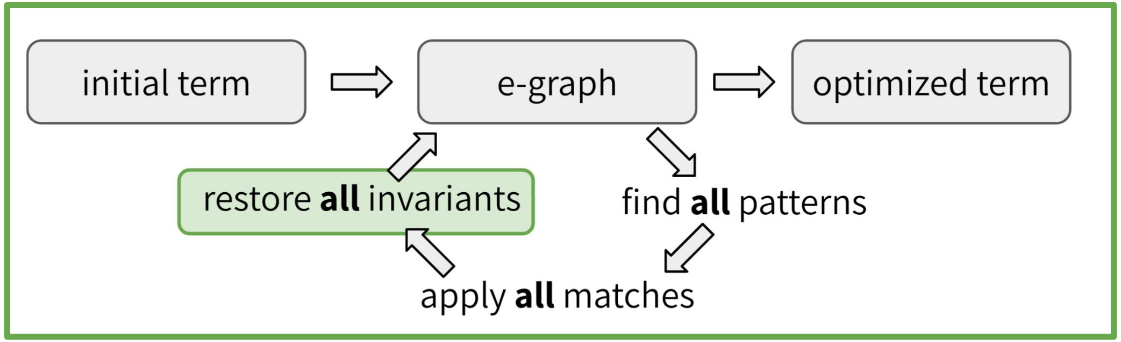
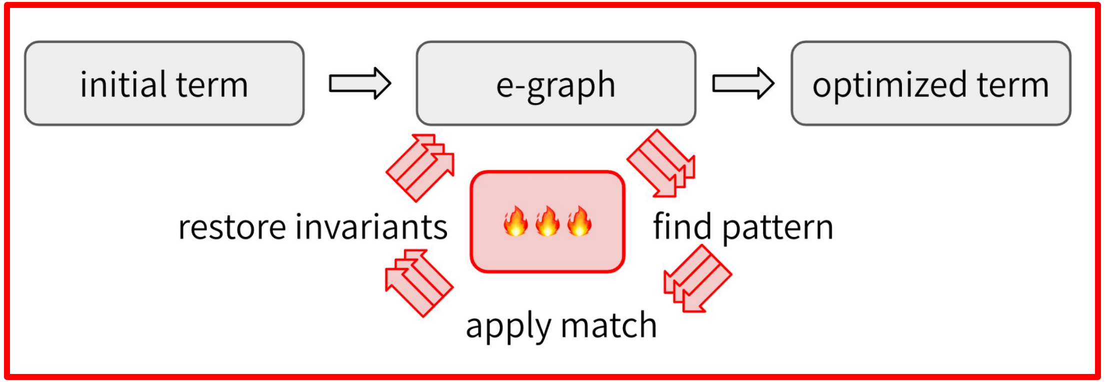
  return egraph.extract_best()
```


initial term
e-graph
optimized term
restore all invariants
find all patterns
apply all matches

versus


initial term
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restore invariants
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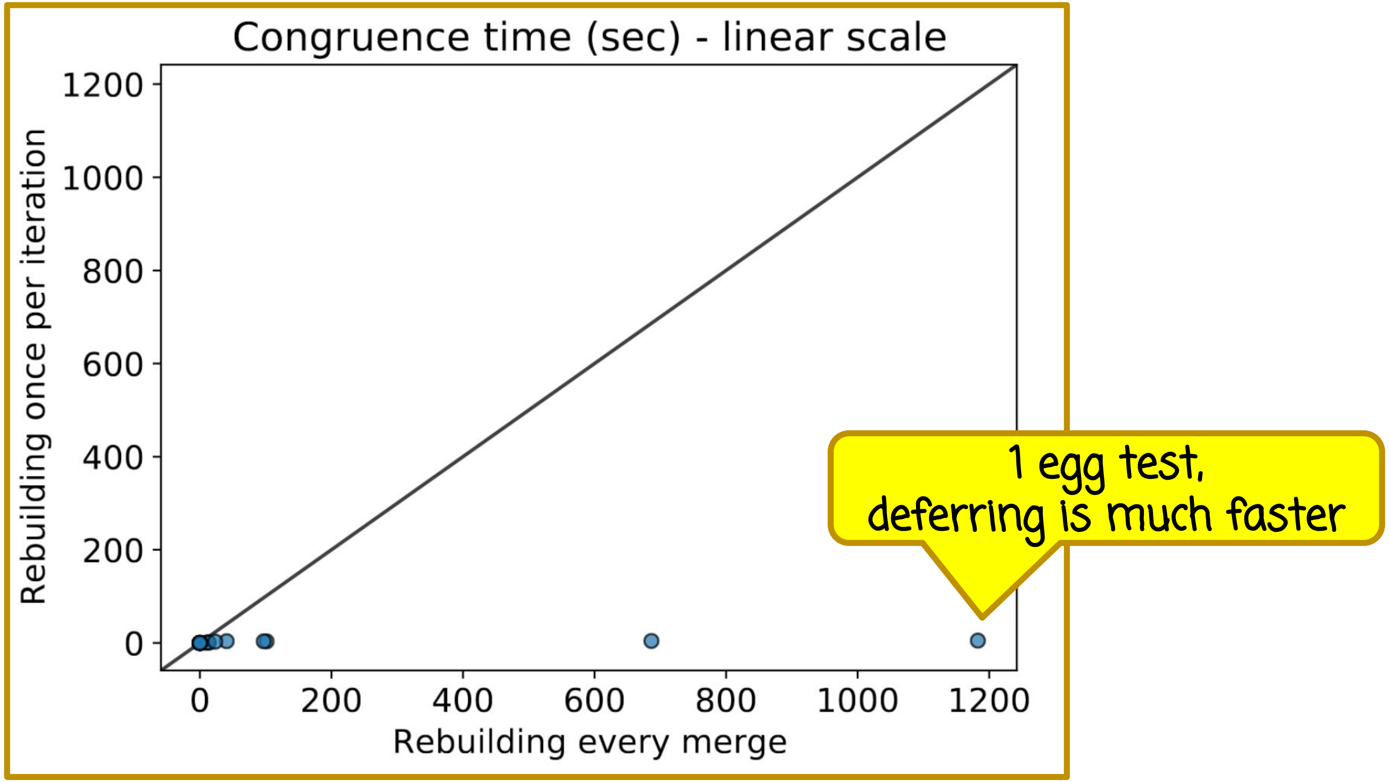
apply match

Rebuilding is faster



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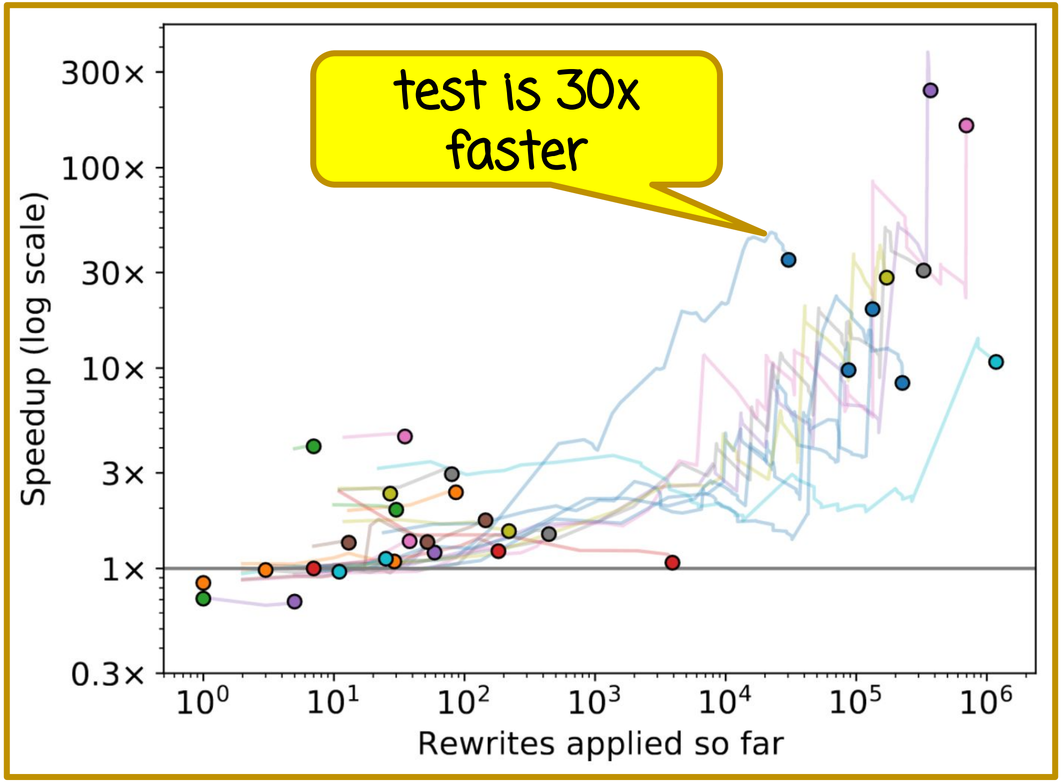
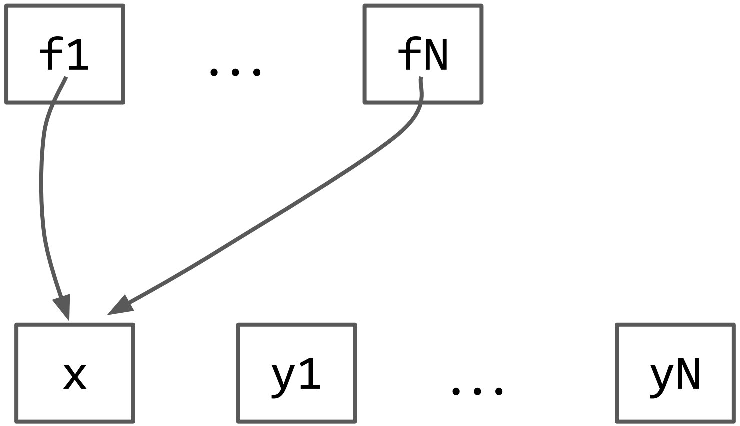


Why is rebuilding is faster?

- Consider $f_1(x)$ … $f_n(x)$ and y_1 … y_n
- Workload: merge(x, y_1) … merge(x, y_n)
- Traditional: $O(n^2)$ hashcons updates
- Deferred only does $O(n)$ updates

Downey, Sethi, Tarjan 1980

Why is rebuilding is faster?



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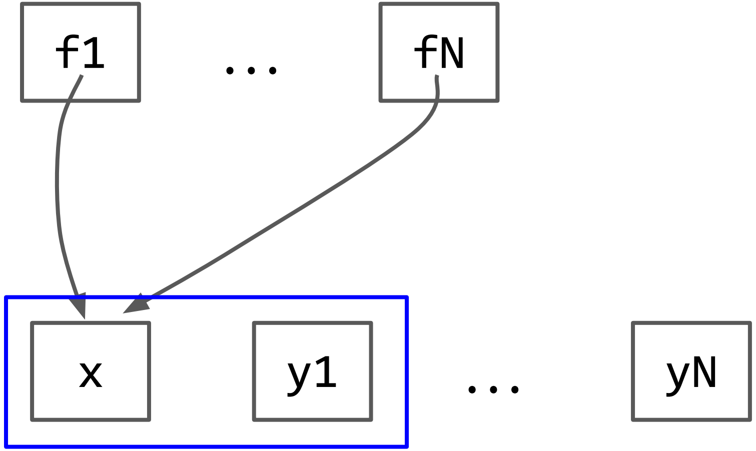

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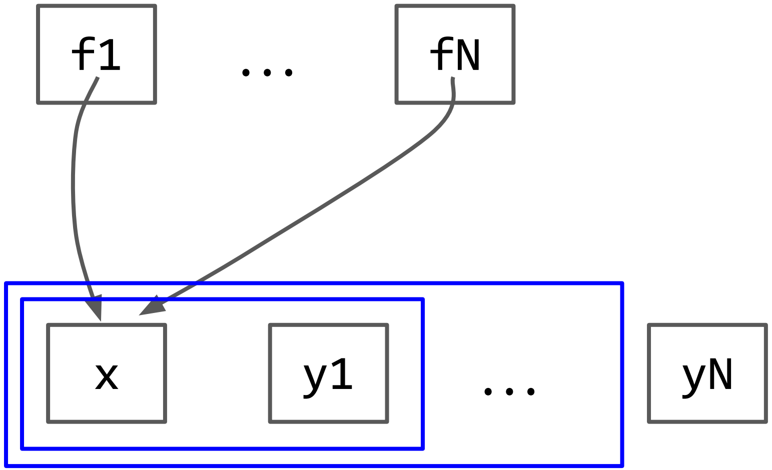

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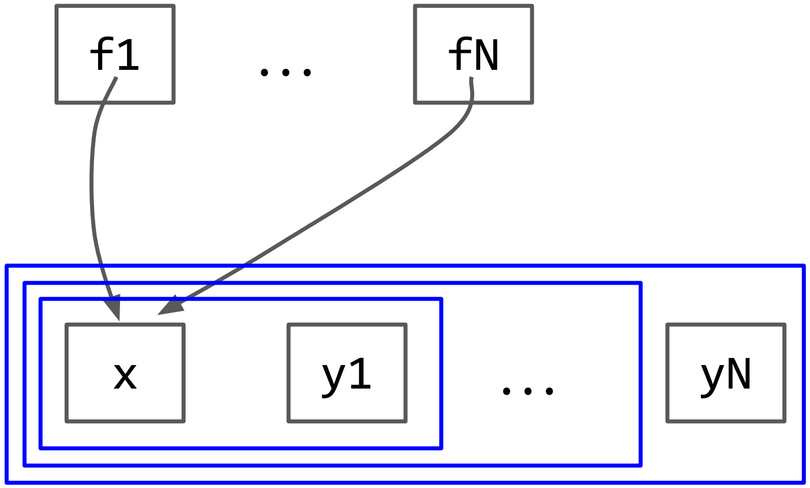
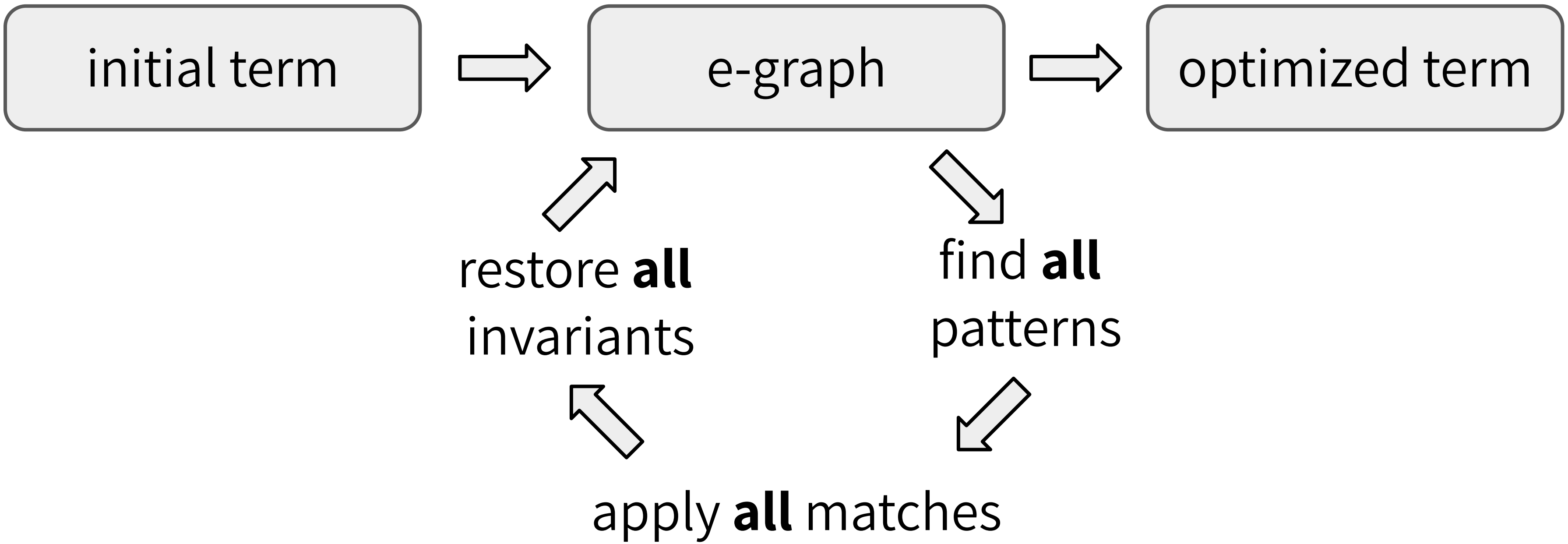


More amortization via *batching* in egg



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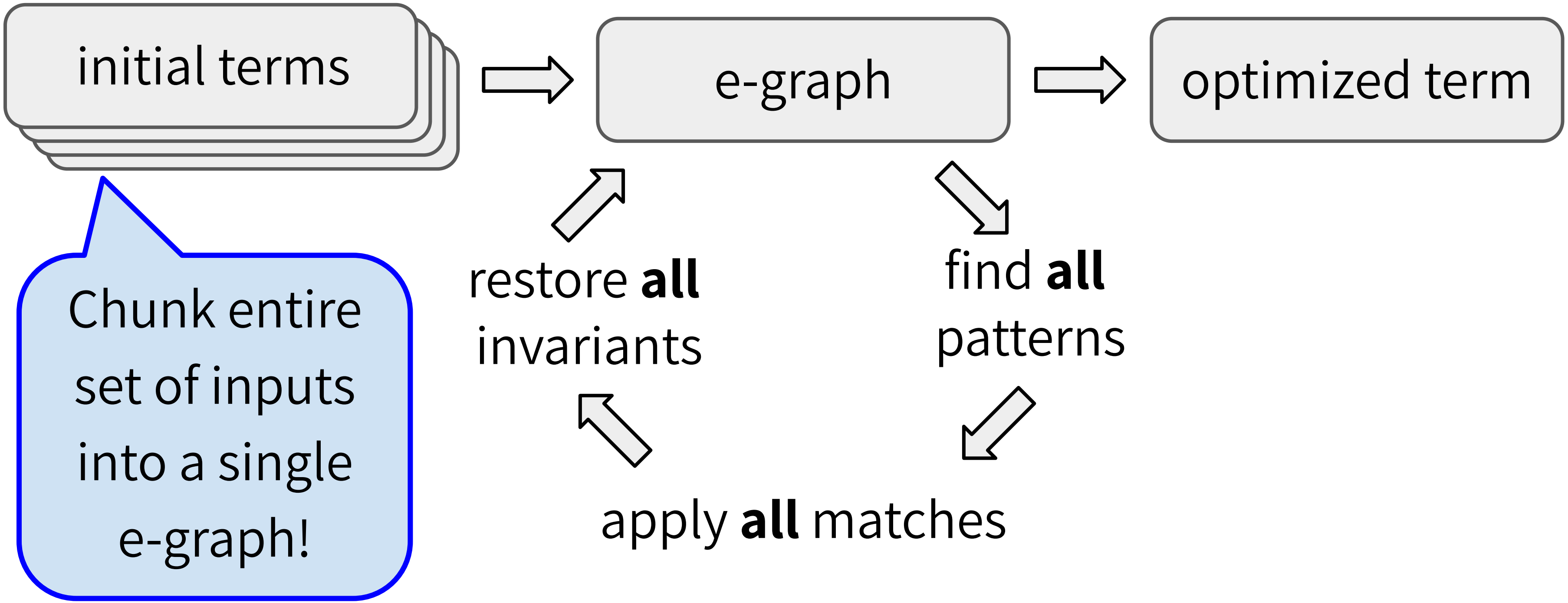

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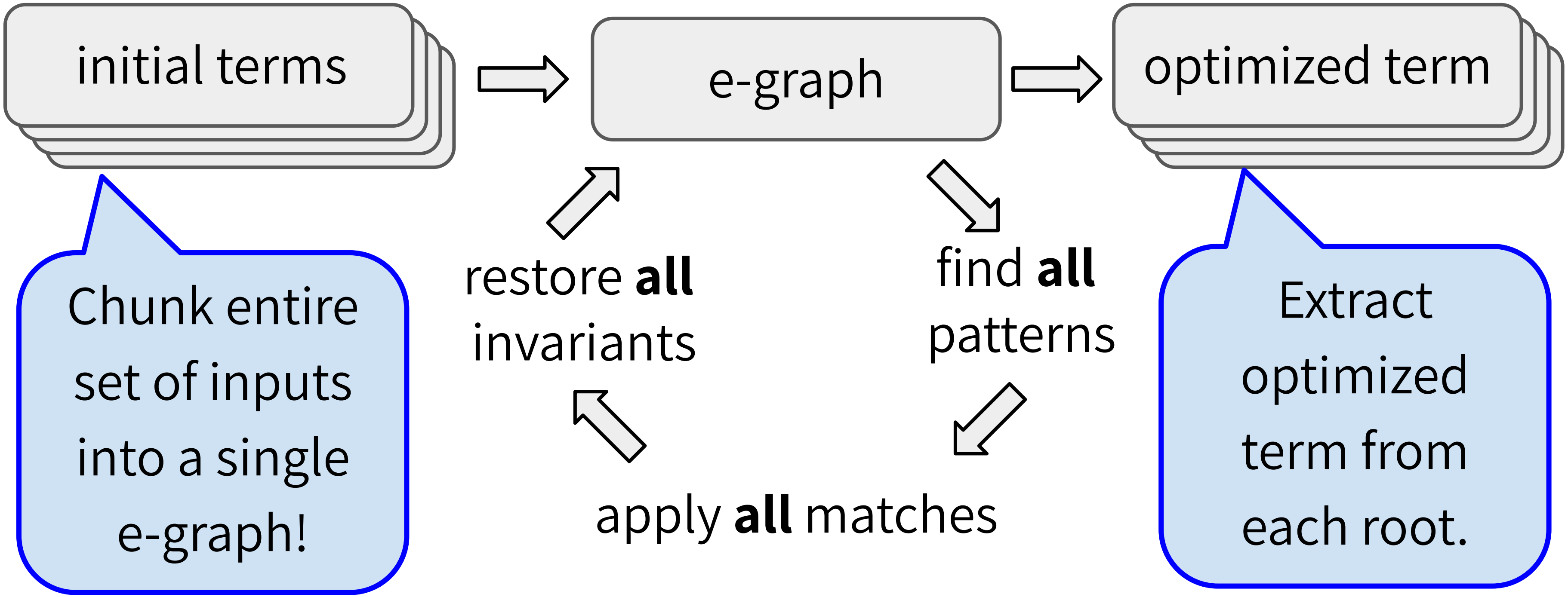
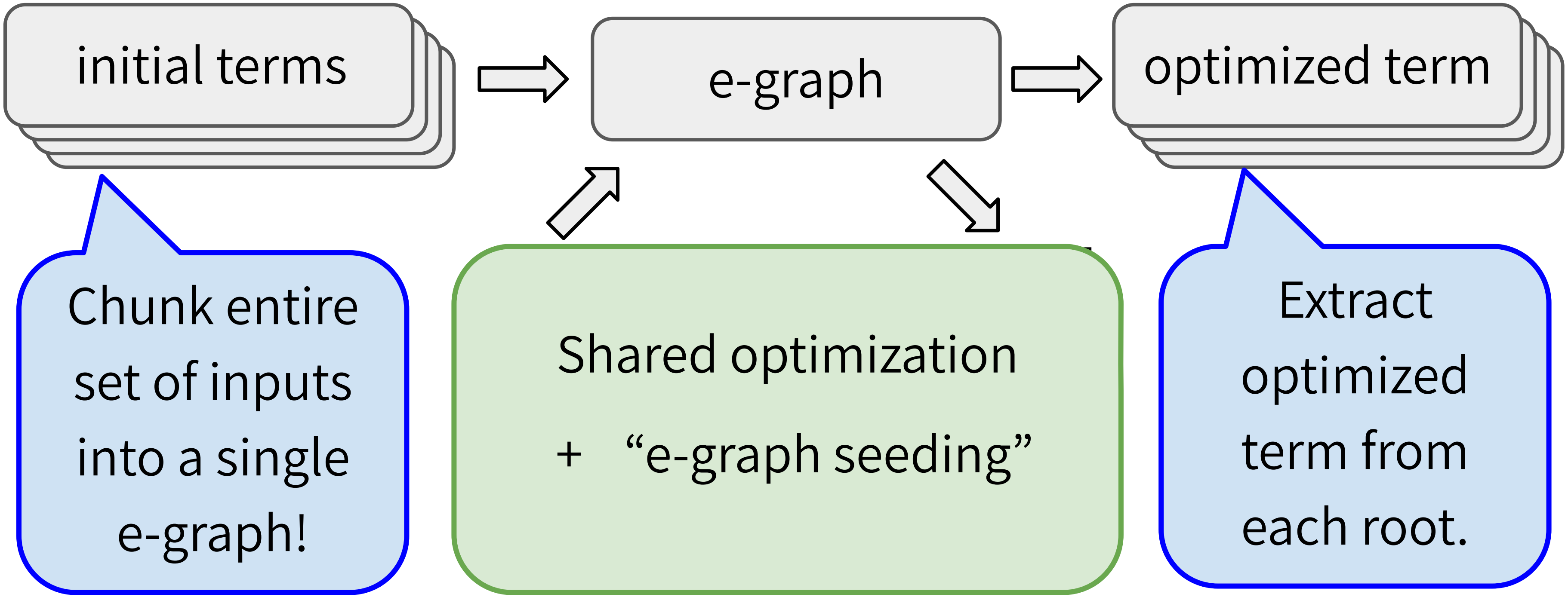


More amortization via *batching* in egg



E-graphs in Herbie

$$\frac{(-b) + \sqrt{b \cdot b - 4 \cdot (a \cdot c)}}{2 \cdot a}$$

E-graphs in Herbie

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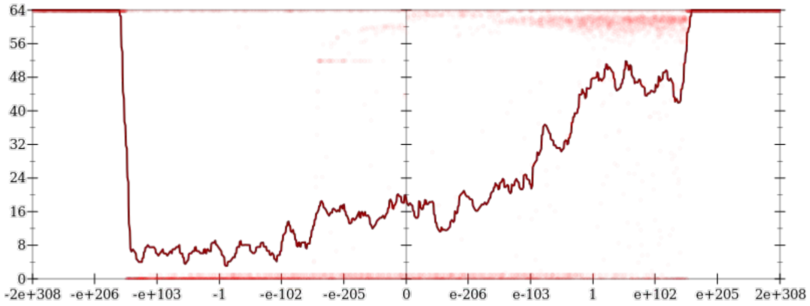


E-graphs in Herbie

$$\frac{(-b) + \sqrt{b \cdot b - 4 \cdot (a \cdot c)}}{2 \cdot a}$$

$$\downarrow$$

if $b \leq -2.1714197031320663 \cdot 10^{+114}$:

$$-\frac{b}{a}$$

elif $b \leq 2.9809086538561536 \cdot 10^{-153}$:

$$\frac{\sqrt{\mathsf{fma}(b, b, c \cdot (a \cdot -4))} - b}{a \cdot 2}$$

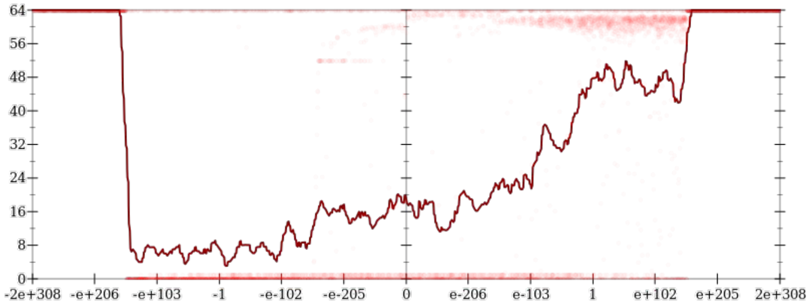
elif $b \leq 3.095118518558678 \cdot 10^{+20}$:

$$t_0 := 4 \cdot (a \cdot c)$$

$$\frac{t_0 \cdot \frac{0.5}{a}}{(-b) - \sqrt{b \cdot b - t_0}}$$

else :

$$-\frac{c}{b}$$



E-graphs in Herbie

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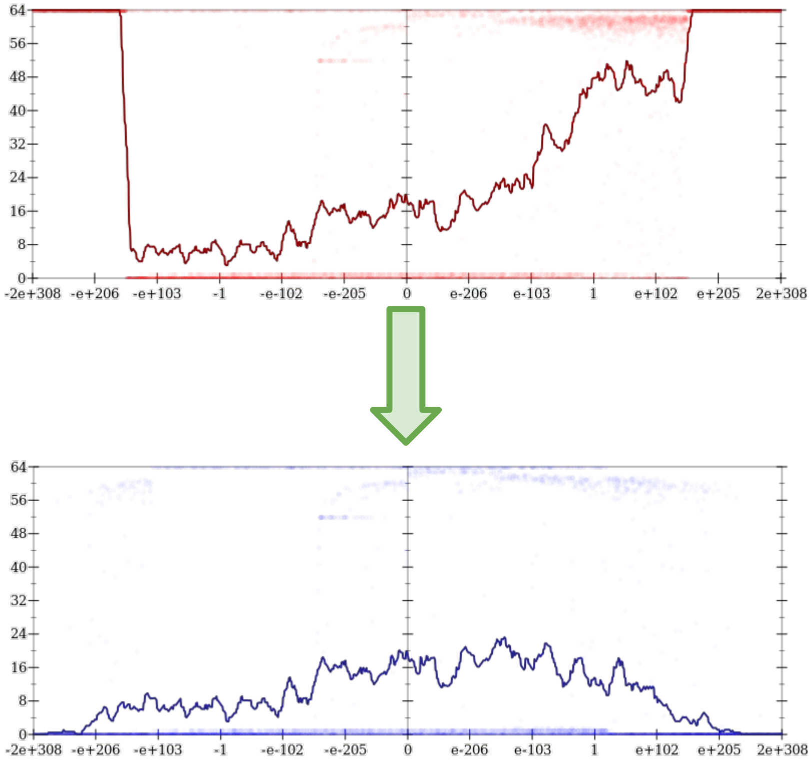
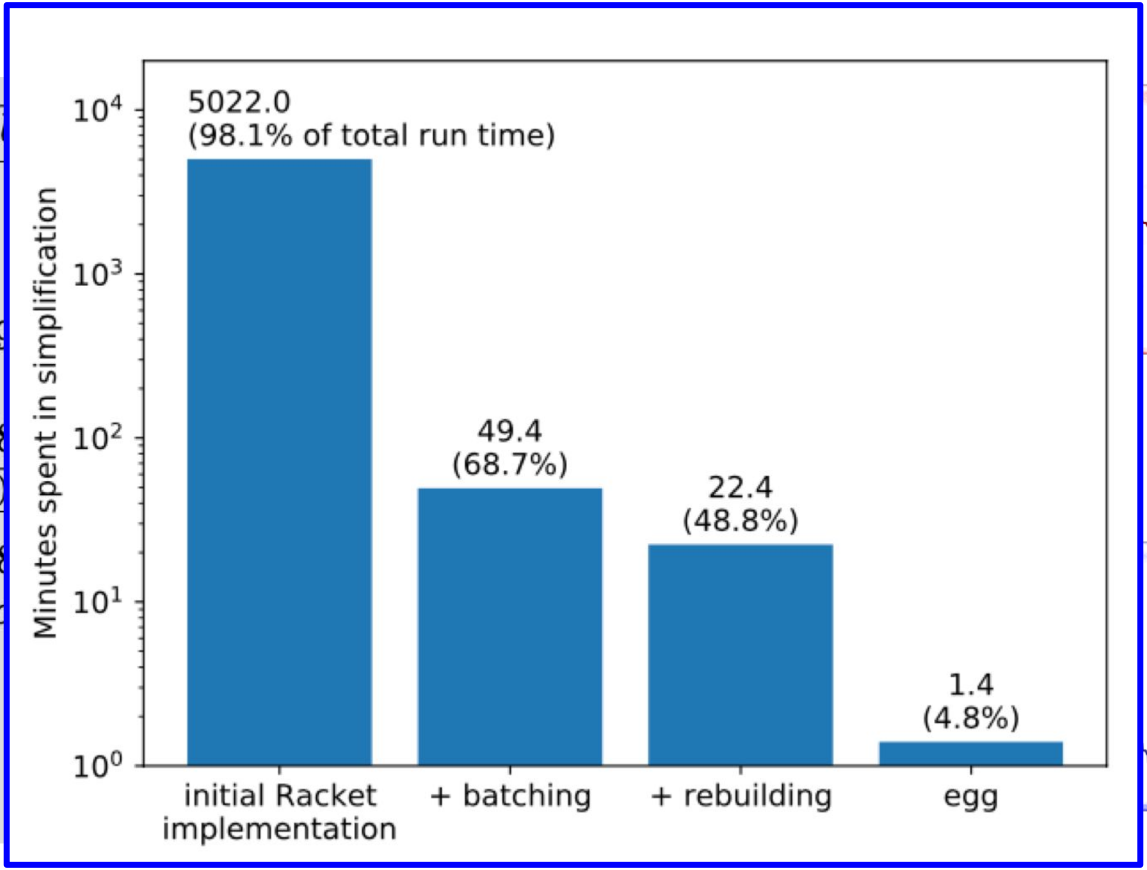
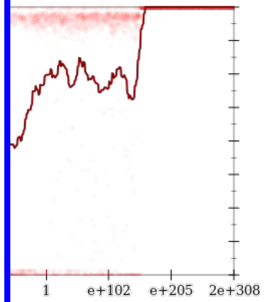


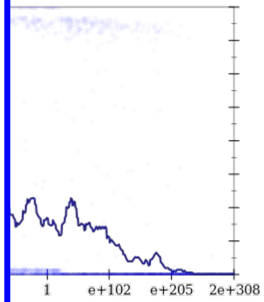
egg Herbie

$$\frac{(-b) + \sqrt{\cdots}}{\cdots}$$

if $b \leq -2.171419\ldots$
$-\frac{b}{a}$
elif $b \leq 2.980908\ldots$
$\frac{\sqrt{\mathsf{fma}(b, b, c \cdot (a \cdot -4))}\ldots}{a \cdot 2}$
elif $b \leq 3.095118\ldots$
$t_0 := 4 \cdot (a \cdot c\ldots$
$\frac{t_0 \cdot \frac{0.5}{a}}{(-b) - \sqrt{b \cdot b - t_0}}$
else :
$-\frac{c}{b}$



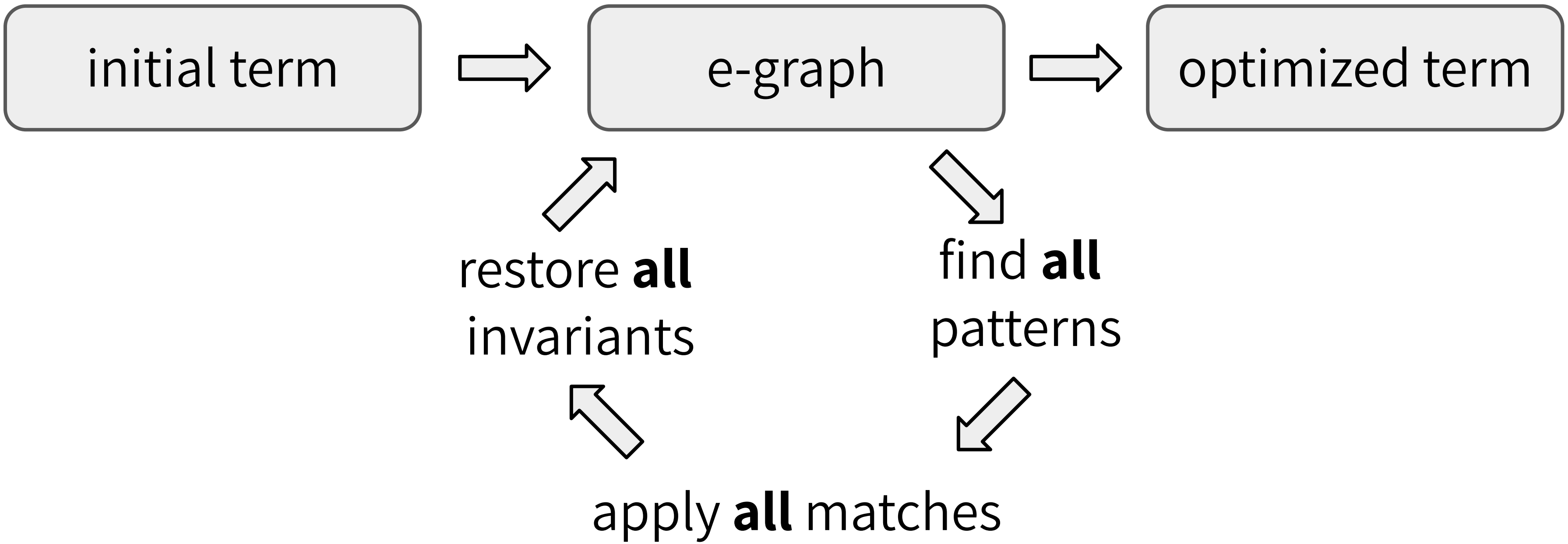




egg EqSat Toolkit [POPL 2021, Distinguished Paper]

- ✓ Deferred invariant maintenance & batching
- ❏ Relational e-matching [POPL 2022]
- ❏ E-class analyses
- ❏ Rewrite rule synthesis with Ruler [OOPSLA 2021, Distinguished Paper]
- ❏ Applications
  - ❏ 3D CAD in Szalinski, FP Accuracy in Herbie, Lib Learning in Babble, …
  - ❏ EVM simplify @ Certora, wasm JIT @ Fastly, datapath optimize @ Intel, …

egg's Equality Saturation



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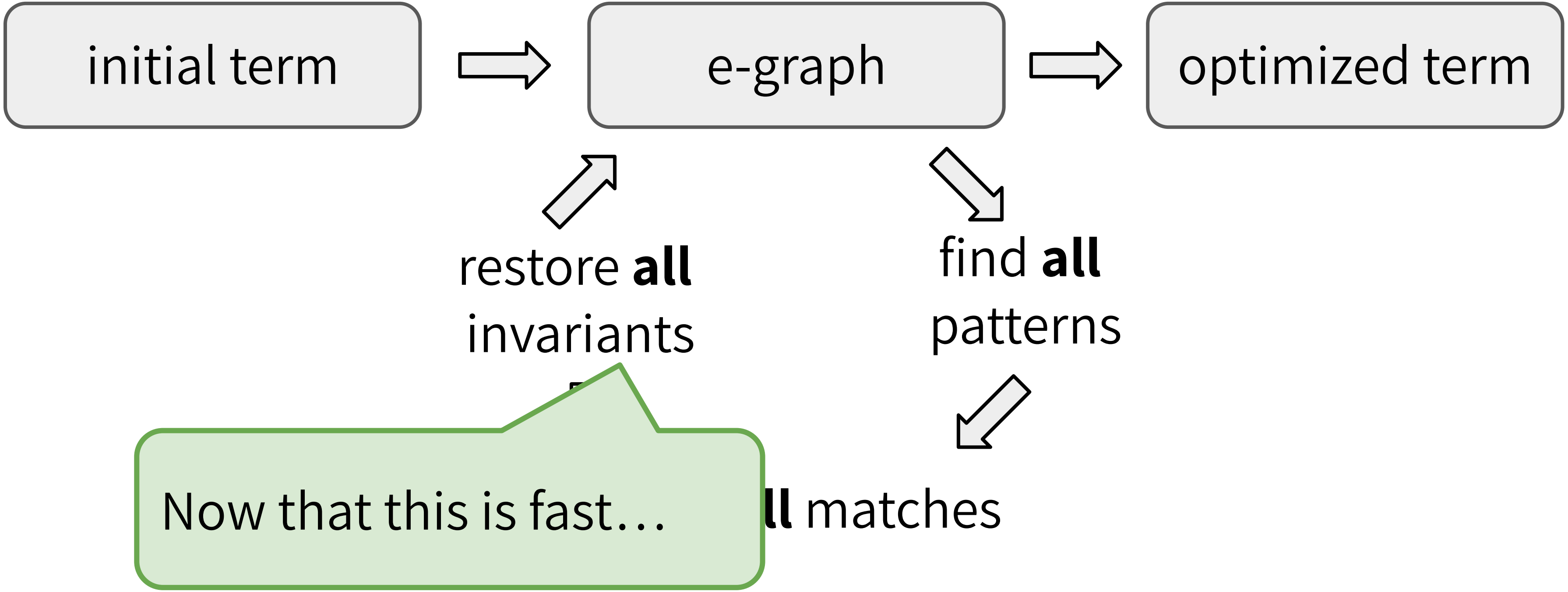
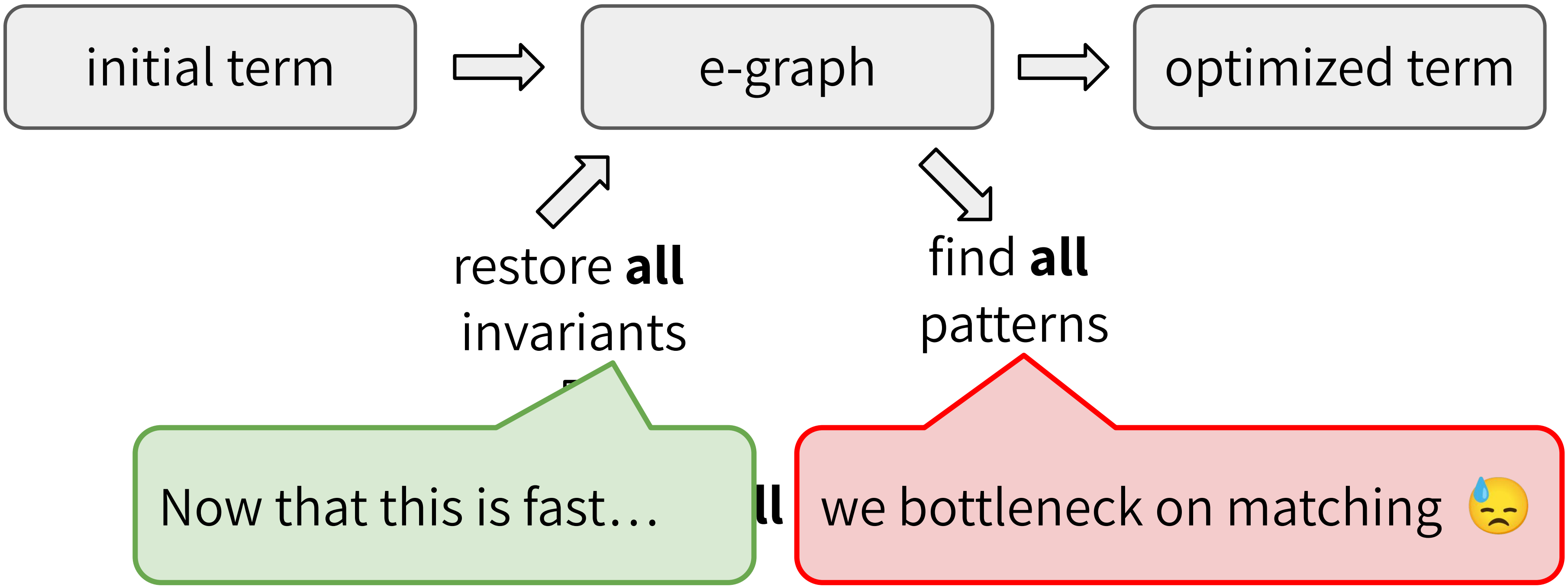


egg's Equality Saturation



E-matching: pattern matching over e-graphs

- *E-matching* : find substs from pattern variables to e-classes
- Substs guaranteed to be represented by the matched e-graph

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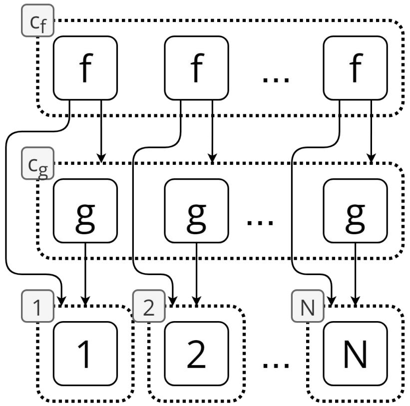

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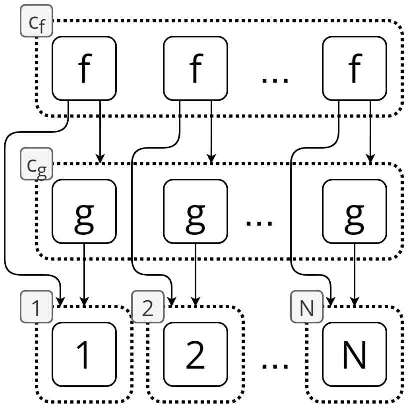
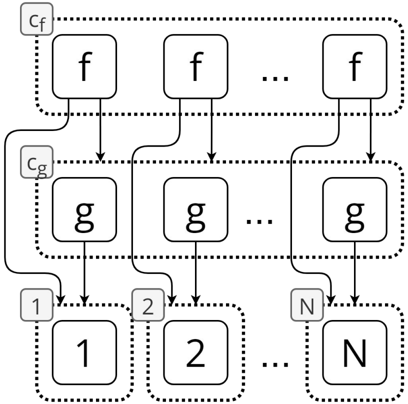


$$f(\alpha, g(\alpha)) \text{ will match } \begin{matrix} f(1, g(1)) \\ f(2, g(2)) \\ \ldots \\ f(N, g(N)) \end{matrix}, \text{ witnessed by } \begin{matrix} \{\alpha \mapsto 1\} \\ \{\alpha \mapsto 2\} \\ \ldots \\ \{\alpha \mapsto N\} \end{matrix}.$$

E-matching: pattern matching over e-graphs

- *E-matching* : find substs from pattern variables to e-classes
- Substs guaranteed to be represented by the matched e-graph



f(α, g(α)) will match

f(1, g(1))
f(2, g(2))
…
f(N, g(N))

, witnessed by

{*α* ↦ *1*}
{*α* ↦ *2*}
…
{*α* ↦ *N*}

.

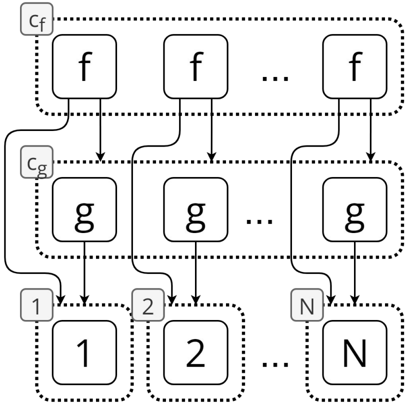
f(1, α) will match

f(1, g(1))
f(1, g(2))
…
f(1, g(N))

, witnessed by {*α* ↦ c_g}.

E-matching: pattern matching over e-graphs

- *E-matching* : find substs from pattern variables to e-classes
- Substs guaranteed to be represented by the matched e-graph
- NP-complete wrt to pattern size (Kozen 1977) 😱



f(a, g(a)) will match *f(1, g(1))*, *f(2, g(2))*, ..., *f(N, g(N))*, witnessed by *{a ↦ 1}*, *{a ↦ 2}*, ..., *{a ↦ N}*.

f(1, a) will match *f(1, g(1))*, *f(1, g(2))*, ..., *f(1, g(N))*, witnessed by *{a ↦ c_g}*.

E-matching: pattern matching over e-graphs

- *E-matching*: find substs from pattern variables to e-classes
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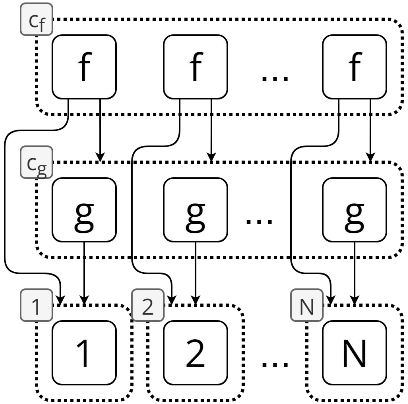
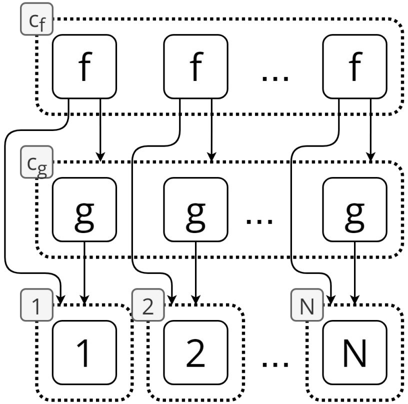


But patterns are often small…

$f(1, \alpha)$ will match $f(1, g(1))$, $f(1, g(2))$, … $f(1, g(N))$, witnessed by $\{\alpha \mapsto c_g\}$.

E-matching: pattern matching over e-graphs

- *E-matching* : find substs from pattern variables to e-classes
- Substs guaranteed to be represented by the matched e-graph
- NP-complete wrt to pattern size (Kozen 1977) 😱



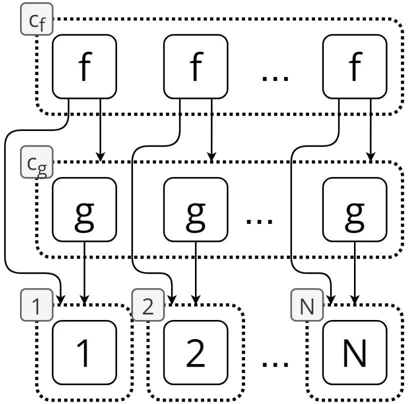
But patterns are often small…

…

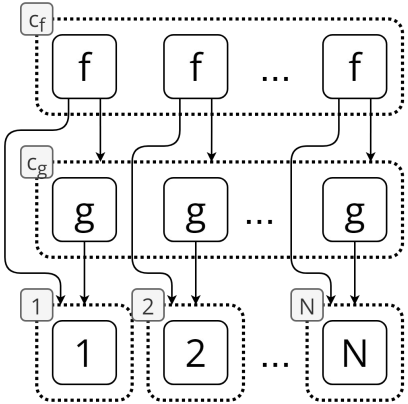
f(1, g(N))

💡 # of matches is much better metric!

Traditional e-matching via backtracking

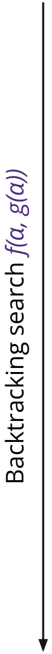


Traditional e-matching via backtracking

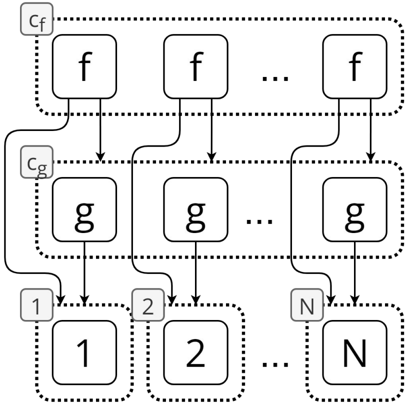


f(α, g(α))

```
for e-class c in e-graph E:
```

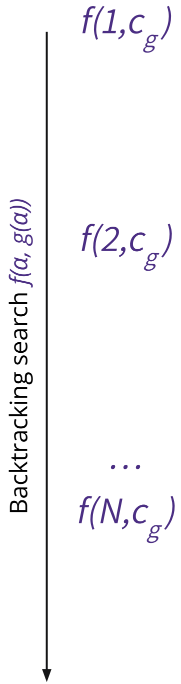


Traditional e-matching via backtracking



$$f(\alpha, g(\alpha))$$

```
for e-class c in e-graph E:
  for f-node n_1 in c:
```



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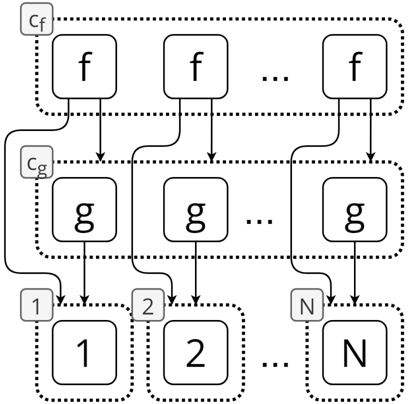
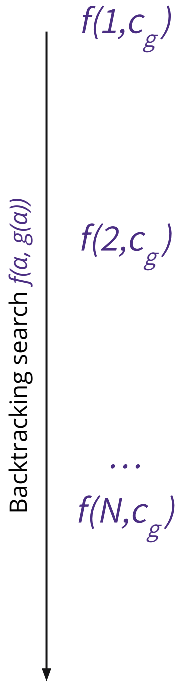
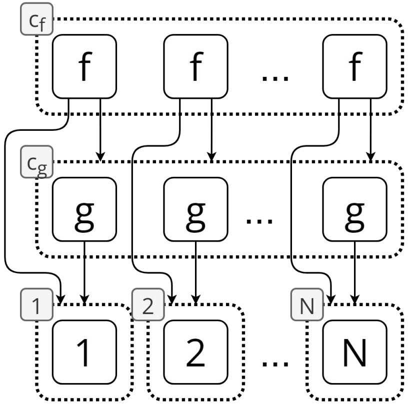


$$f(\alpha, g(\alpha))$$

```
for e-class c in e-graph E:
  for f-node n₁ in c:
    subst = {root ↦ c, α ↦ n₁.child₁}
```



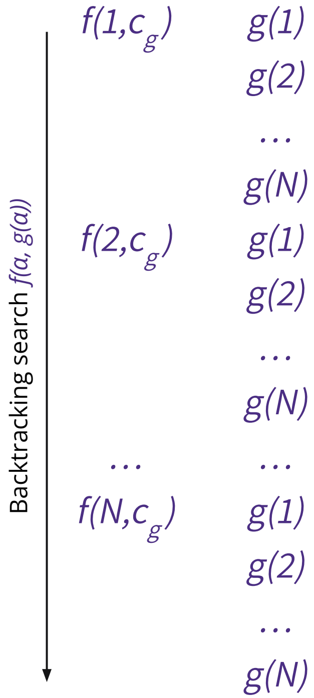
Traditional e-matching via backtracking



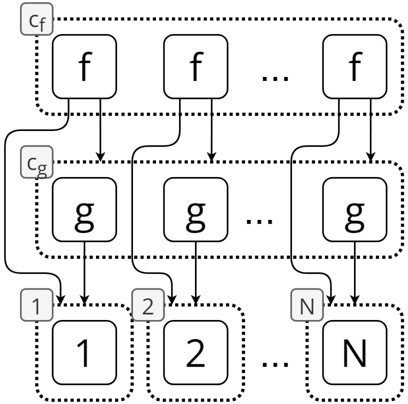
f(α, g(α))

⇩

```
for e-class c in e-graph E:
  for f-node n1 in c:
    subst = {root ↦ c, α ↦ n1.child1}
    for g-node n2 in n1.child2:
```

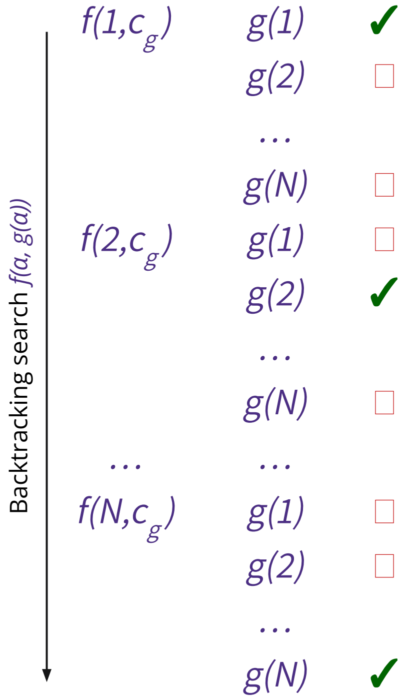


Traditional e-matching via backtracking



$$f(\alpha, g(\alpha))$$

```
for e-class c in e-graph E:
  for f-node n1 in c:
    subst = {root ↦ c, α ↦ n1.child1}
    for g-node n2 in n1.child2:
      if subst[α] = n2.child1:
```



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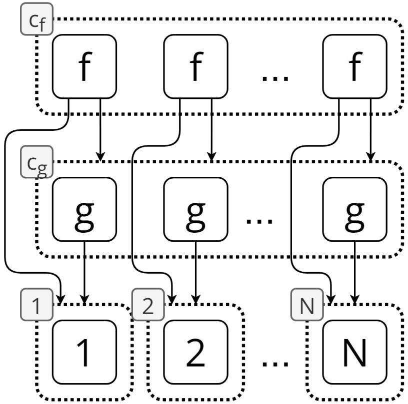
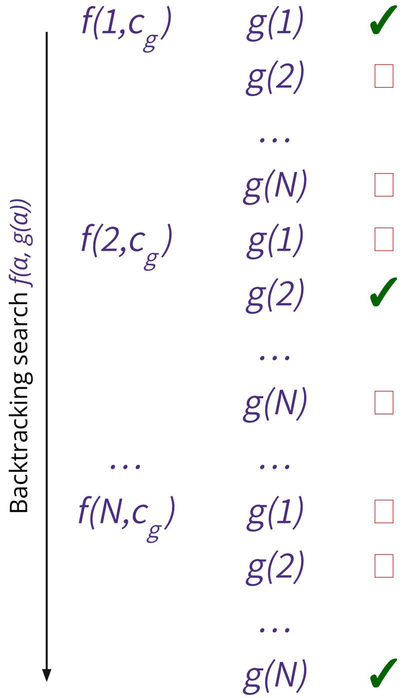


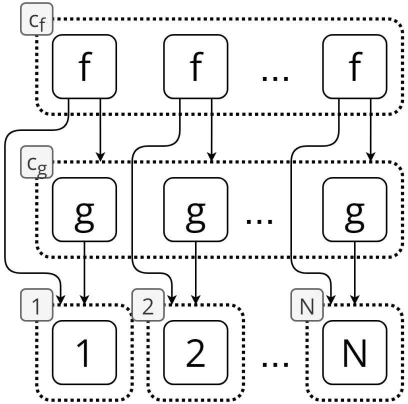
f(α, g(α))

⇩

```
for e-class c in e-graph E:
  for f-node n₁ in c:
    subst = {root ↦ c, α ↦ n₁.child₁}
    for g-node n₂ in n₁.child₂:
      if subst[α] = n₂.child₁:
        yield subst
```



Traditional e-matching via backtracking



f(α, g(α))

⇓

```
for e-class c in e-graph E:
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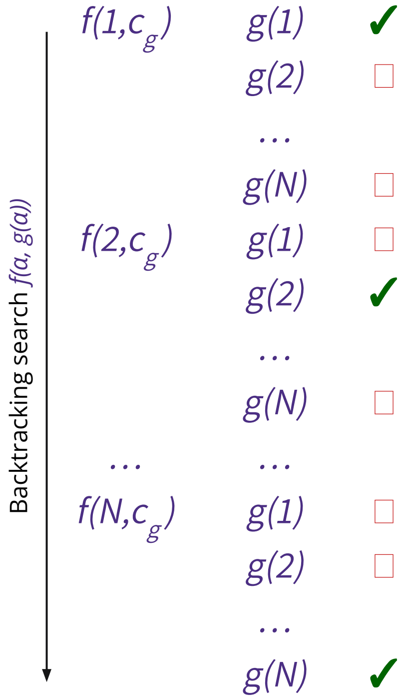
    subst = {root ↦ c, α ↦ n1.child1}
    for g-node n2 in n1.child2:
      if subst[α] = n2.child1:
```

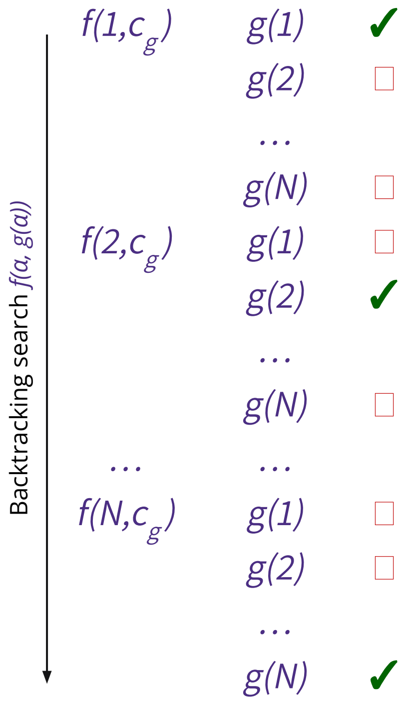
O(N^2), yet at most O(N) matches 😫



Traditional e-matching via backtracking

- Many optimizations in literature
  - custom VMs for “CSE”
  - specific patterns
  - mod-time analysis

- No data complexity bounds!



💡 Key insight: e-matching is a DB problem!

E-matching in e-graphs

Finding substitutions such that substituted terms are represented in an e-graph.

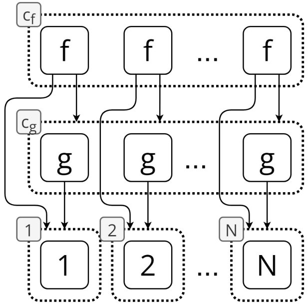
Conjunctive queries in DBs

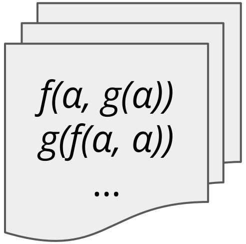
Finding substitutions such that substituted atoms are present in a relational DB.

egg’s relational e-matching

egg’s relational e-matching

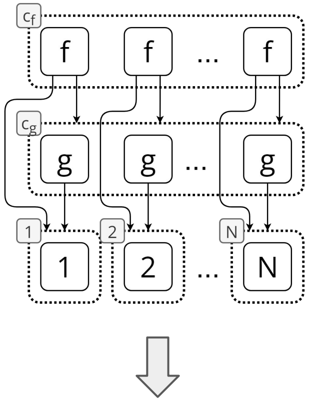
- Given e-graph + patterns





egg’s relational e-matching

- Given e-graph + patterns
- Transform e-graph to tables

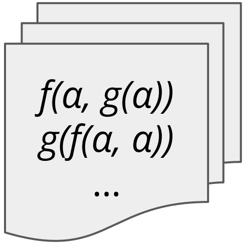


R_f

id	arg_1	arg_2
c_f	1	c_g
c_f	2	c_g
…	…	…
c_f	N	c_g

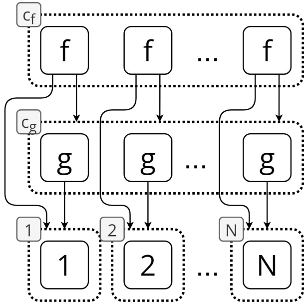
R_g

id	arg_1
c_g	1
c_g	2
…	…
c_g	N



egg's relational e-matching

- Given e-graph + patterns
- Transform e-graph to tables
- Compile patterns to queries

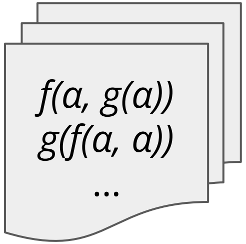


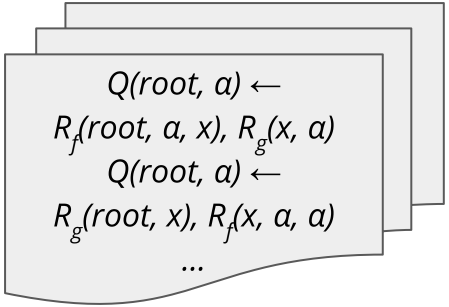
R_f

id	arg_1	arg_2
c_f	1	c_g
c_f	2	c_g
...	...	...
c_f	N	c_g

R_g

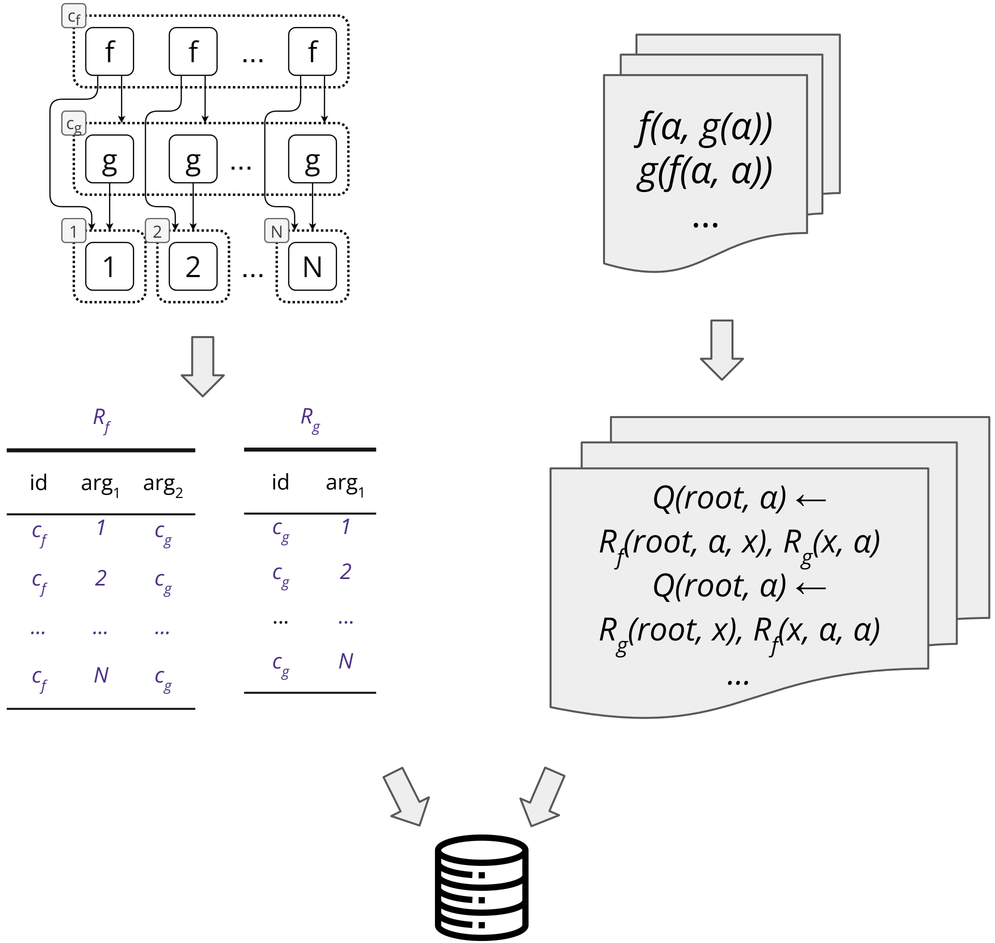
id	arg_1
c_g	1
c_g	2
...	...
c_g	N





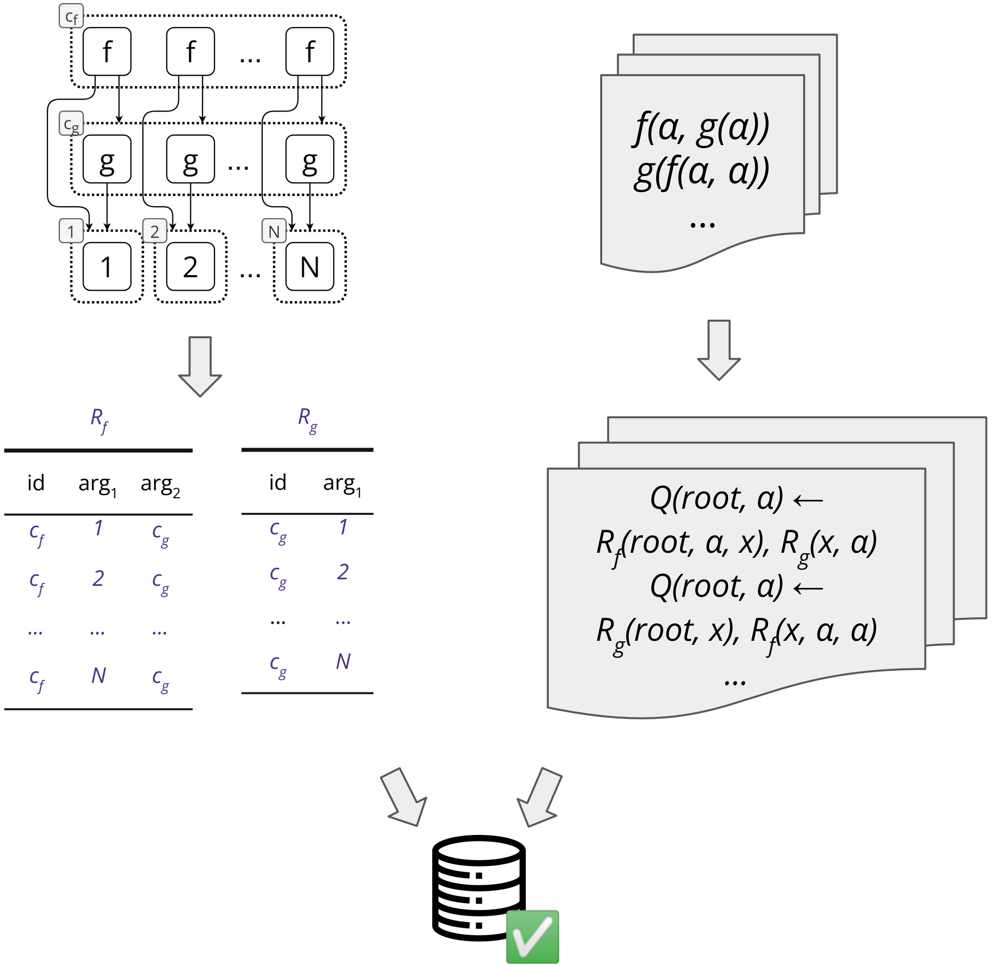
egg's relational e-matching

- Given e-graph + patterns
- Transform e-graph to tables
- Compile patterns to queries
- Use DB query engine to e-match!

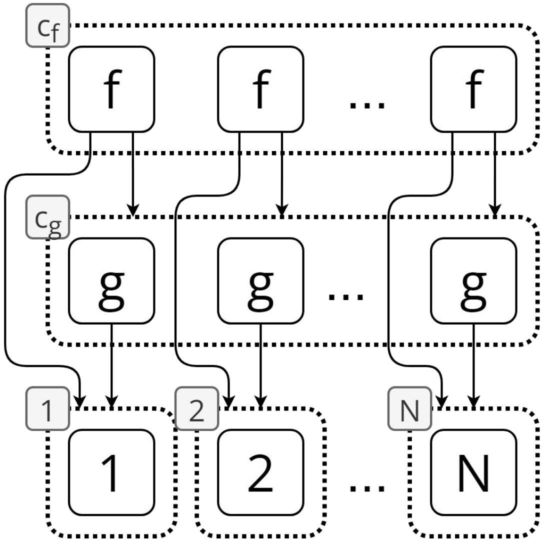


egg’s relational e-matching

- Given e-graph + patterns
- Transform e-graph to tables
- Compile patterns to queries
- Use DB query engine to e-match!
- Derive bounds from DB theory!



E-graphs as tables (relational DBs)



R_f

id	arg_1	arg_2
c_f	*1*	c_g
c_f	*2*	c_g
…	…	…
c_f	*N*	c_g

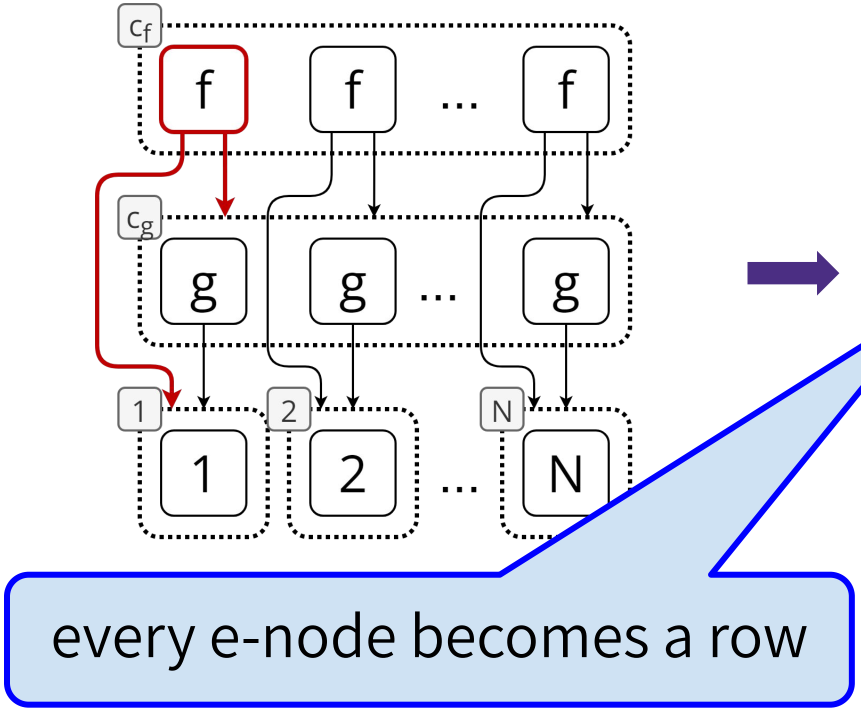
R_g

id	arg_1
c_g	*1*
c_g	*2*
…	…
c_g	*N*

$R_{i=1...N}$

id
i

E-graphs as tables (relational DBs)



R_f

id	arg_1	arg_2
c_f	1	c_g
c_f	2	c_g
...	...	...
c_f	N	c_g

R_g

id	arg_1
c_g	1
c_g	2
...	...
c_g	N

$R_{i=1...N}$

id
i

E-match patterns as conjunctive queries

$f(\alpha, g(\alpha))$

E-match patterns as conjunctive queries

$f(\alpha, g(\alpha))$

$$Q(root, \alpha) \leftarrow R_f(root, \alpha, x), R_g(x, \alpha)$$

E-match patterns as conjunctive queries

$f(\alpha, g(\alpha))$

$$Q(root, \alpha) \leftarrow$$
$$R_f(root, \alpha, x), R_g(x, \alpha)$$

```
ind = {}
for (x, α) in R_g: # build index
  ind.insert((x, α))
```

build hash

$R_g(c_g, 1)$
$R_g(c_g, 2)$
...
$R_g(c_g, N)$

E-match patterns as conjunctive queries

$f(\alpha, g(\alpha))$

$$Q(root, \alpha) \leftarrow R_f(root, \alpha, x), R_g(x, \alpha)$$

```
ind = {}
for (x, α) in R_g: # build index
  ind.insert((x, α))
for (root, α, x) in R_f: # probe
  if (α, x) in ind:
    yield {root ↦ root, α ↦ α}
```

build hash

$R_g(c_g, 1)$
$R_g(c_g, 2)$
…
$R_g(c_g, N)$

probe

$R_f(c_f, 1, c_f)$ ✓
$R_f(c_f, 2, c_f)$ ✓
…
$R_f(c_f, N, c_f)$ ✓

Why is relational e-matching faster?

$f(\alpha, g(\alpha))$

Enum all terms of shape $f(\alpha, g(\beta))$

Check if $\alpha = \beta$ only before yielding

$Q(root, \alpha) \leftarrow$
$R_f(root, \alpha, x), R_g(x, \alpha)$

Build indices on both α and x.

Only enum terms where constraints on both x and α are satisfied.

structural constraints

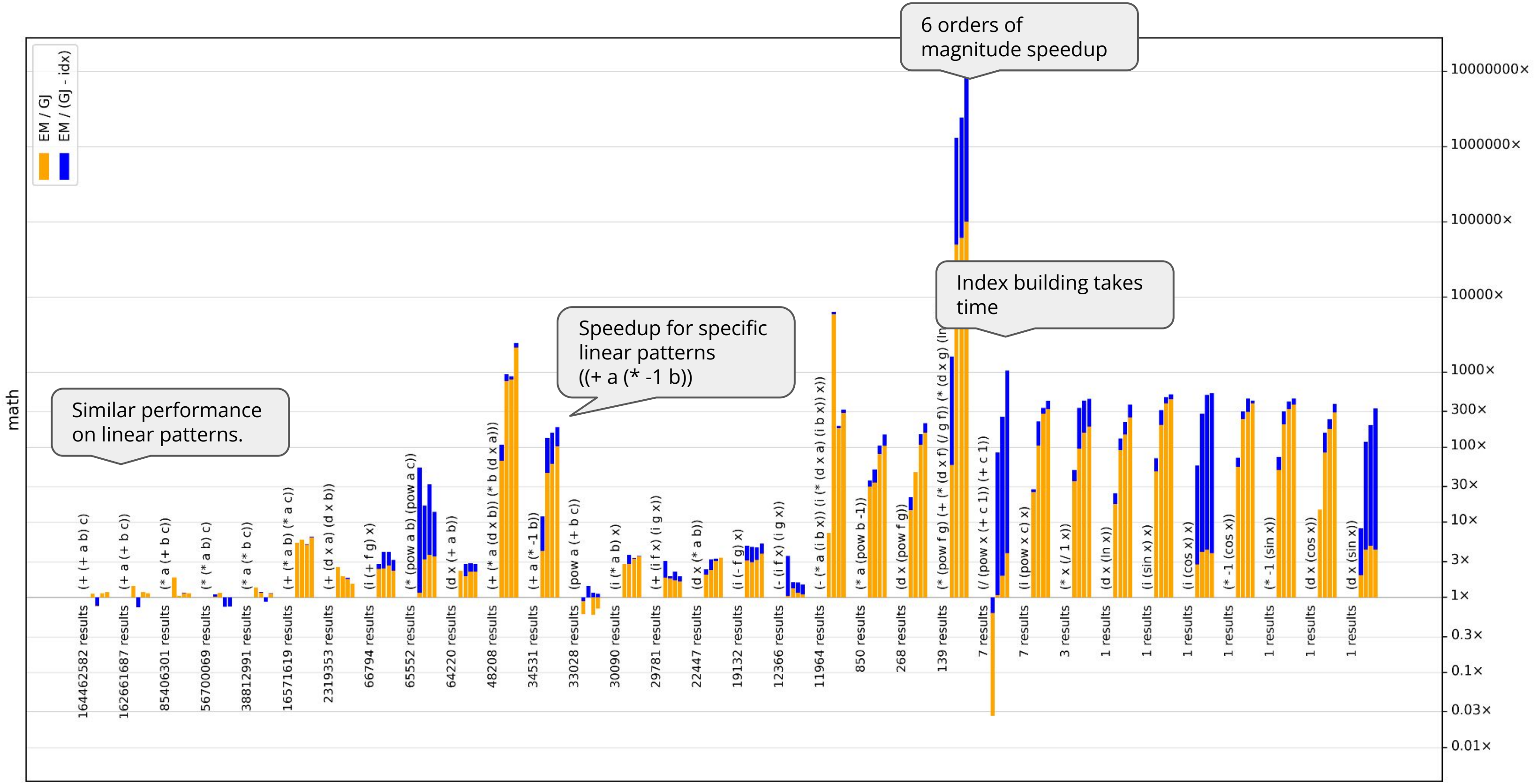
equality constraints

Data complexity results (see paper)

THEOREM 9. *Relational e-matching is worst-case optimal; that is, fix a pattern p, let $M(p, E)$ be the set of substitutions yielded by e-matching on an e-graph E with N e-nodes, relational e-matching runs in time $O(\max_E(|M(p, E)|))$.*

THEOREM 10. *Fix an e-graph E with N e-nodes that compiles to a database I, and a fix pattern p that compiles to conjunctive query $Q(\overline{X}) \leftarrow R_1(\overline{X_1}), \ldots, R_m(\overline{X_m})$. Relational e-matching p on E runs in time $O\left(\sqrt{|Q(I)| \times \Pi_i |R_i|}\right) \leq O\left(\sqrt{|Q(I)| \times N^m}\right)$.*

Relational e-matching : asymptotic speedup



New Capabilities: *Multi-patterns*

```
x = matmul(a, b),
y = matmul(a, c)
```

➡

```
x = split1(matmul(a,concat(b, c))),
y = split2(matmul(a,concat(b, c)))
```

New Capabilities: *Multi-patterns*

```
x = matmul(a, b),
y = matmul(a, c)
```

search for two patterns *anywhere* in the e-graph

➡

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search for two patterns *anywhere* in the e-graph

➡

```
x = split1(matmul(a,concat(b, c))),
y = split2(matmul(a,concat(b, c)))
```

perform two merges, each on a separate e-class!

egg EqSat Toolkit [POPL 2021, Distinguished Paper]

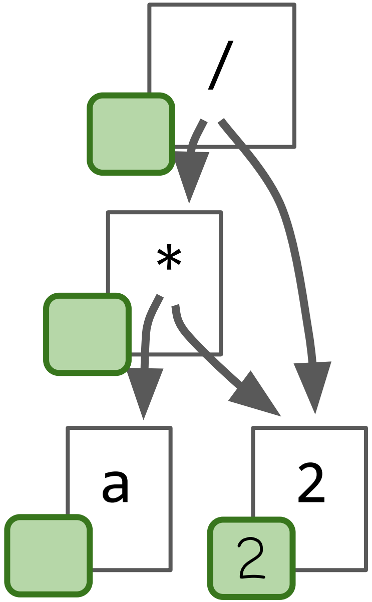
- ✓ Deferred invariant maintenance & batching
- ✓ Relational e-matching [POPL 2022]
- ❏ E-class analyses
- ❏ Rewrite rule synthesis with Ruler [OOPSLA 2021, Distinguished Paper]
- ❏ Applications
  - ❏ 3D CAD in Szalinski, FP Accuracy in Herbie, Lib Learning in Babble, …
  - ❏ EVM simplify @ Certora, wasm JIT @ Fastly, datapath optimize @ Intel, …

Syntactic rewriting is not enough…

- How many rules do we need for constant folding?
  - `2 + 2 → 4,  3 + 4 → 6,  4 + 6 → 10, …` *a lot!*
- What about satisfying guards for conditional rules?
  - `x / x → 1` only ok if `x <> 0`
- In general, many optimizations depend on analyses!
  - nullability, tensor shape, intervals, free variables, …

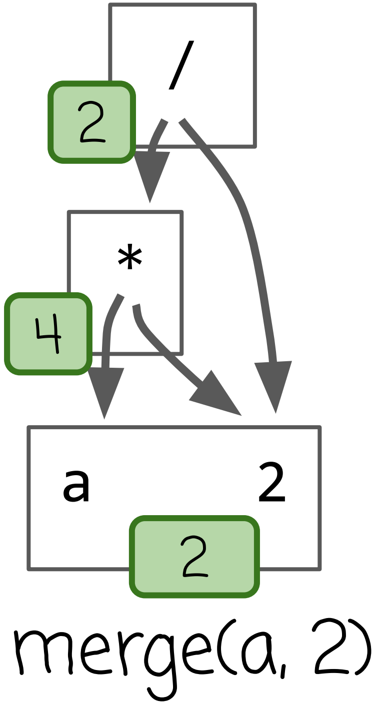
Constant folding

- Option<Number> per eclass
- try to eval new e-nodes
- Option “or” on merge



Constant folding

- Option<Number> per eclass
- try to eval new e-nodes
- Option “or” on merge
- it propagates up!



E-class analyses

- One fact per e-class from a join-semilattice D
- make(n) $\rightarrow d_c$
  - make a new analysis value for a new e-node
- join(d_{c1}, d_{c2}) $\rightarrow d_c$
  - combine two analysis values
- modify(c) $\rightarrow$ c’
  - change the e-class (optionally)

E-class analysis invariant

for each e-class

fixed point

$$\forall c \in G. \quad d_c = \bigvee_{n \in c} \mathrm{make}(n) \quad \text{and} \quad \mathrm{modify}(c) = c$$

Analysis data is LUB
(lattice properties)

Program analysis modulo equivalence

- Tightest summary over all equivalent represented terms!

To demonstrate an advantage of this approach, consider the following example, for $x \in [0, 1], y \in [1, 2]$, where the following concrete-equivalences are discovered via rewriting:

$$1 - \frac{2y}{x+y} \qquad \in \left[-3, \frac{1}{3}\right] \qquad (5)$$

$$\cong \frac{x-y}{x+y} \qquad \in [-2, 0] \qquad (6)$$

$$\cong \frac{2x}{x+y} - 1 \qquad \in [-1, 1]. \qquad (7)$$

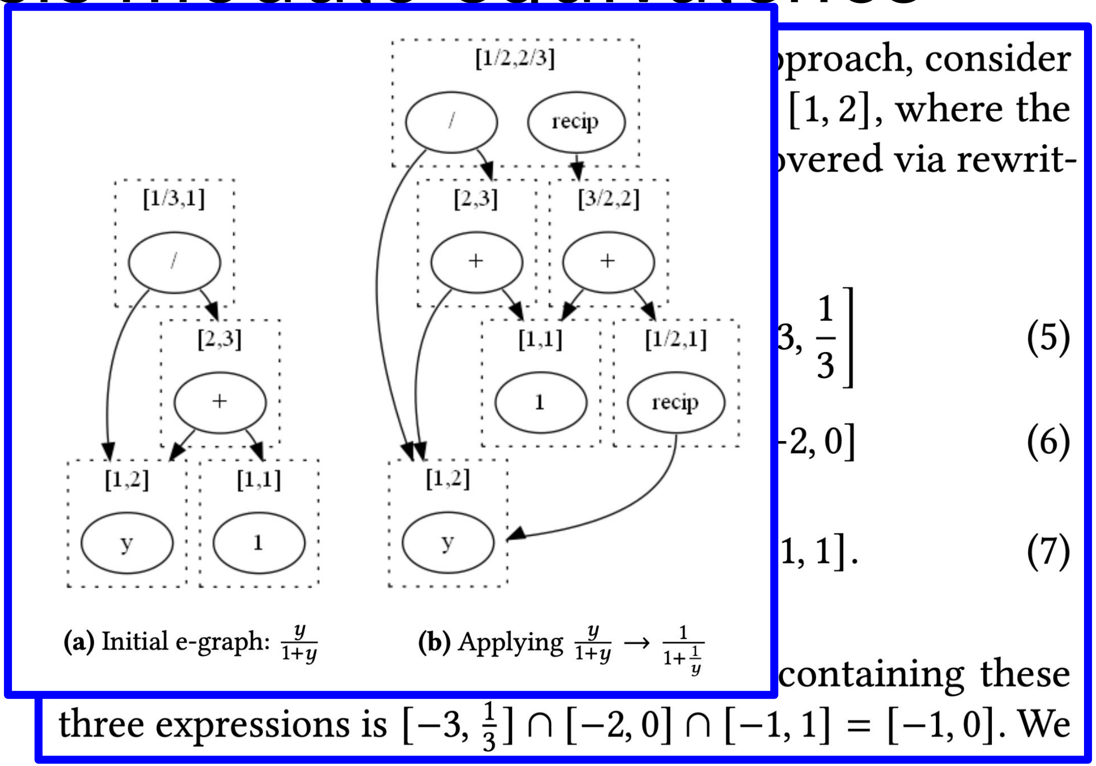
The interval associated with the e-class containing these three expressions is $[-3, \frac{1}{3}] \cap [-2, 0] \cap [-1, 1] = [-1, 0]$. We

Sam Coward et al. (2022)

Program analysis modulo equivalence

- Tightest summary over all equivalent represented terms!

- *Virtuous cycle:*
  facts enable rewrites,
  rewrites improve facts!



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egg EqSat Toolkit [POPL 2021, Distinguished Paper]

- ✓ Deferred invariant maintenance & batching
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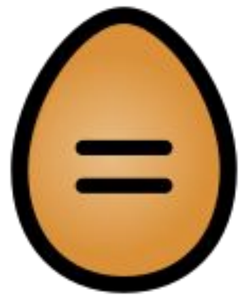
🐓 & 

🐓 & 🥚

- EqSat and egg can only be as good as user's rules…

🐓 & 🥚

- EqSat and egg can only be as good as user's rules…

Where do rules come from?

- Typically hand written by experts
- Time consuming, often takes years
- Too few / too many / unsound rules

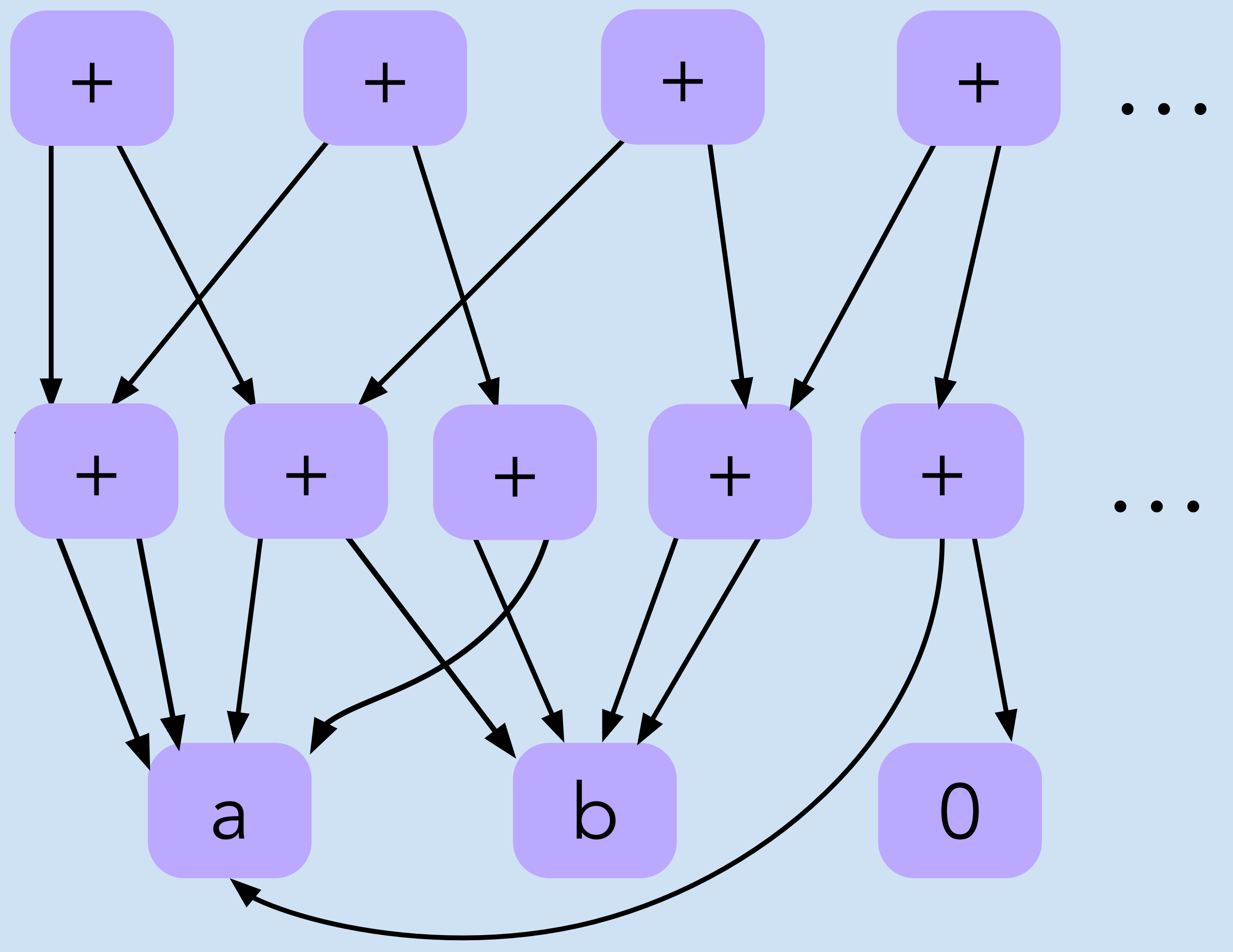
A 3-step approach for inferring rewrite rules

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-step approach for inferring rewrite rules

Enumerate terms from a grammar

a, b, 0, +, …



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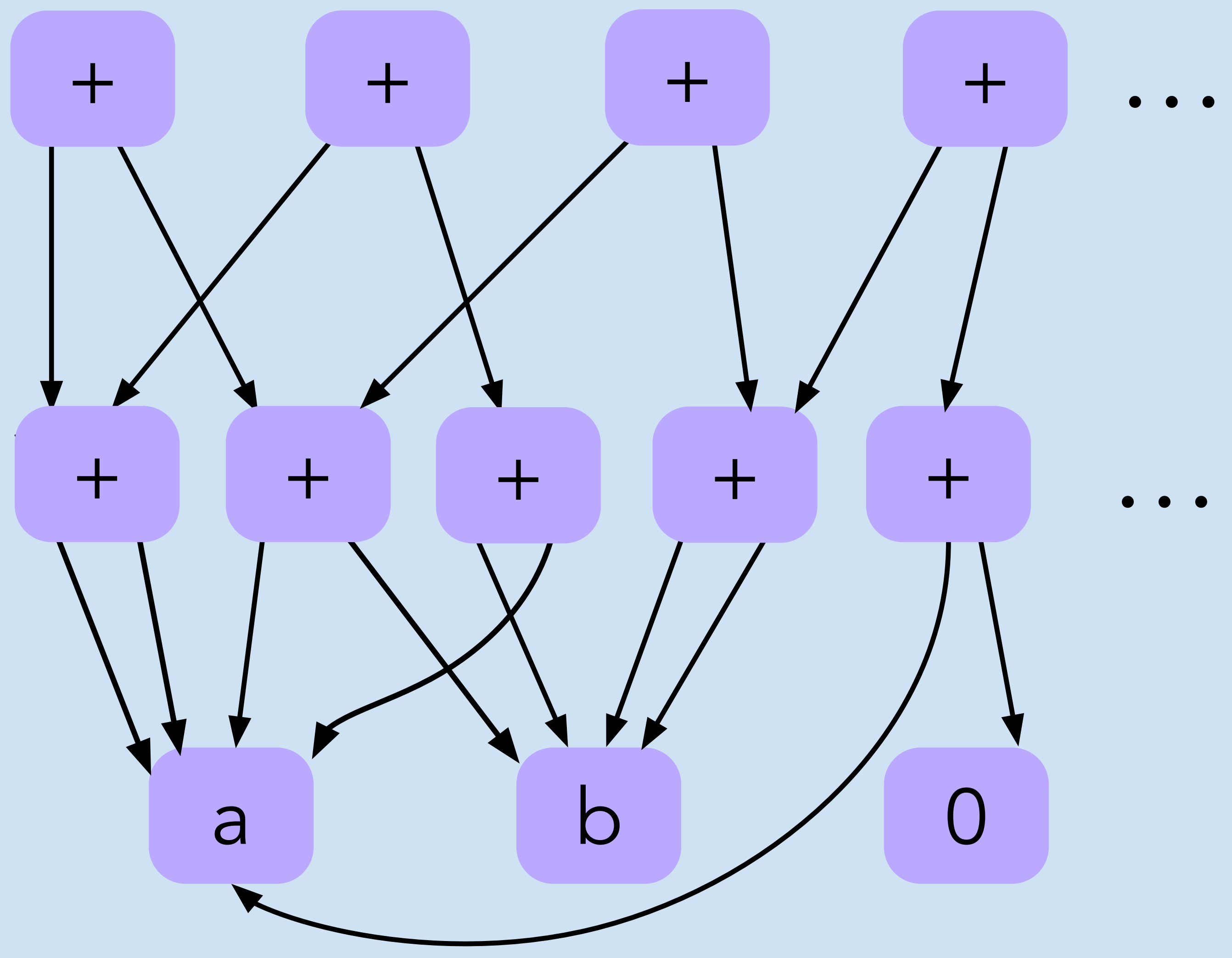
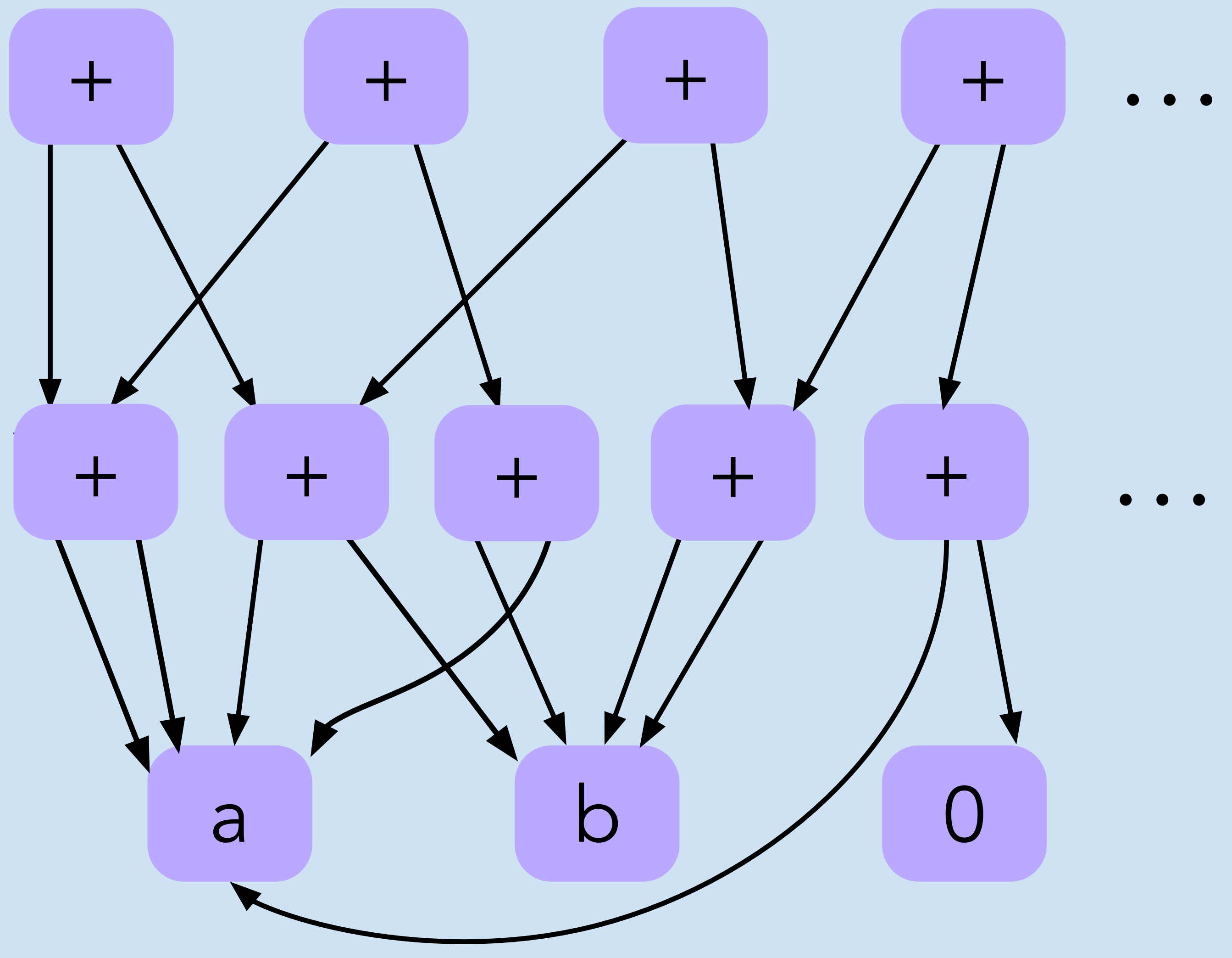


Find candidates: interpret over concrete inputs

“Fingerprints”



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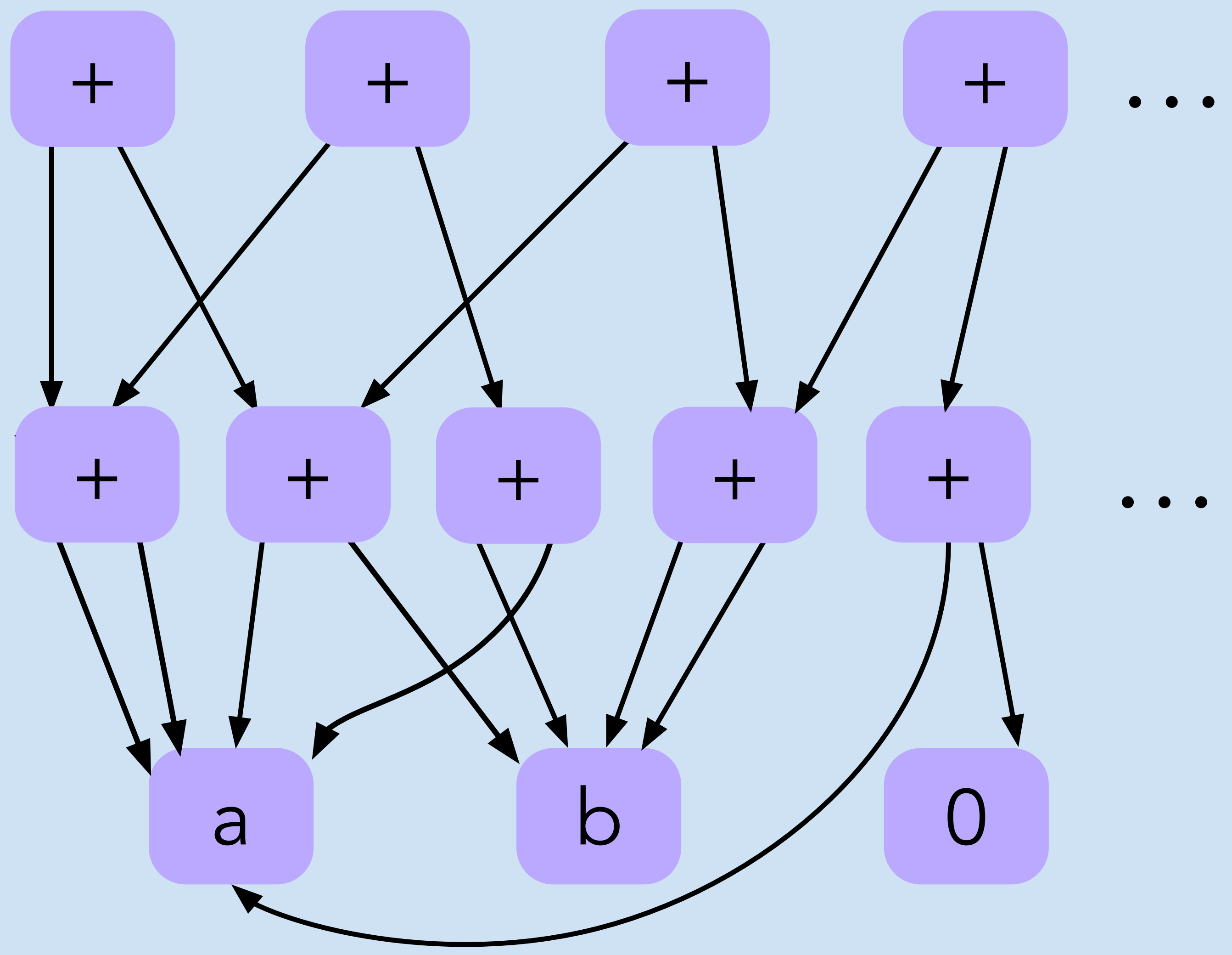

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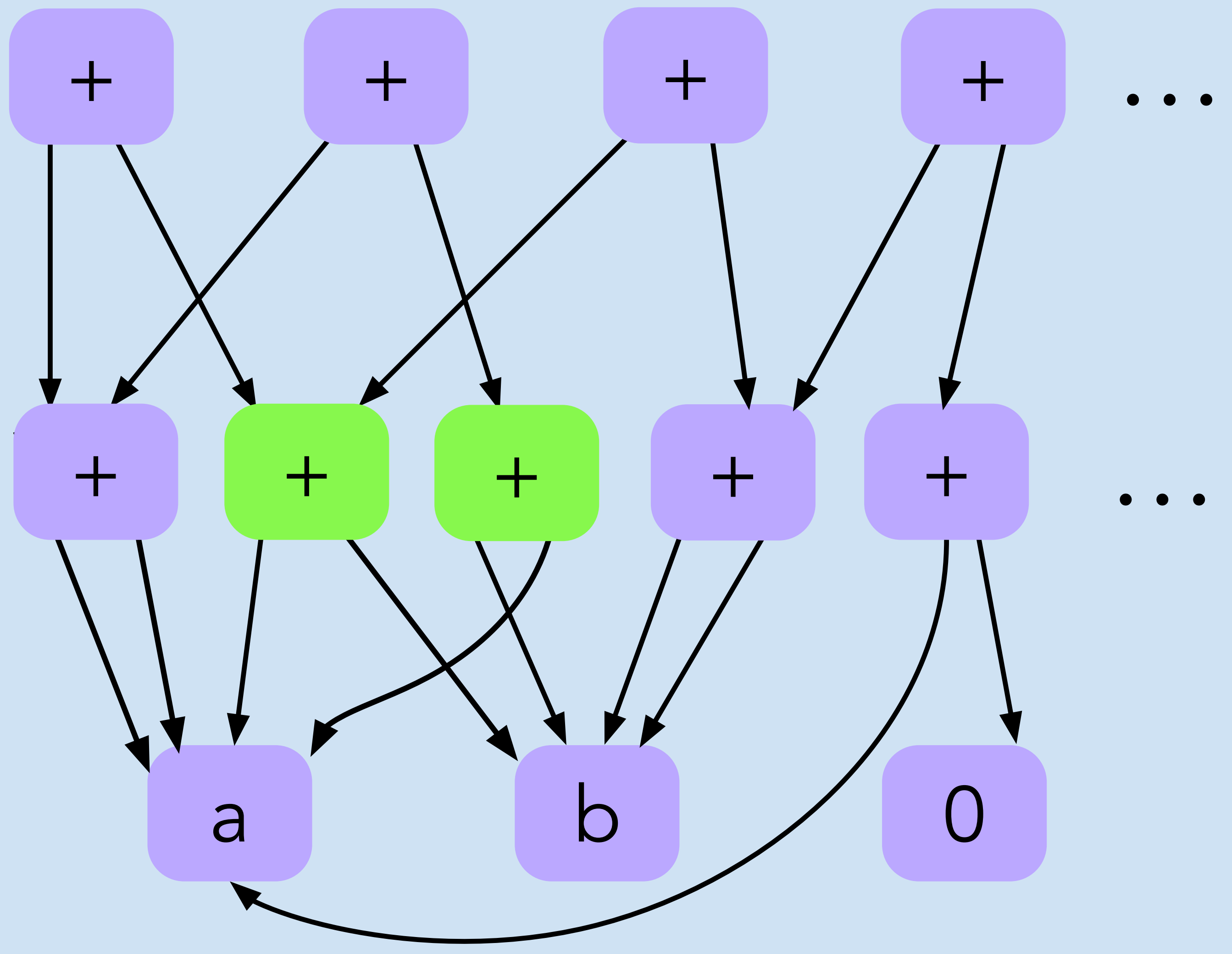


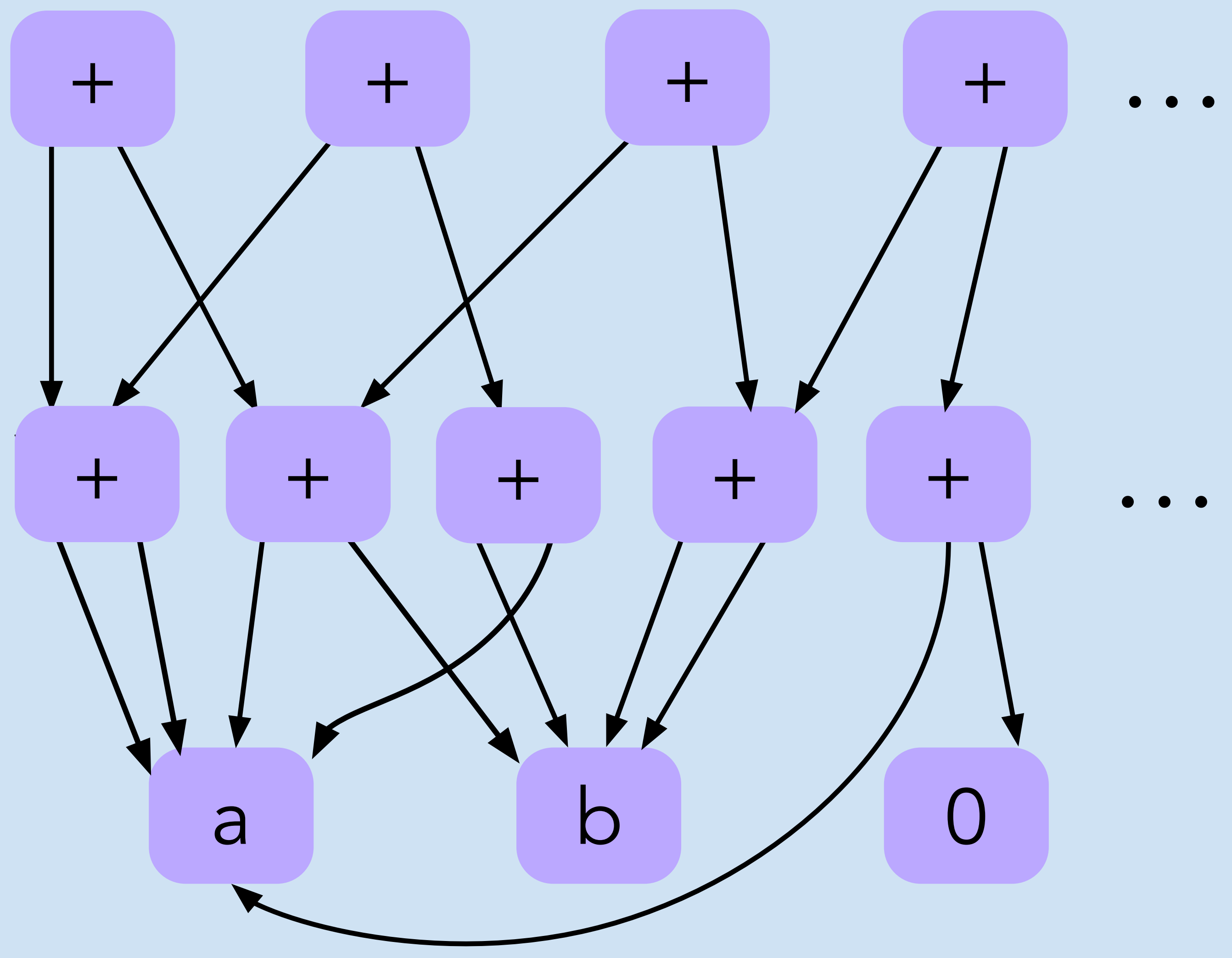
(x + y) ↔ (y + x)

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-step approach for inferring rewrite rules

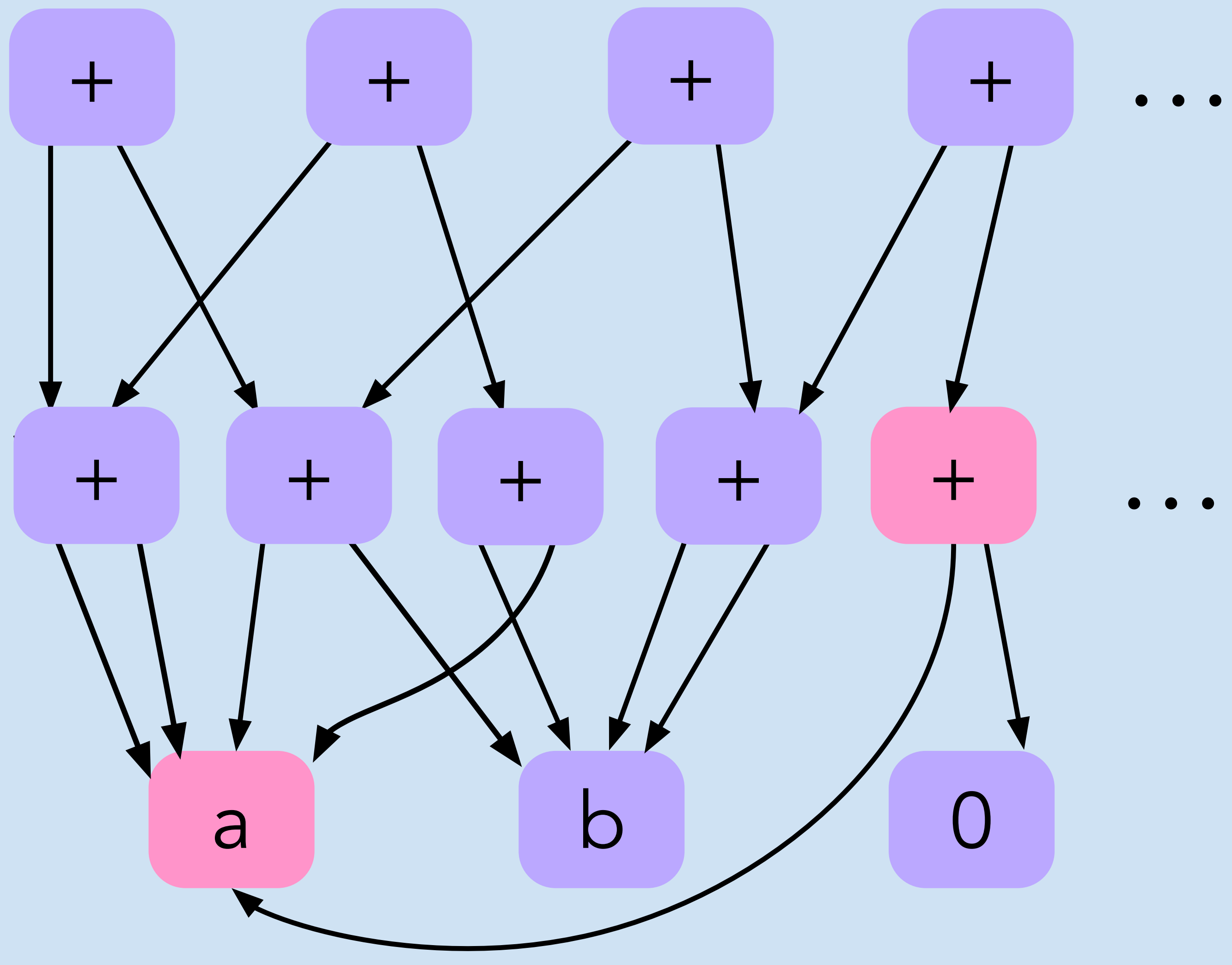
Enumerate terms from a grammar

a, b, 0, +, …



Find candidates: interpret over concrete inputs

"Fingerprints"



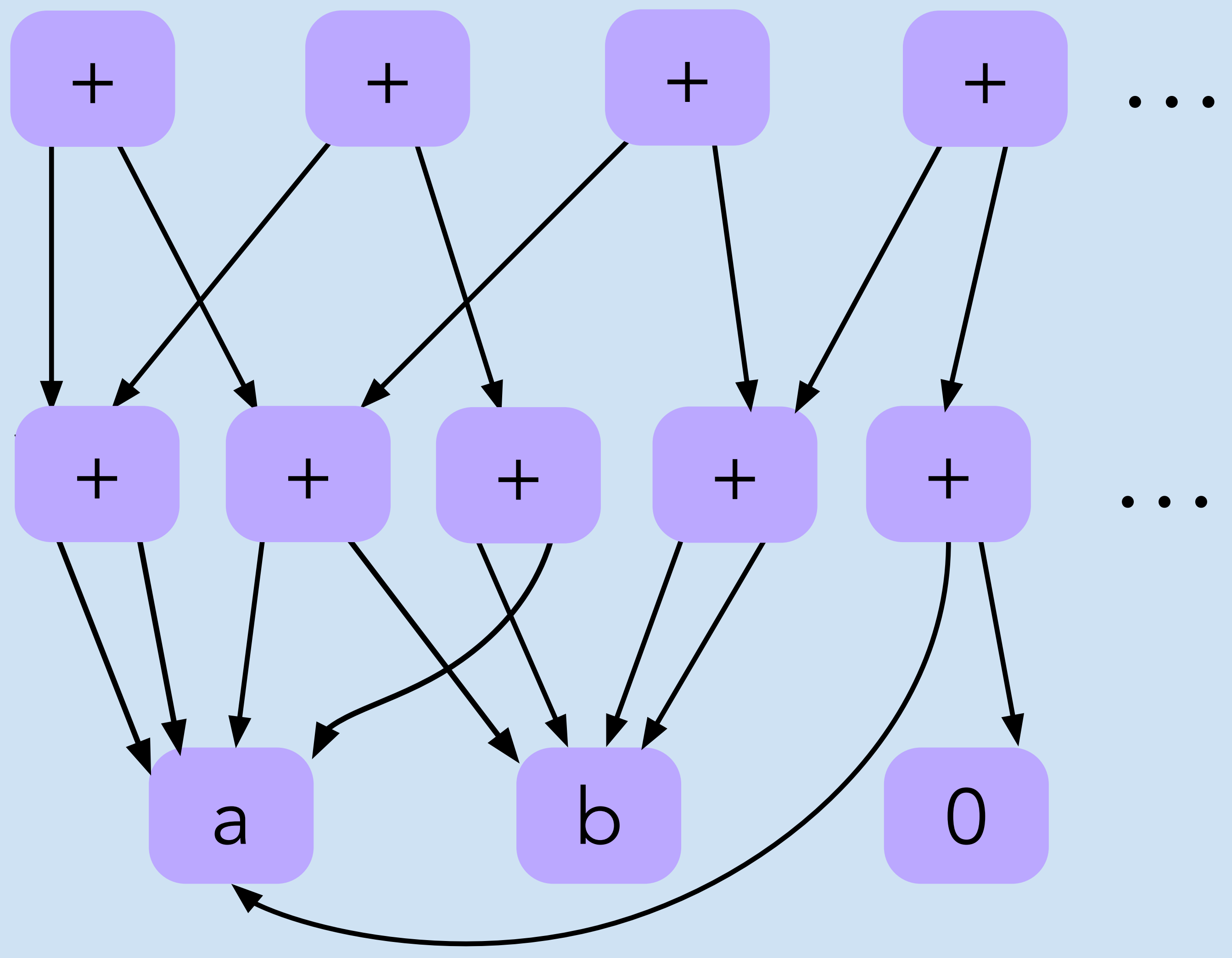
(x + 0) ⟷ x

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-step approach for inferring rewrite rules

Enumerate terms from a grammar

a, b, 0, +, …



Find candidates: interpret over concrete inputs

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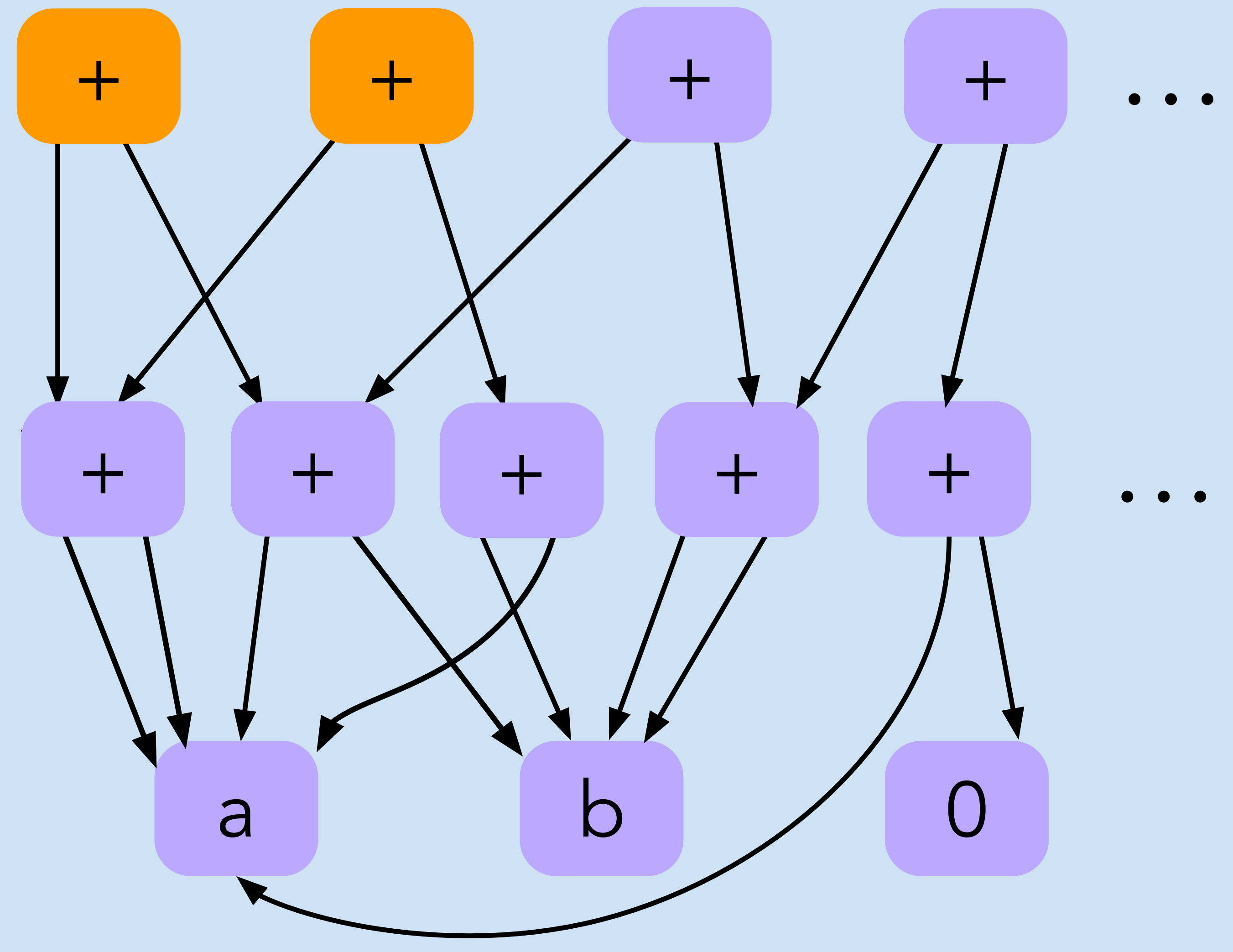


(x + x) + (x + y)

↕

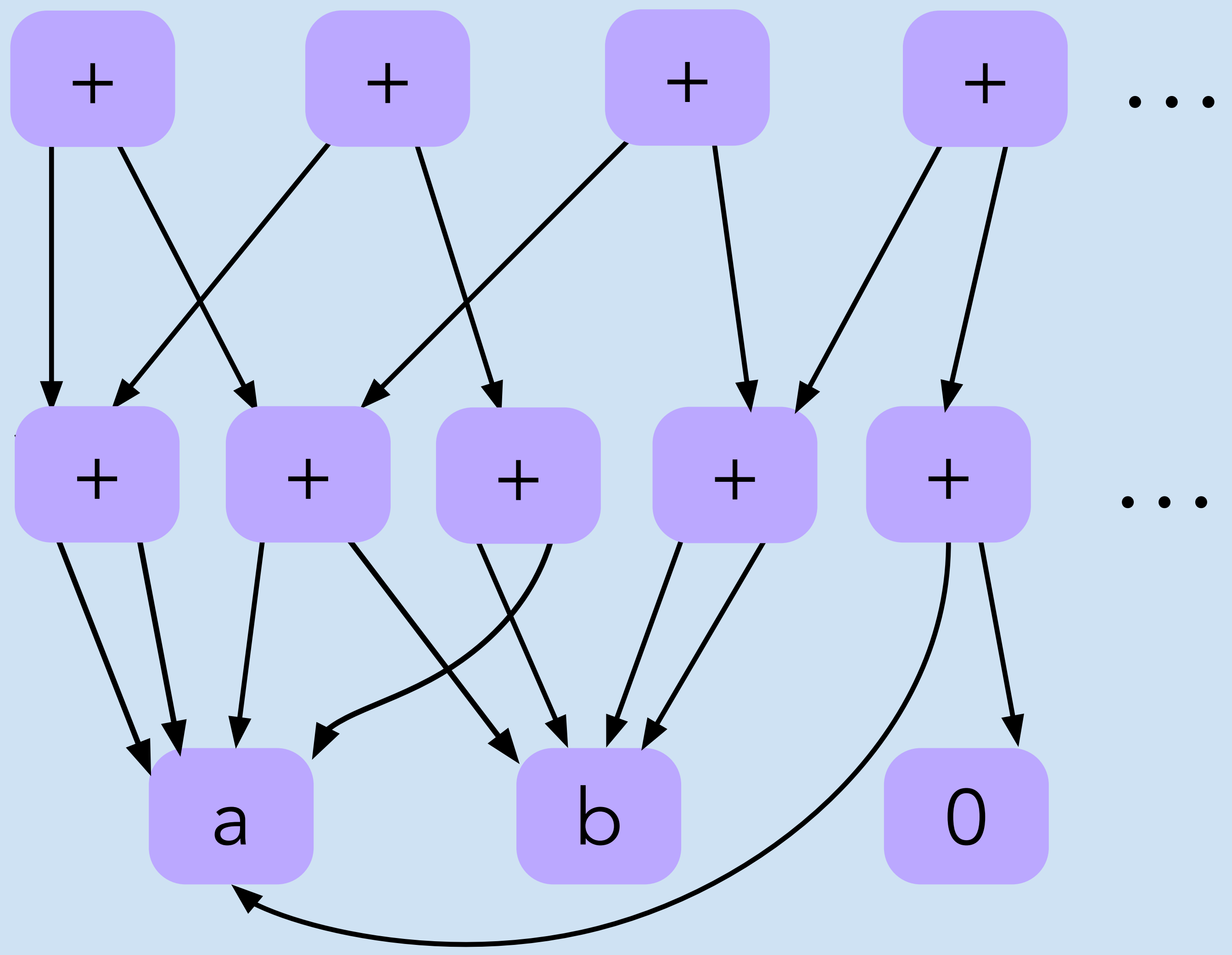
(x + x) + (y + x)

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-step approach for inferring rewrite rules

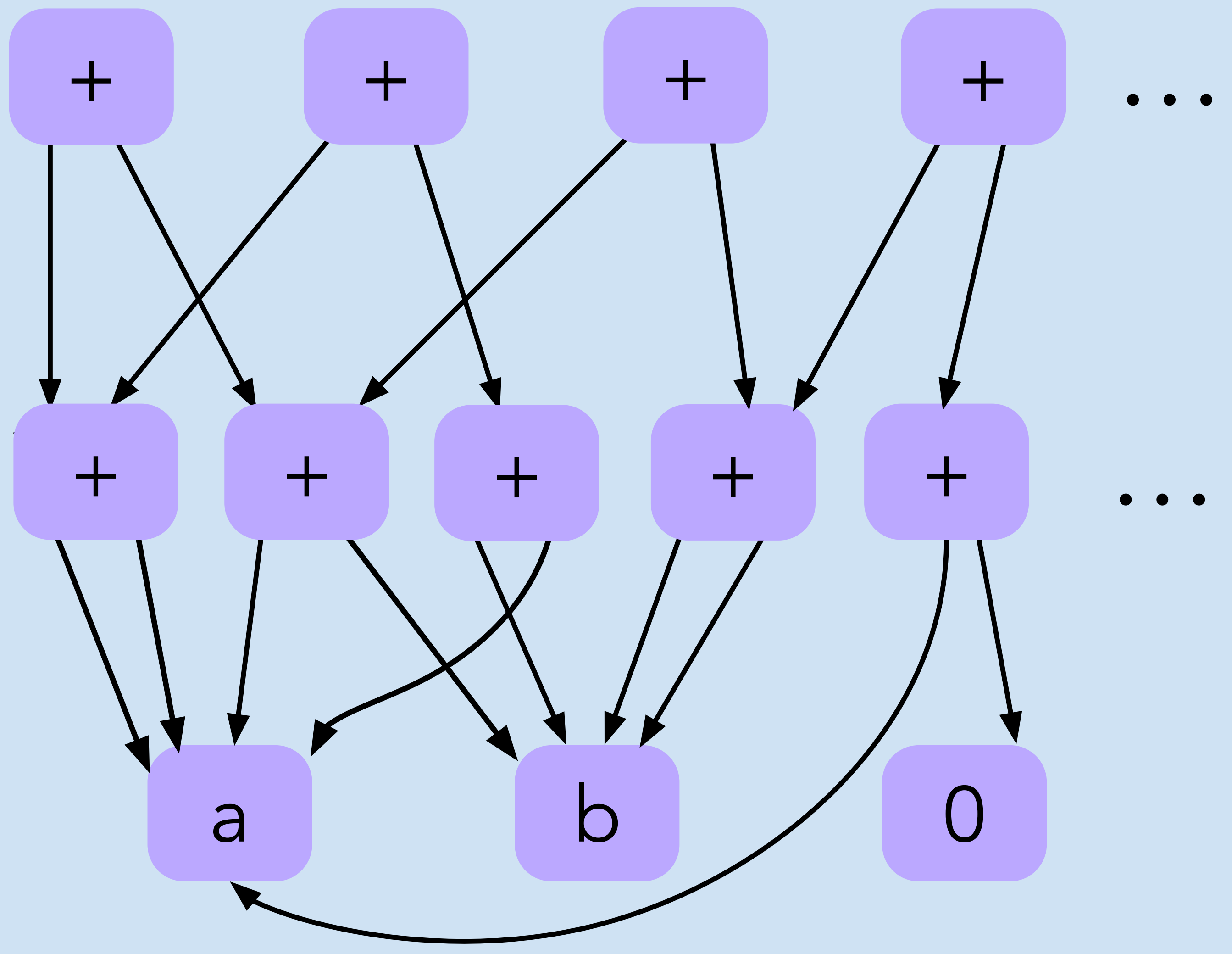
Enumerate terms from a grammar

a, b, 0, +, …



Find candidates: interpret over concrete inputs

 "Fingerprints"



Filter candidates to get final ruleset

Remove redundant rules

x + 0 ⟷ 0 + x

y + 0 ⟷ 0 + y

x + y ⟷ y + x

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-step approach for inferring rewrite rules

Enumerate terms from a grammar

Exponentially many terms!

Find candidates: interpret over concrete inputs

Too many candidates, some potentially unsound!

Filter candidates to get final ruleset

Hard to find a small, useful ruleset

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

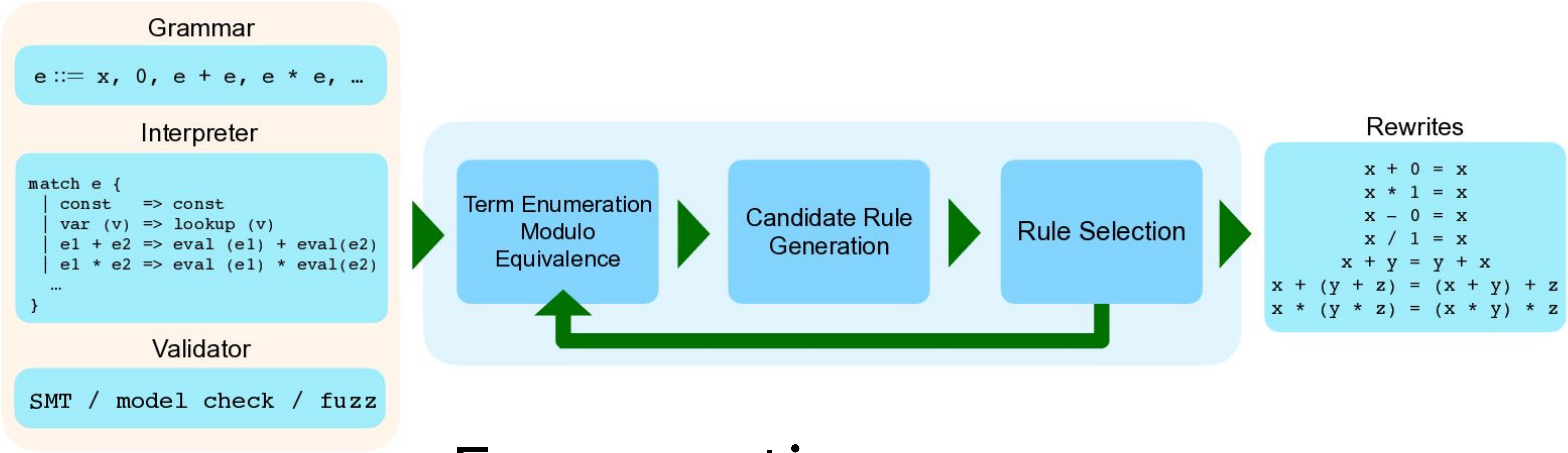
A 3-step approach for inferring rewrite rules

Inferring ***Small, Useful*** Rulesets ***Faster*** using Equality Saturation!

Equality Saturation for not just applying rewrites, but also ***inferring*** them!

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

Ruler

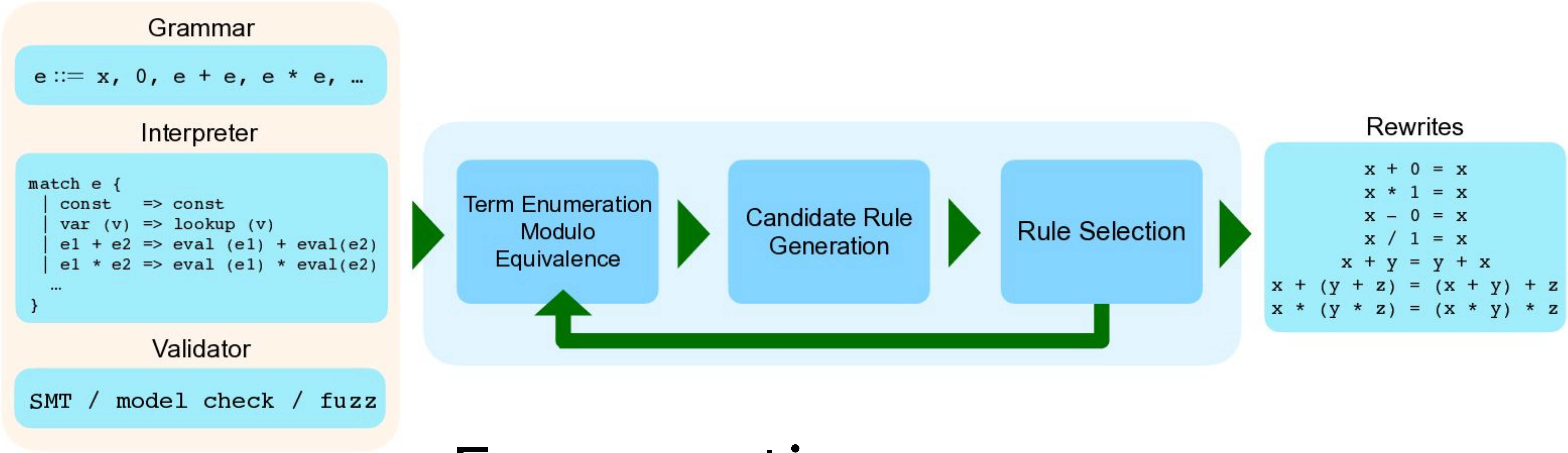


Enumeration

Candidate Generation

Rule Selection

Ruler



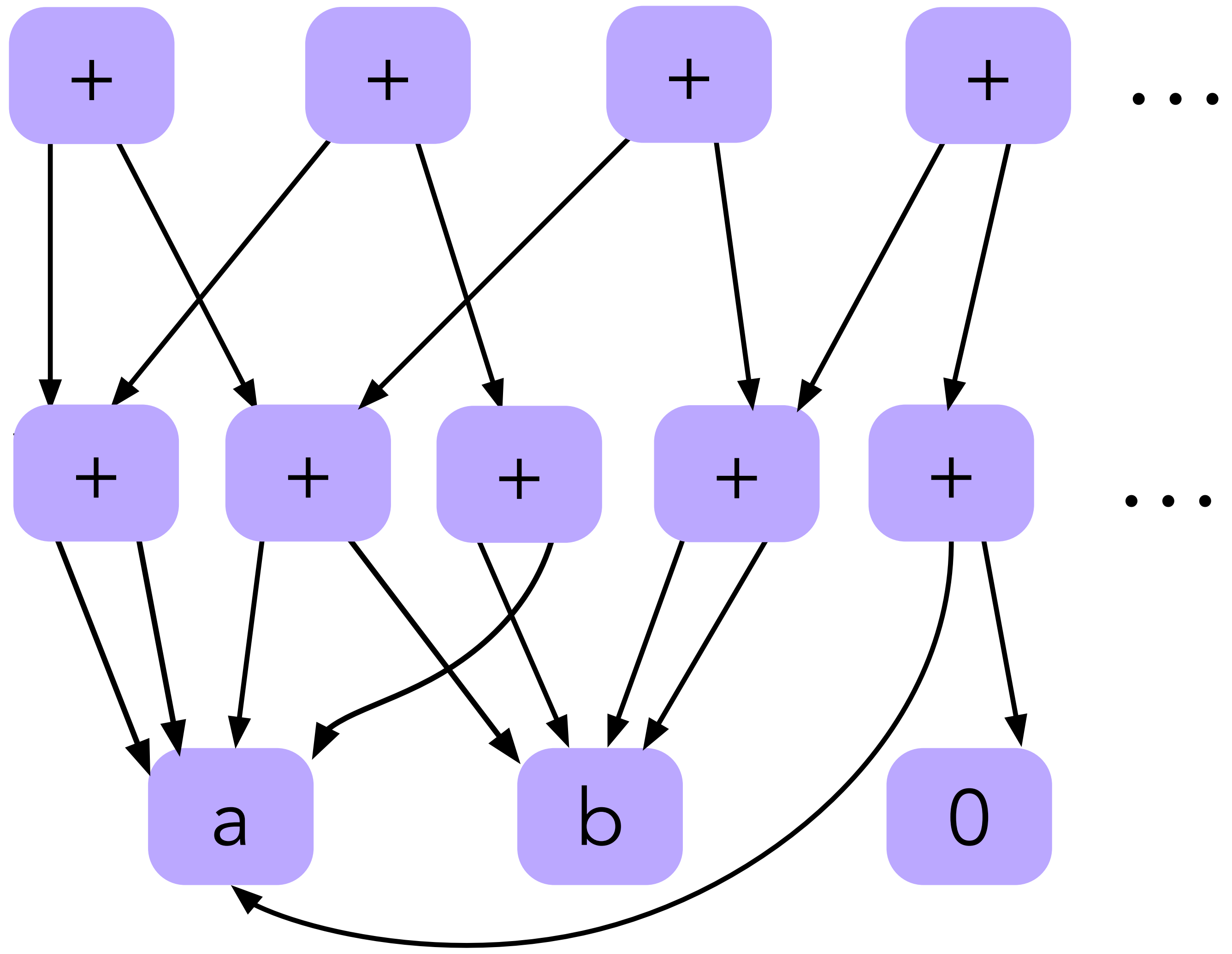
Enumeration

Candidate Generation

Rule Selection

Enumeration modulo equality saturation

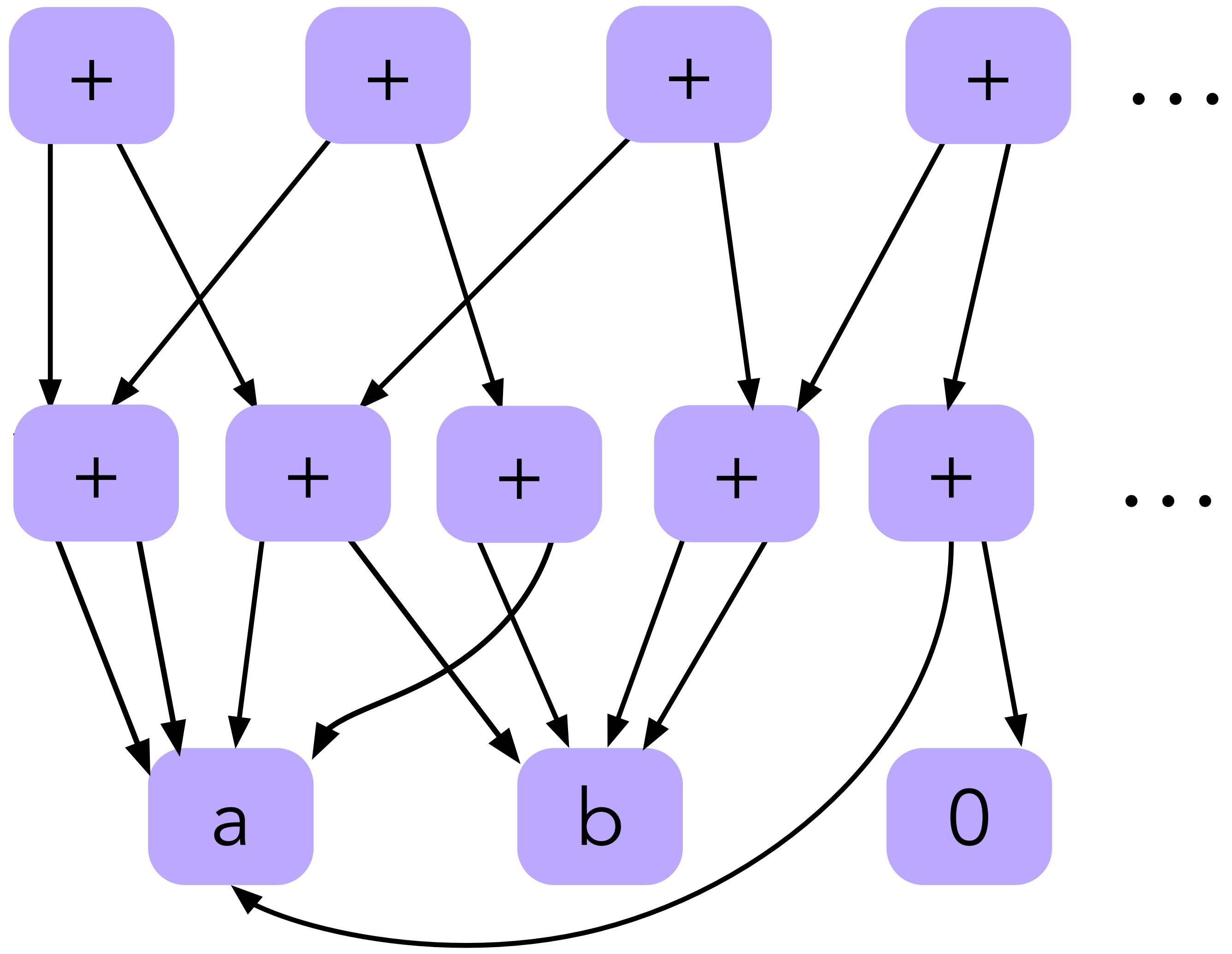
a, b, 0, +, …



Exponentially many terms!

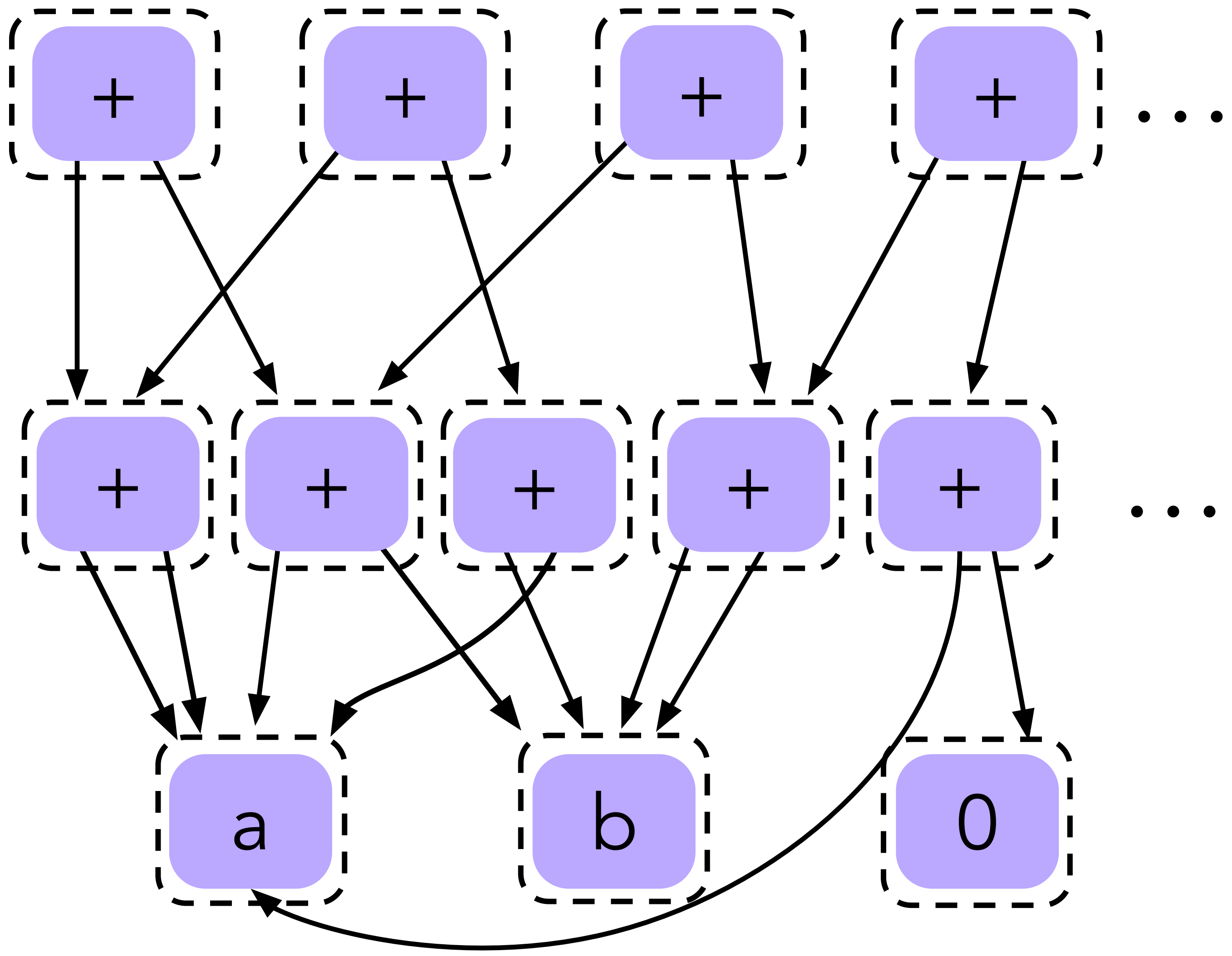
Enumeration modulo equality saturation

a, b, 0, +, …



Exponentially many terms!

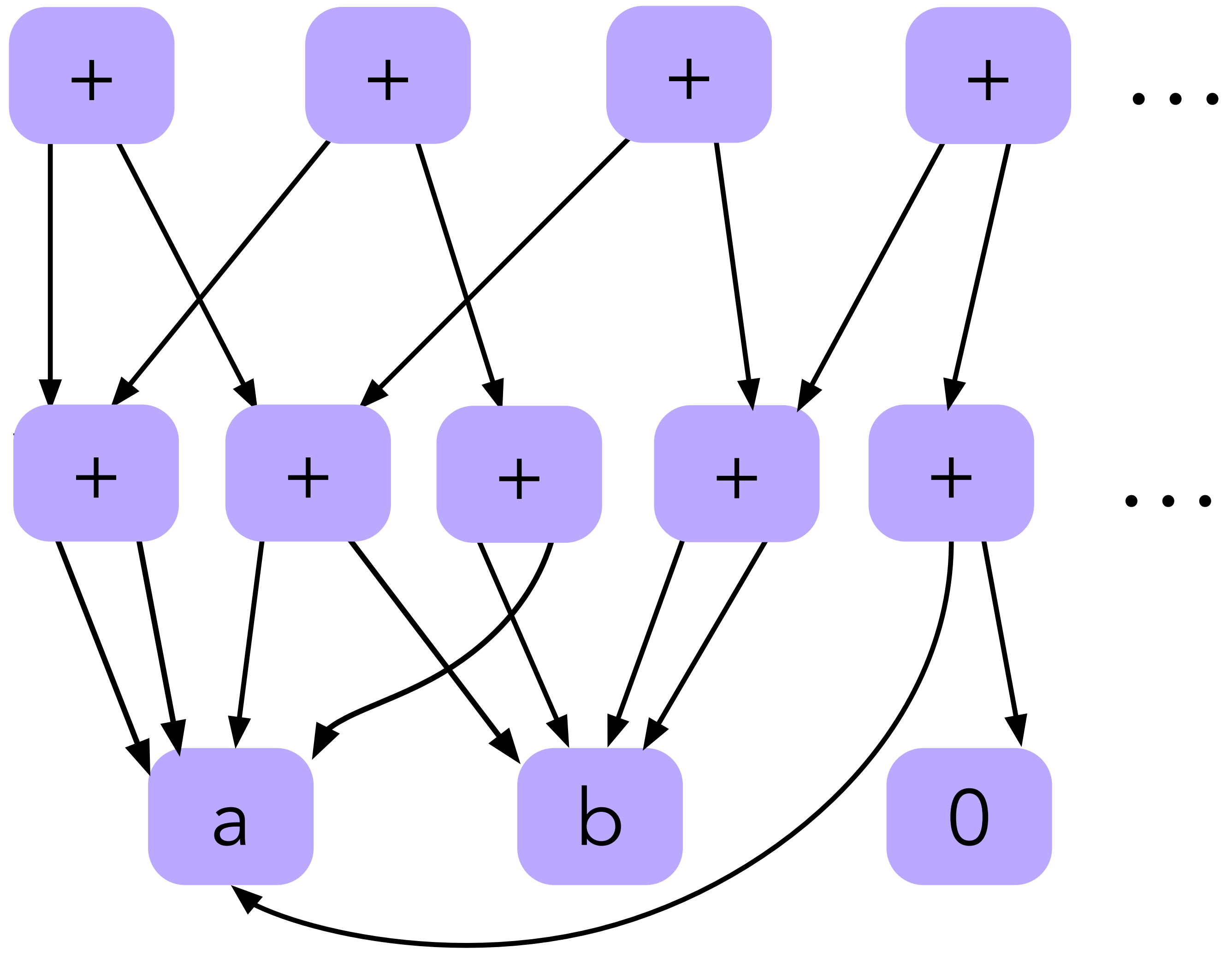
E-classes



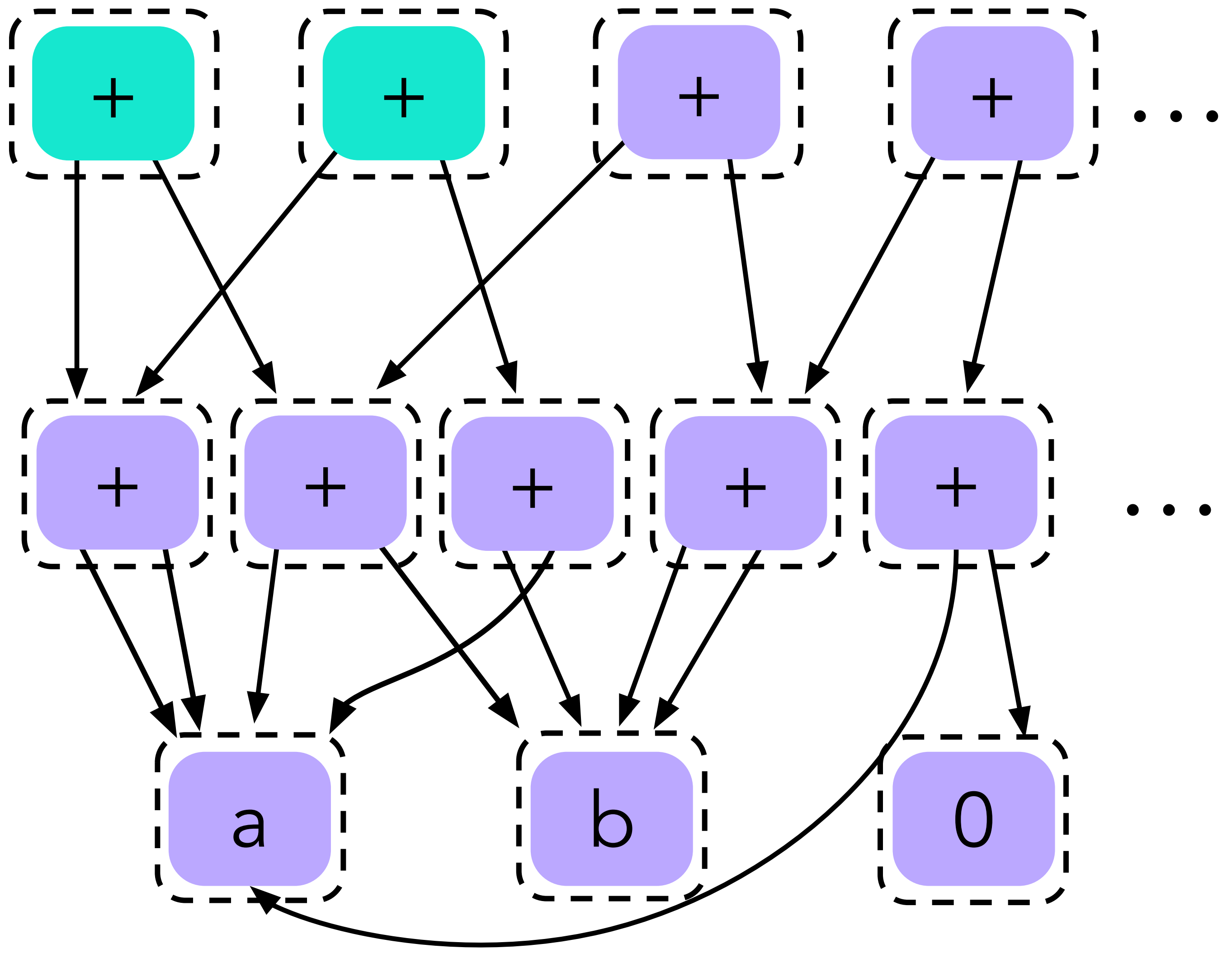
Enumerate over an E-graph

Enumeration modulo equality saturation

a, b, 0, +, …



E-classes



(x + x) + (x + y)

=

(x + x) + (y + x)

Exponentially many terms!

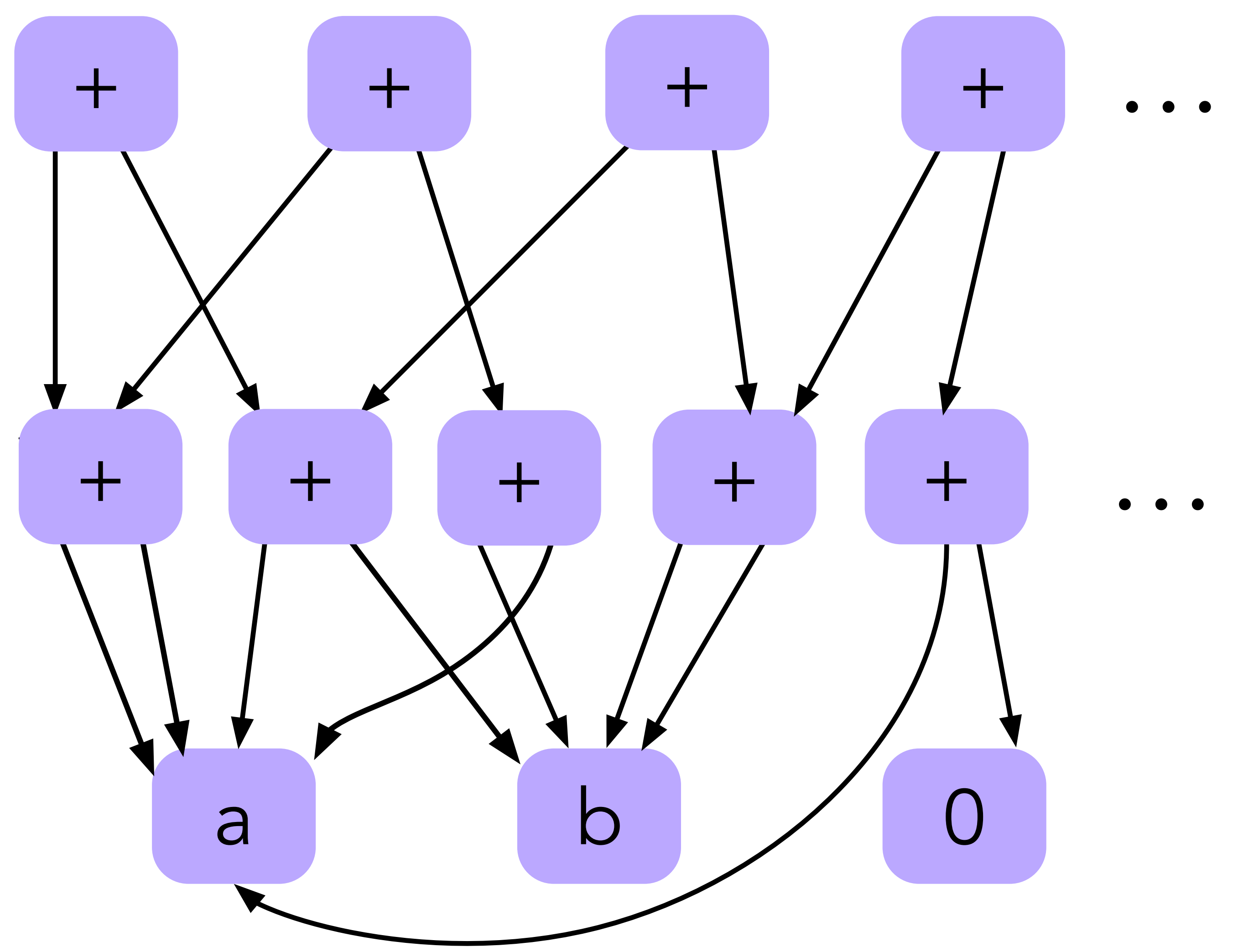
Enumerate over an E-graph

Apply current ruleset

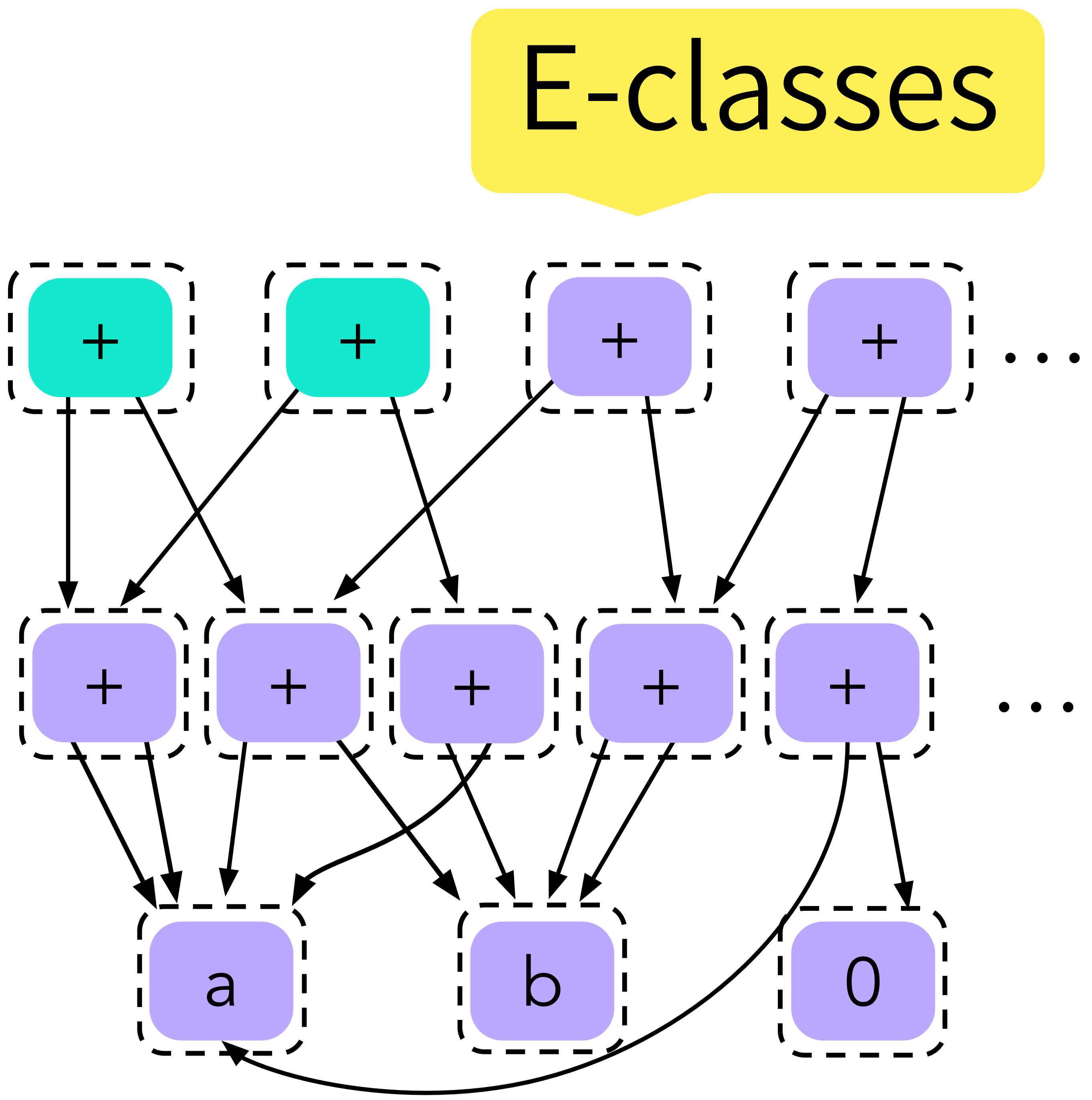
(x + y) ⟷ (y + x)

Enumeration modulo equality saturation

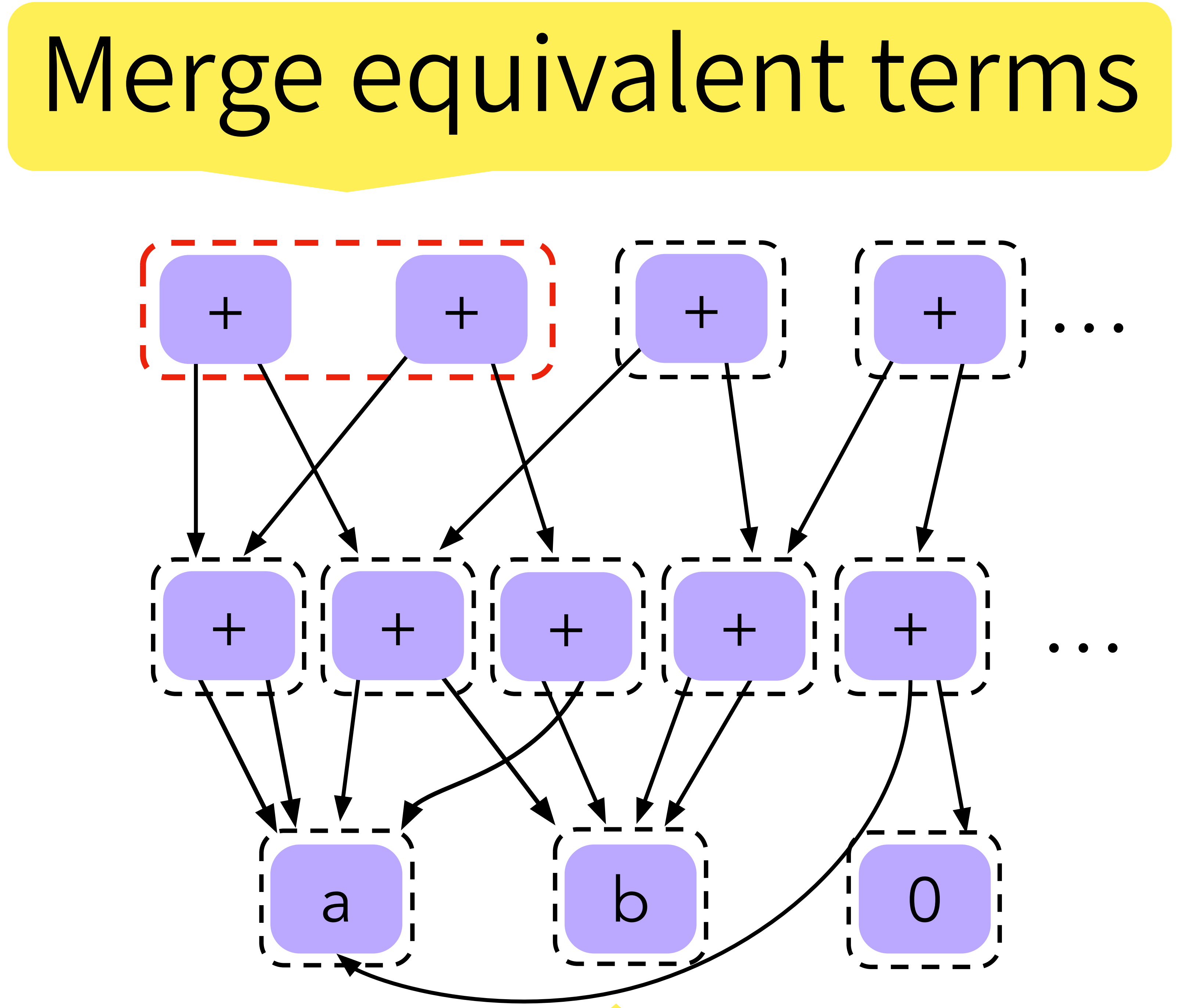
a, b, 0, +, …



Exponentially many terms!



Enumerate over an E-graph



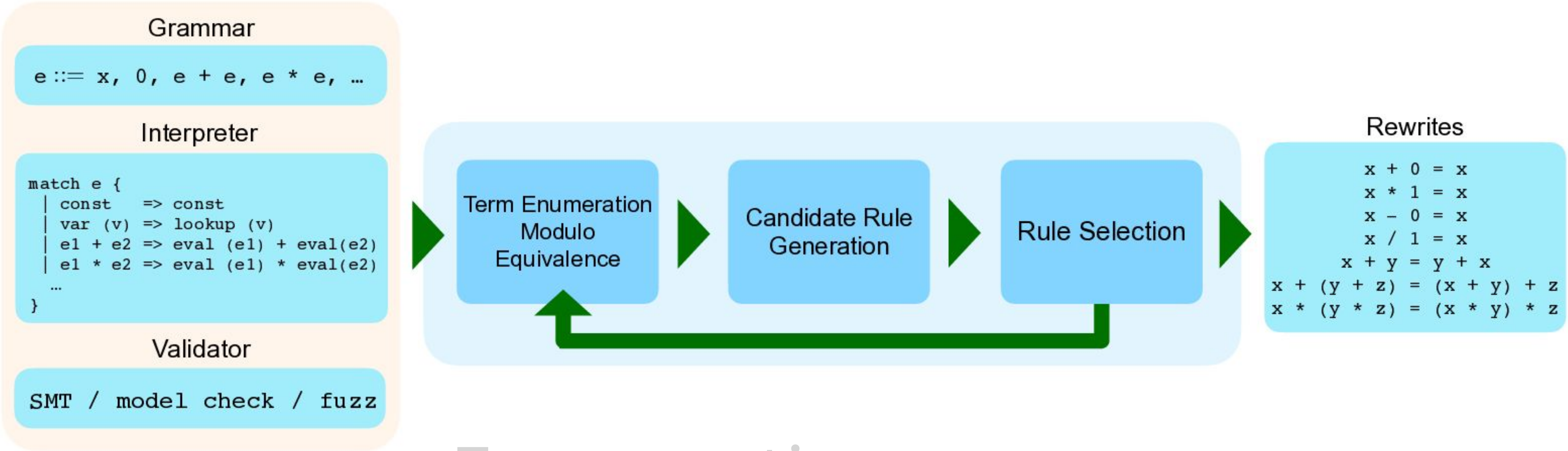
Apply current ruleset

$(x + y) \leftrightarrow (y + x)$

Enumeration modulo equality saturation

Shrinks the term space by applying rewrites as they are learned!

Ruler

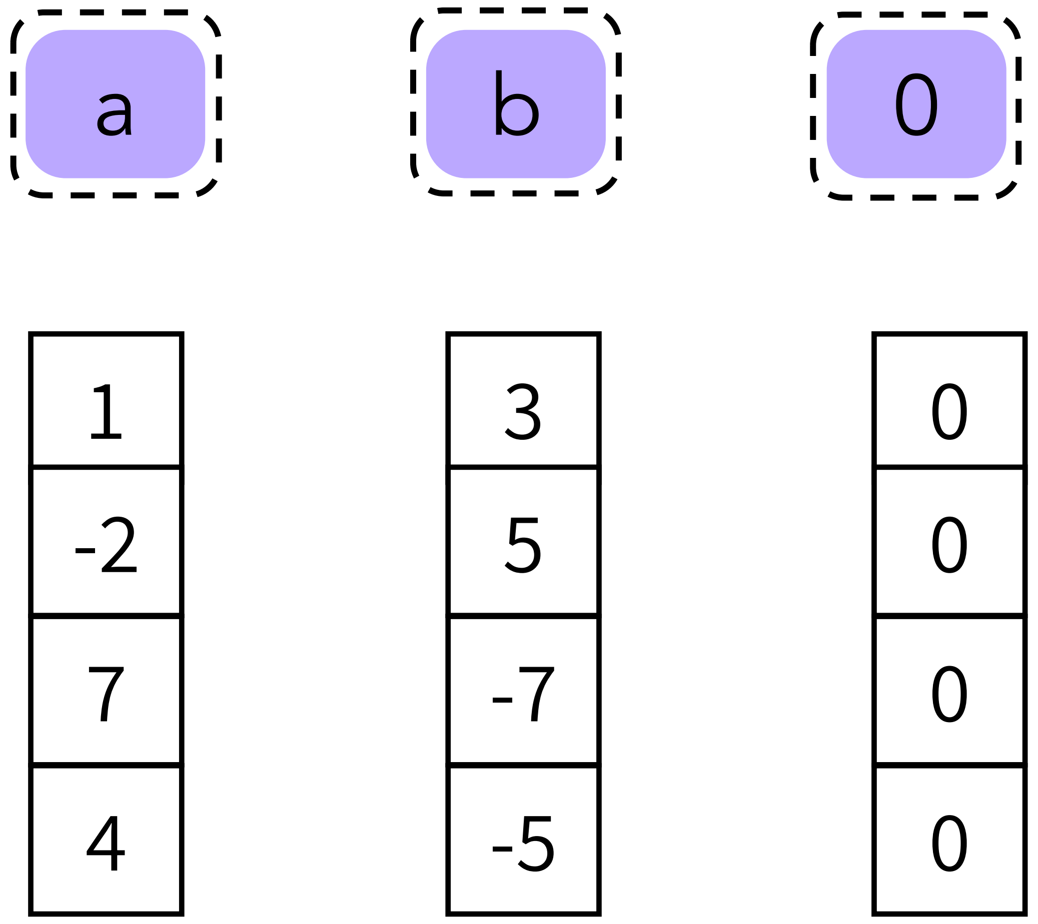


Enumeration

Candidate Generation

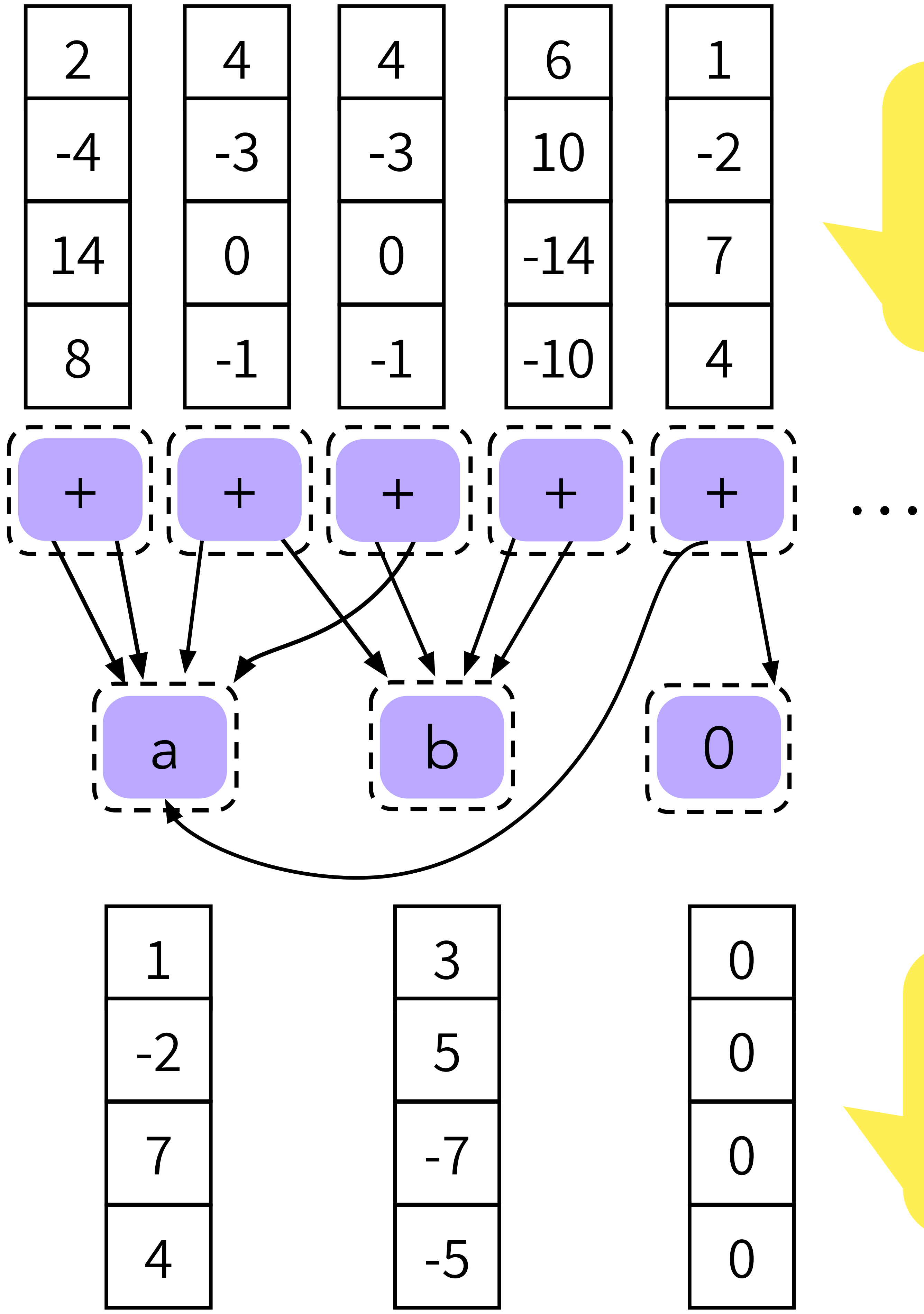
Rule Selection

Candidate generation by characteristic vector matching



Seed initial E-classes with concrete values (cvecs) from the domain

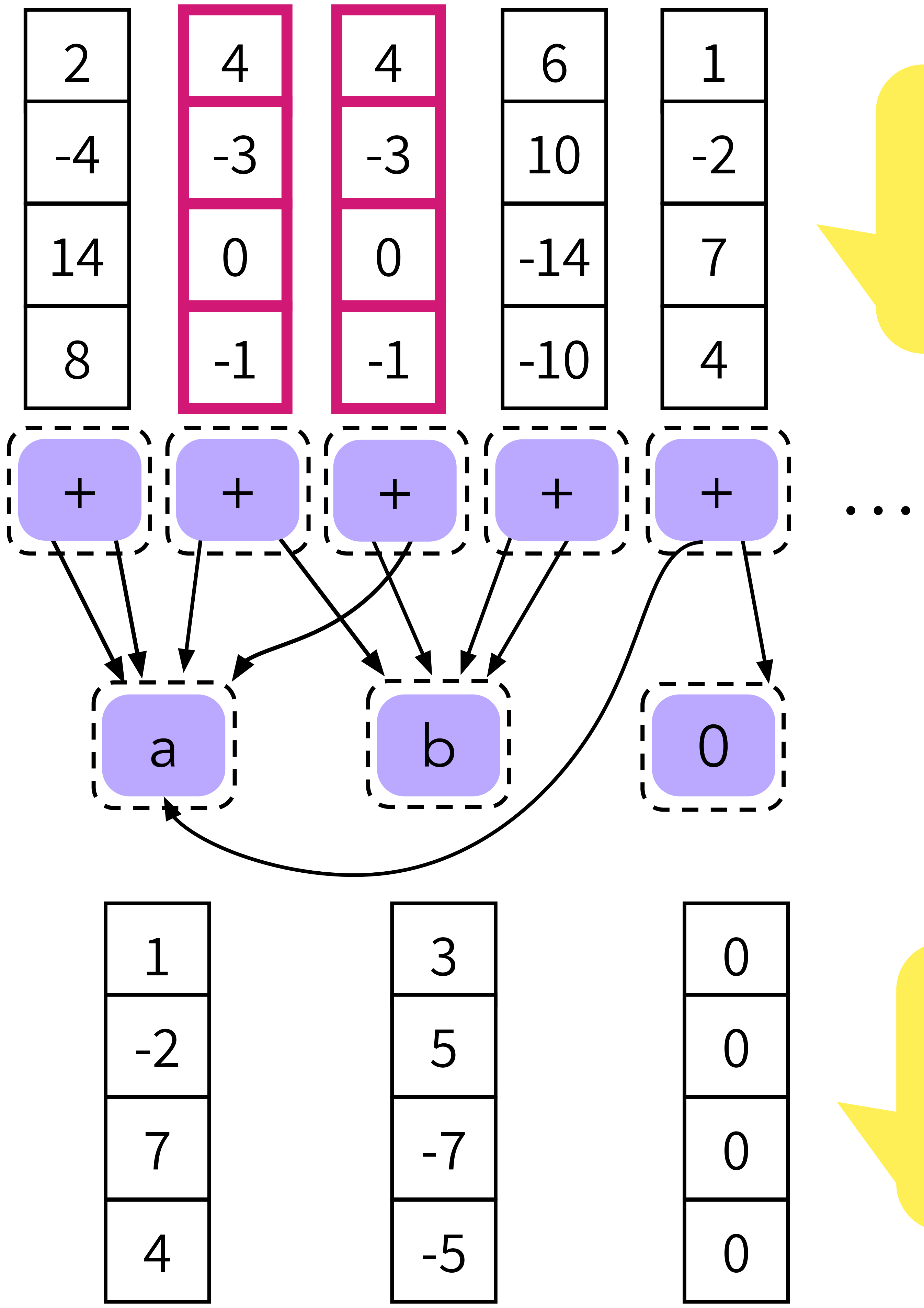
Candidate generation by characteristic vector matching



Compute the cvecs for newly enumerated E-classes

Seed initial E-classes with concrete values (cvecs) from the domain

Candidate generation by characteristic vector matching

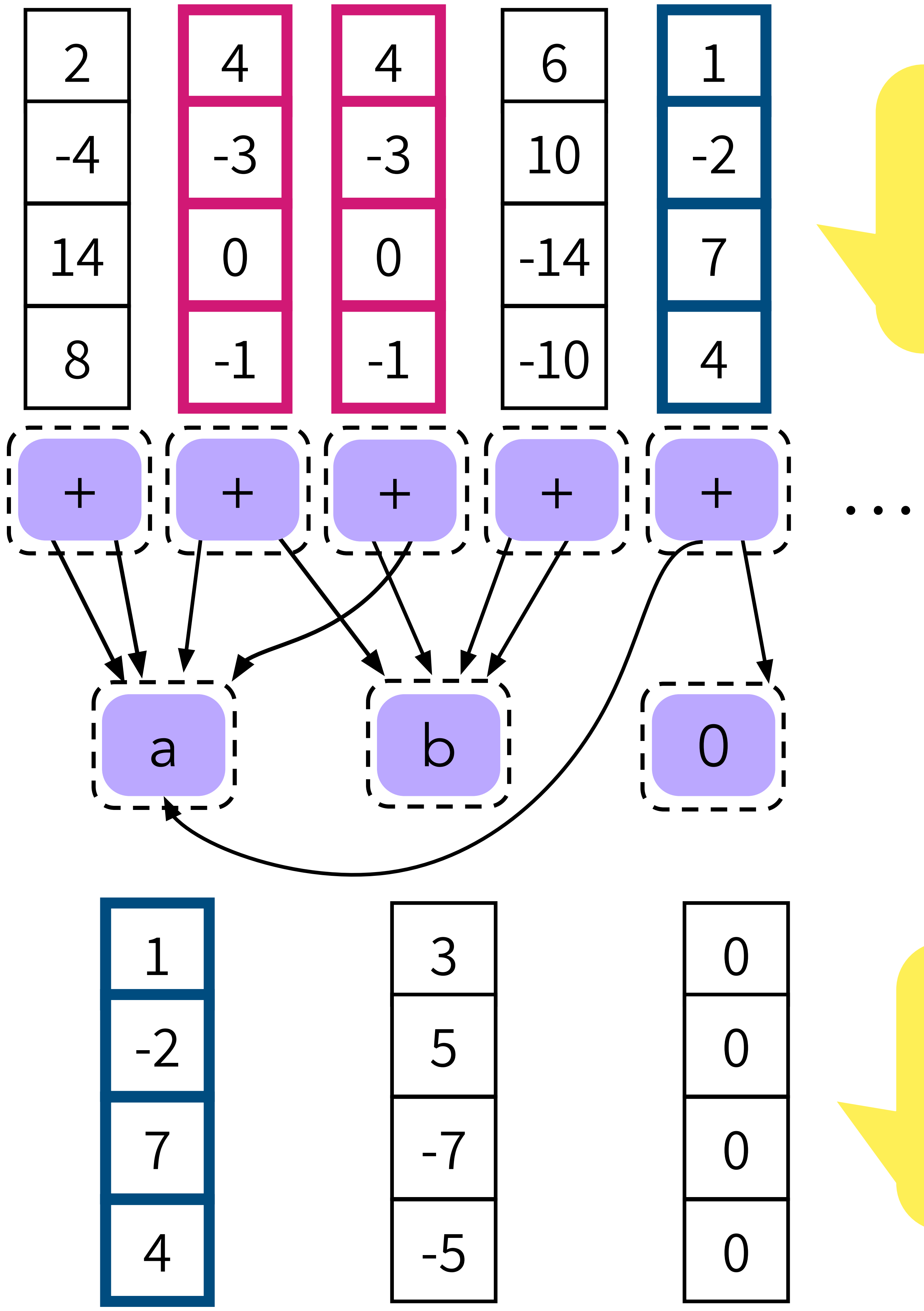


Compute the cvecs for newly enumerated E-classes

Seed initial E-classes with concrete values (cvecs) from the domain

(x + y) ↔ (y + x)

Candidate generation by characteristic vector matching



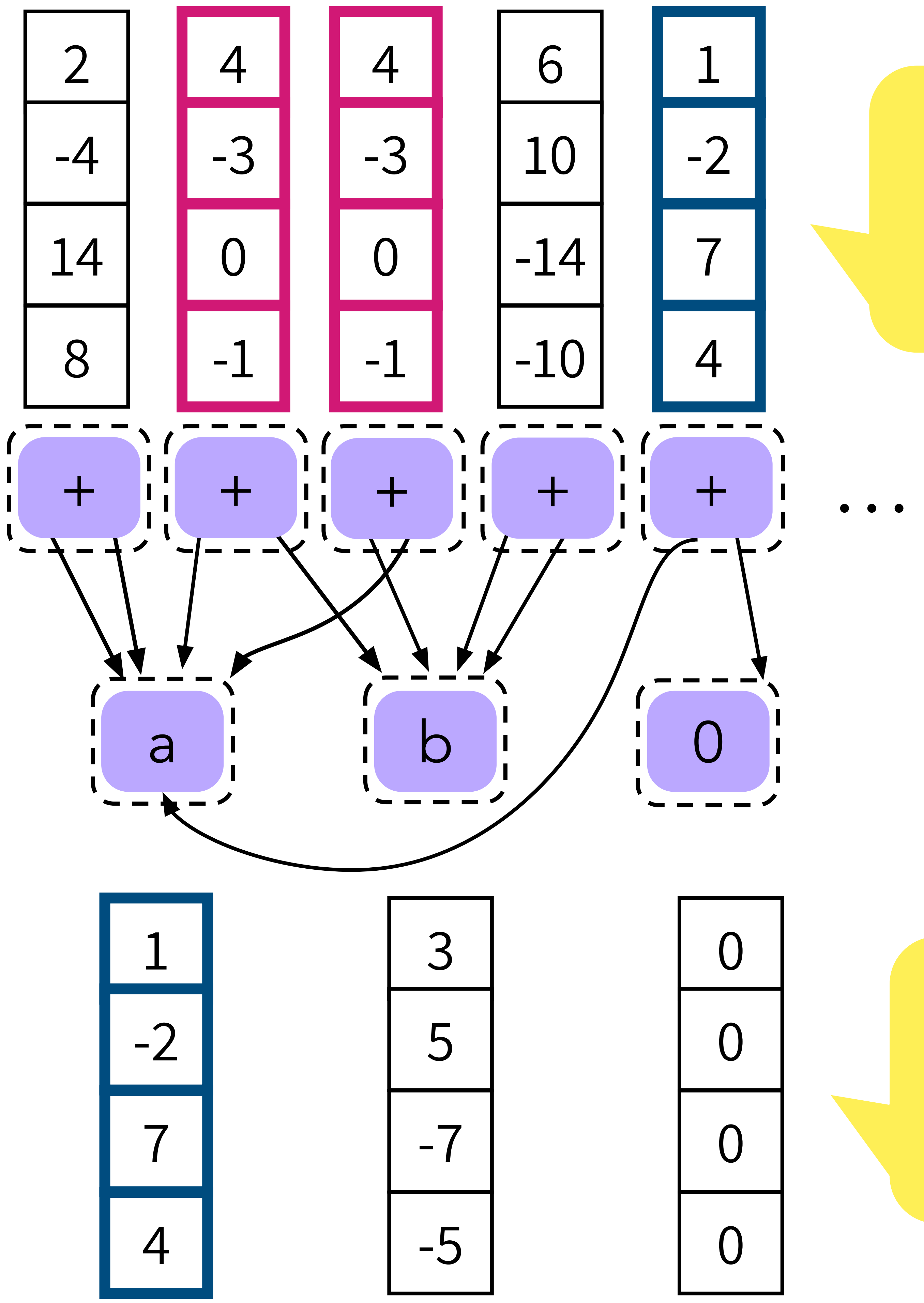
Compute the cvecs for newly enumerated E-classes

Seed initial E-classes with concrete values (cvecs) from the domain

(x + y) ↔ (y + x)

(x + 0) ↔ x

Candidate generation by characteristic vector matching



Compute the cvecs for newly enumerated E-classes

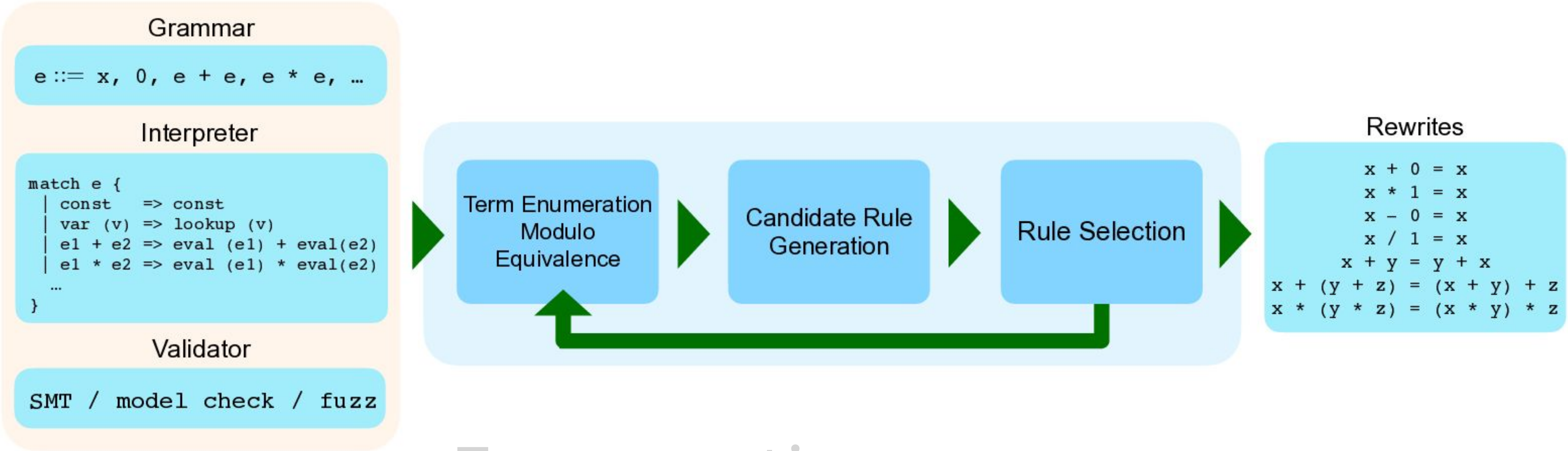
Seed initial E-classes with concrete values (cvecs) from the domain

(x + y) ⟷ (y + x)

(x + 0) ⟷ x

Validate candidates using SMT, fuzzing, model checking

Ruler



Enumeration

Candidate Generation

Rule Selection

Rule selection with equality saturation

C =

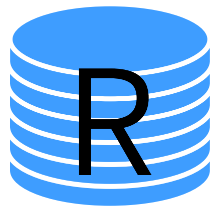
(x + y)	⟷	(y + x)
(x + 0)	⟷	(0 + x)
(y + 0)	⟷	(0 + y)
(x * y)	⟷	(y * x)
(x * 1)	⟷	(1 * x)
(y * 1)	⟷	(1 * y)

Rule selection with equality saturation

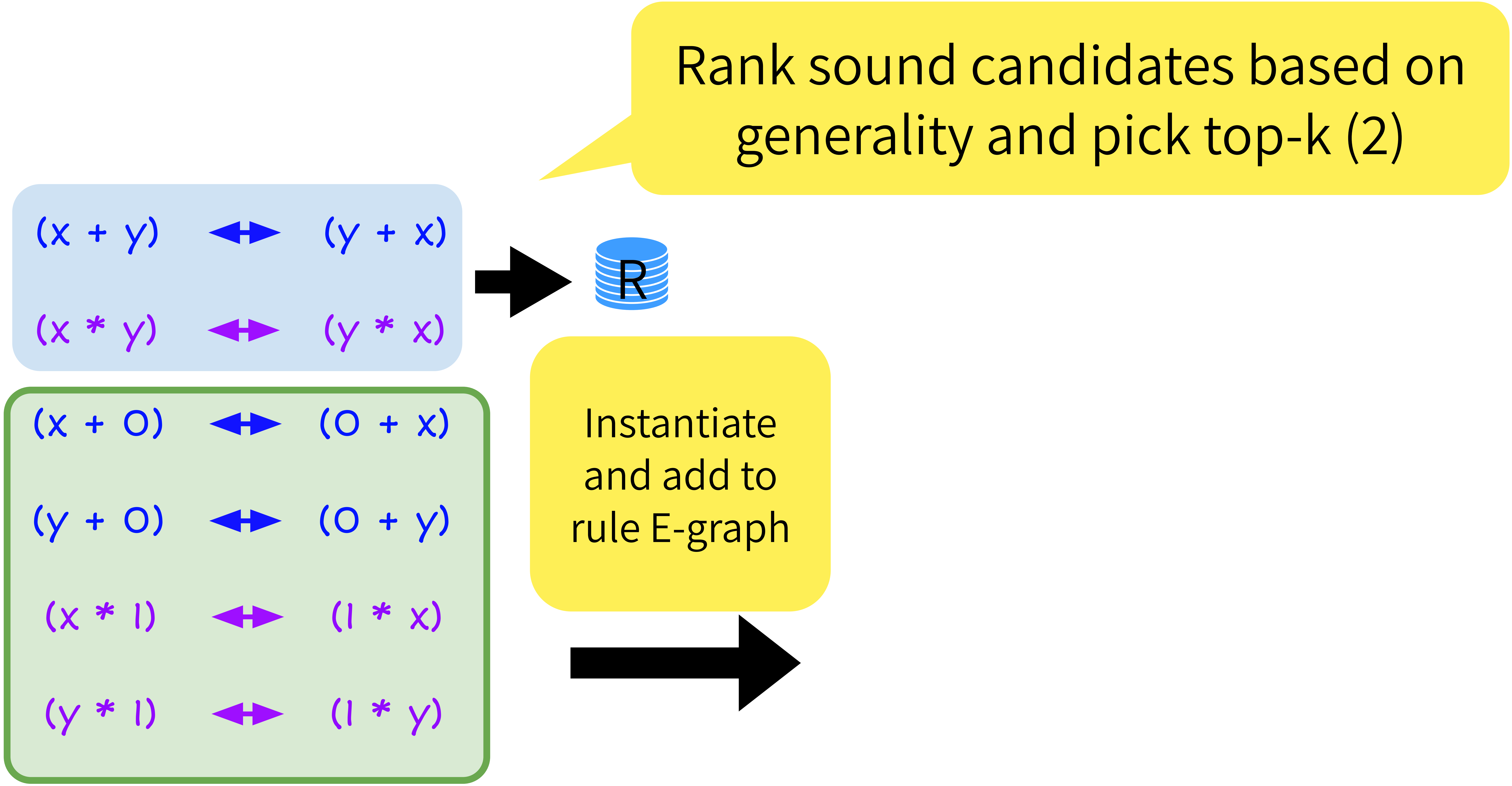
Rank sound candidates based on generality and pick top-k (2)

C =

- (x + y) ↔ (y + x)
- (x * y) ↔ (y * x)
- (x + 0) ↔ (0 + x)
- (y + 0) ↔ (0 + y)
- (x * 1) ↔ (1 * x)
- (y * 1) ↔ (1 * y)

→ 

Rule selection with equality saturation



Rule selection with equality saturation

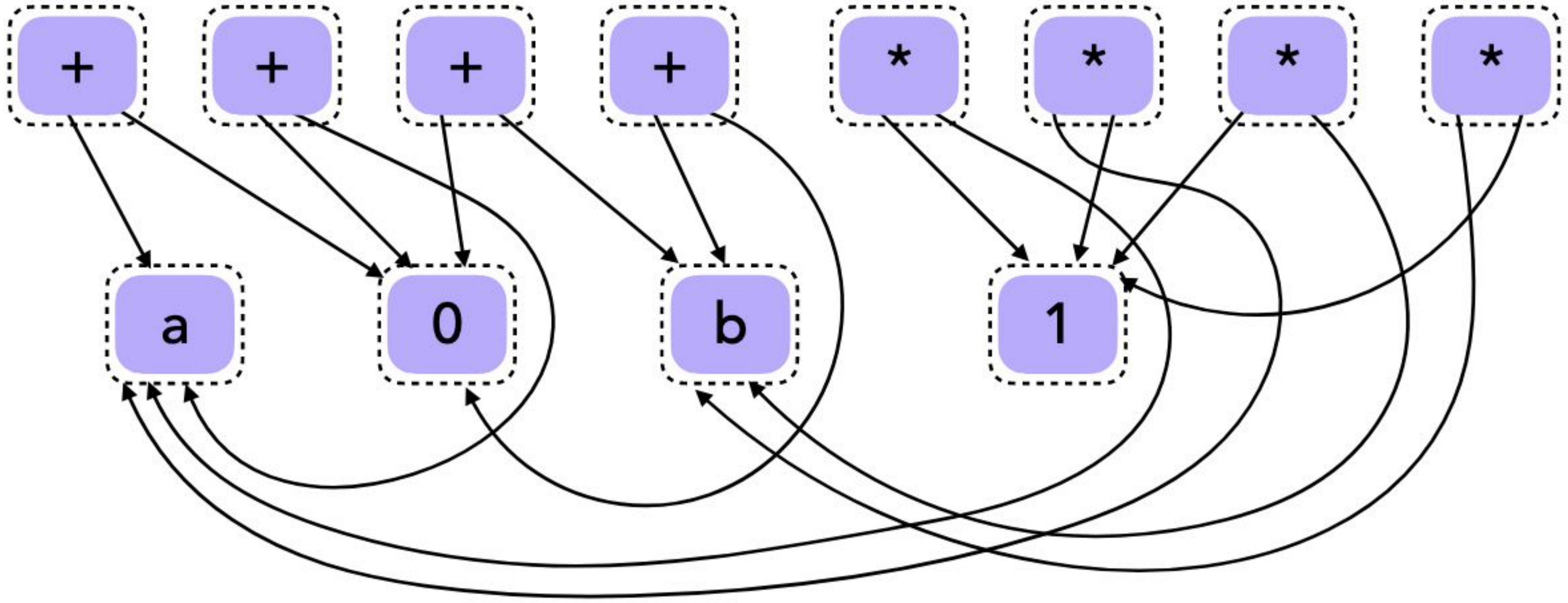
Rank sound candidates based on generality and pick top-k (2)

(x + y) ↔ (y + x)
(x * y) ↔ (y * x)

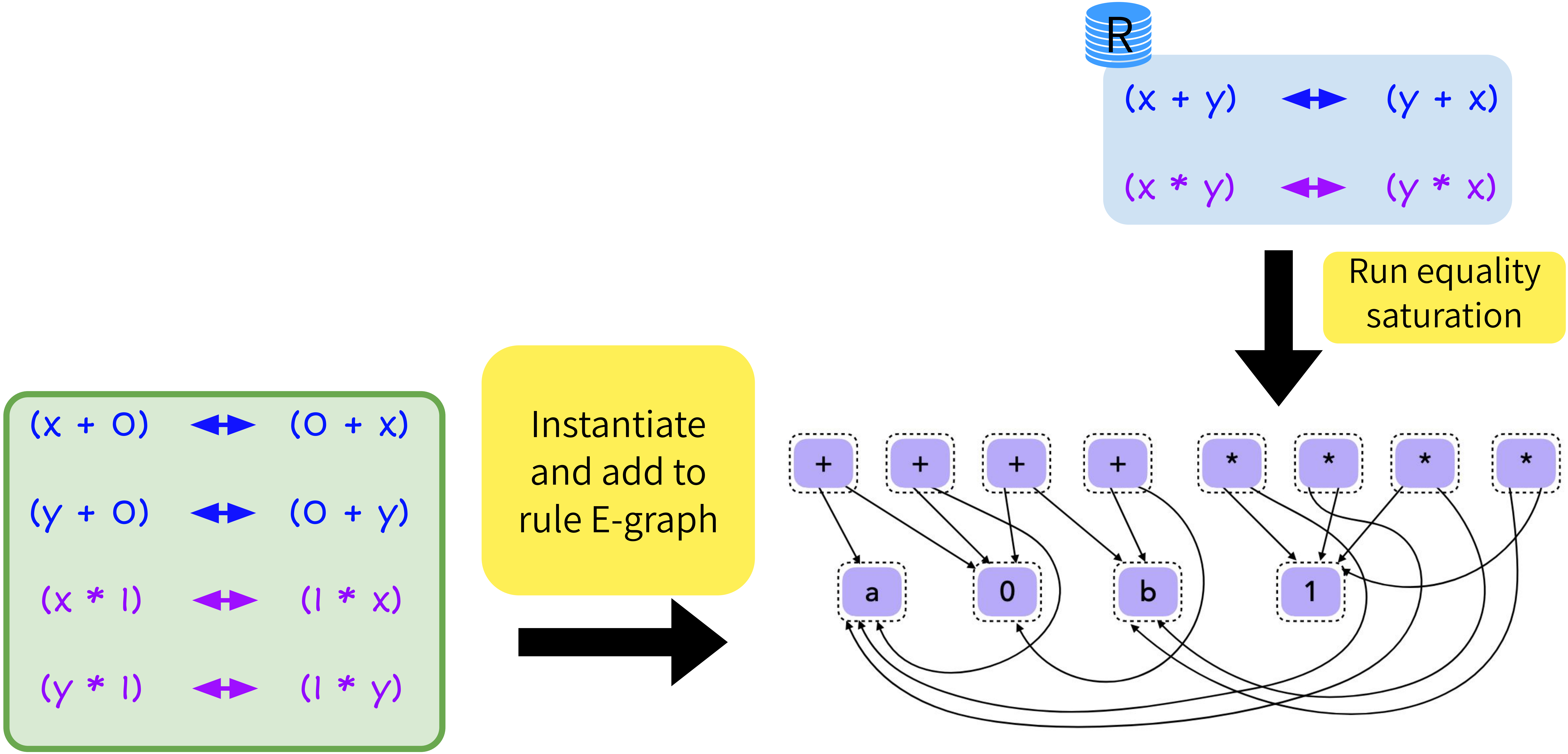
R

(x + 0) ↔ (0 + x)
(y + 0) ↔ (0 + y)
(x * 1) ↔ (1 * x)
(y * 1) ↔ (1 * y)

Instantiate and add to rule E-graph



Rule selection with equality saturation



Rule selection with equality saturation

R

(x + y) ↔ (y + x)

(x * y) ↔ (y * x)

All four rules are redundant and therefore discarded!

Run equality saturation

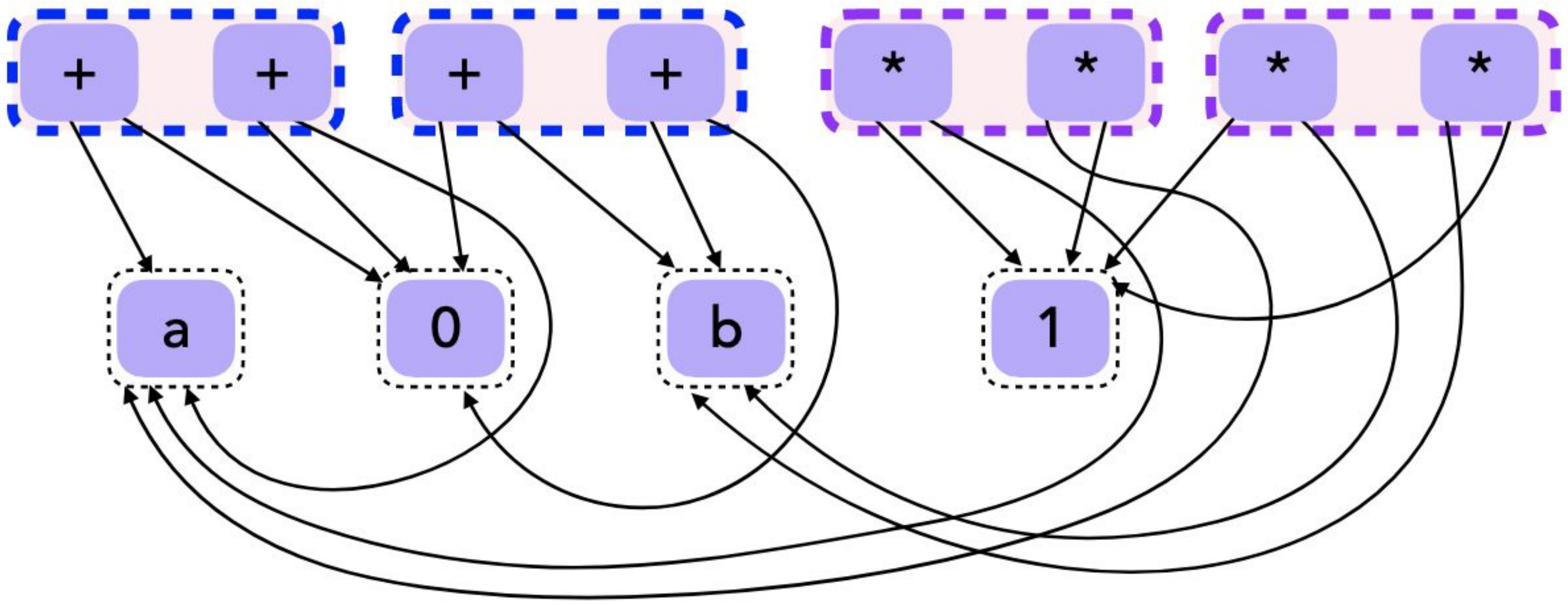
(x + 0) ↔ (0 + x)

(y + 0) ↔ (0 + y)

(x * 1) ↔ (1 * x)

(y * 1) ↔ (1 * y)

Instantiate and add to rule E-graph



Rule selection with equality saturation

Continue processing until candidate set is empty or has only unsound ones left!

R

(x + y) ↔ (y + x)

(x * y) ↔ (y * x)

All four rules are redundant and therefore discarded!

Run equality saturation

(x + 0) ↔ (0 + x)

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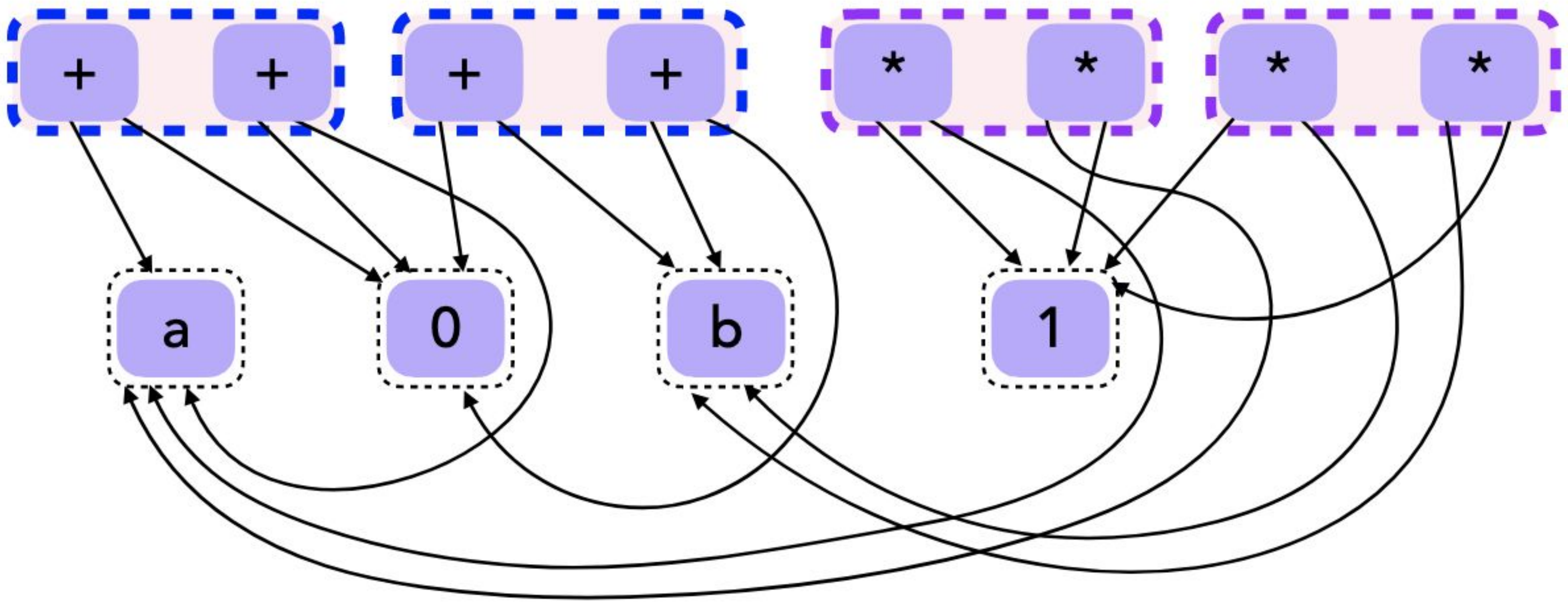

Rule selection with equality saturation

Larger top-k makes Ruler faster

Smaller top-k gives smaller rulesets

See paper for detailed comparison!

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(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

Run equality saturation

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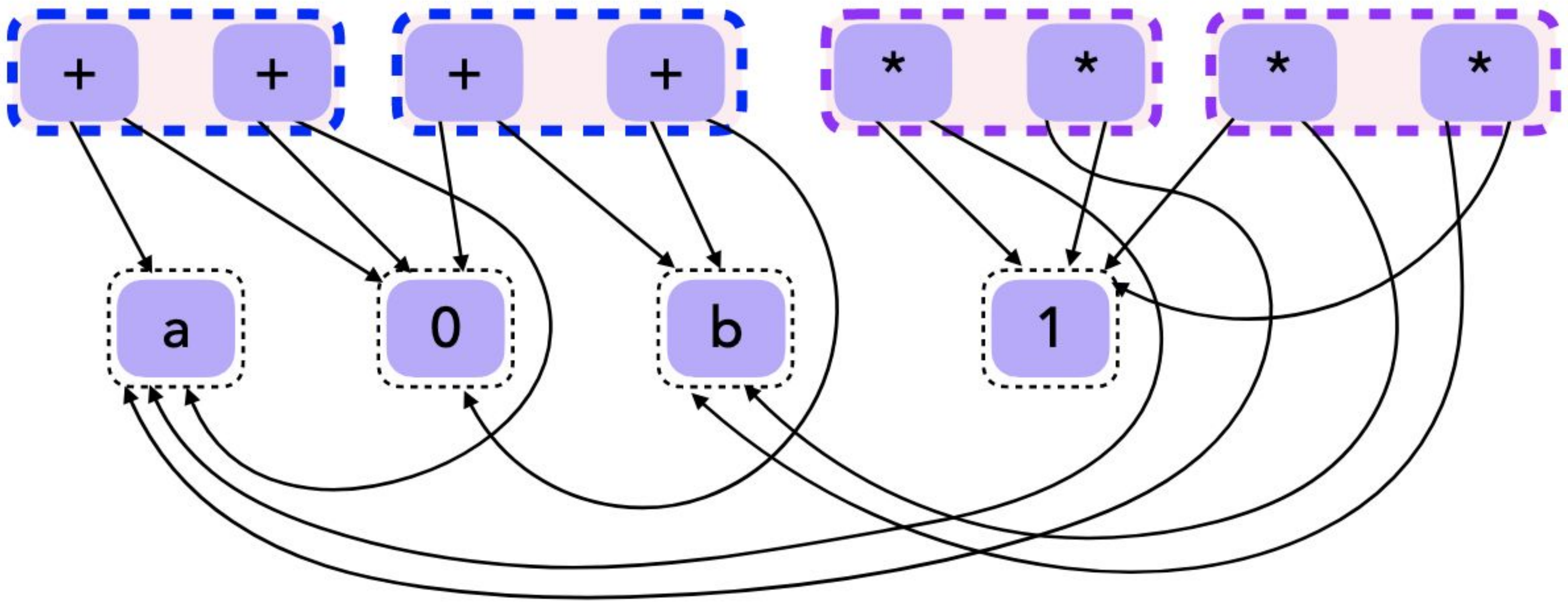
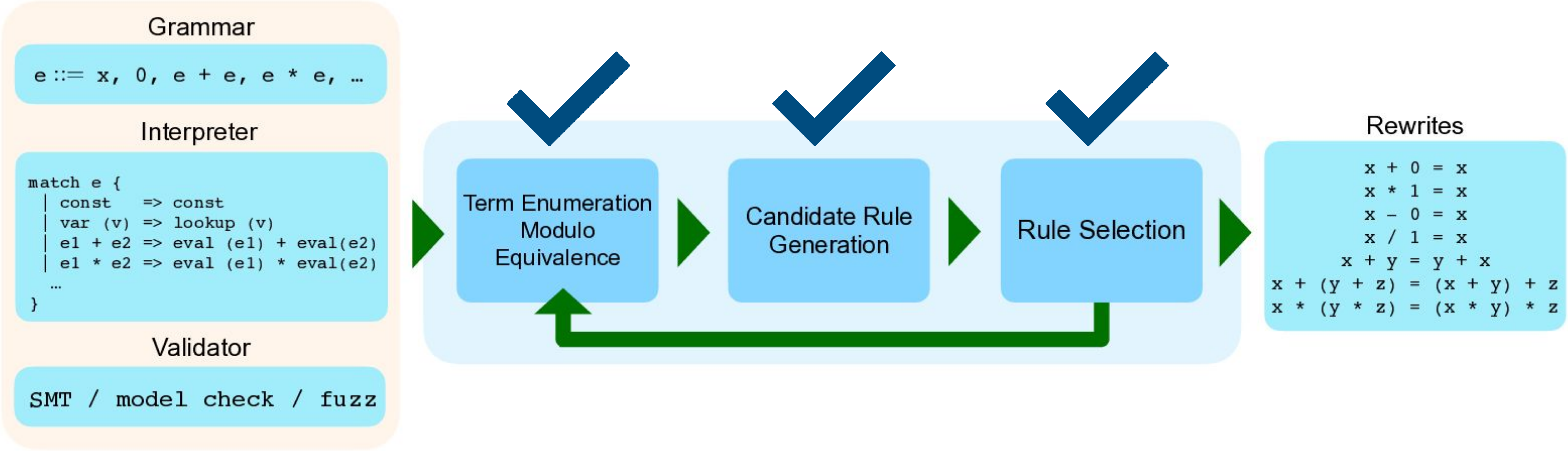


Rule selection with equality saturation

Larger top-k makes Ruler faster

Shrinks the *candidate space* by applying rewrites as they are learned!

Ruler


Grammar
e ::= x, 0, e + e, e * e, …
Interpreter
match e {
| const => const
| var (v) => lookup (v)
| e1 + e2 => eval (e1) + eval(e2)
| e1 * e2 => eval (e1) * eval(e2)
…
}
Validator
SMT / model check / fuzz
Term Enumeration Modulo Equivalence
Candidate Rule Generation
Rule Selection
Rewrites
x + 0 = x
x * 1 = x
x – 0 = x
x / 1 = x
x + y = y + x
x + (y + z) = (x + y) + z
x * (y * z) = (x * y) * z

Equality saturation “*soundiness*”

Equality Saturation *amplifies* unsoundness!

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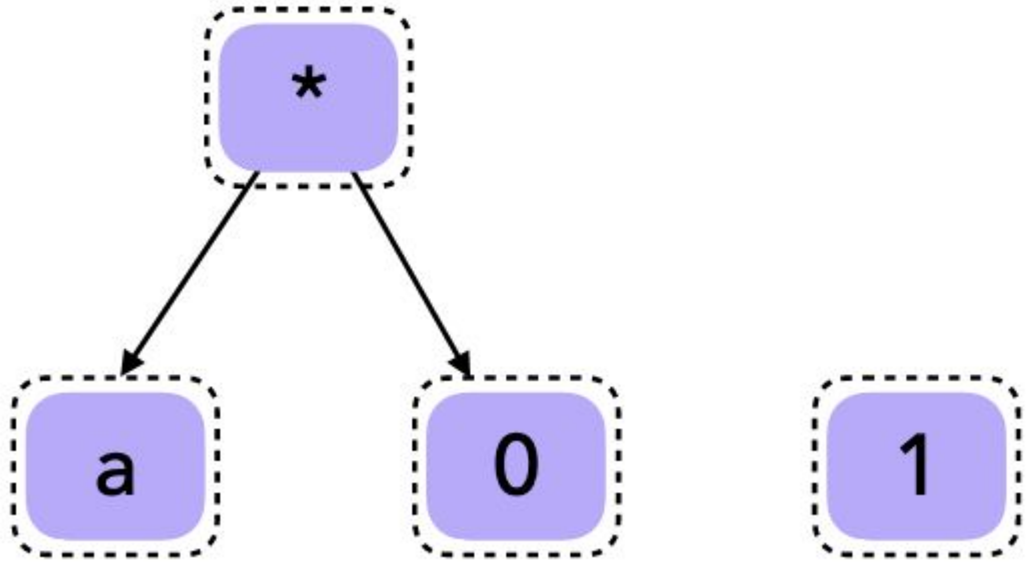


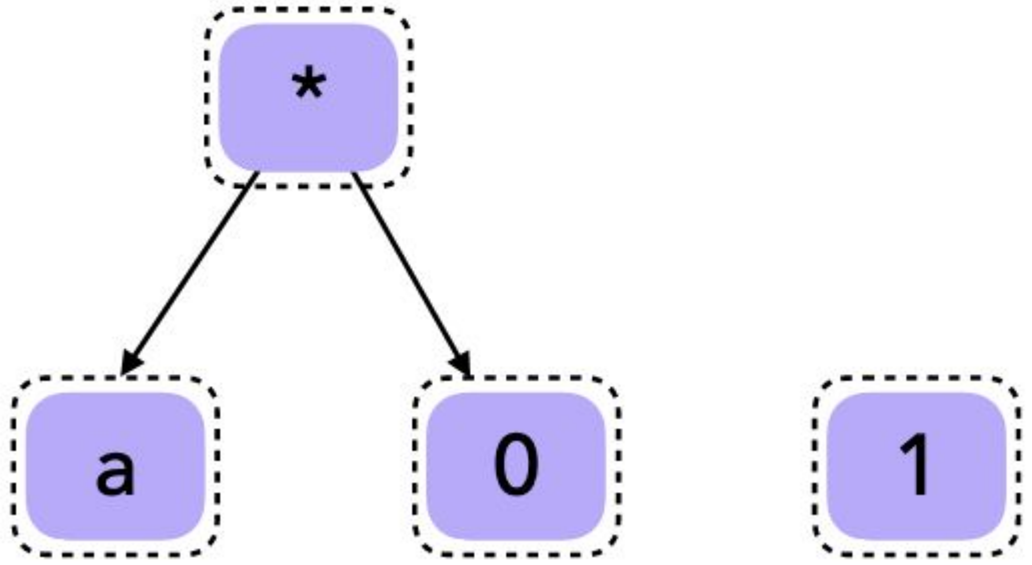
Equality saturation “*soundiness*”

Equality Saturation *amplifies* unsoundness!

current ruleset

(y * 0) ↔ 0

(y * 0) ↔ 1



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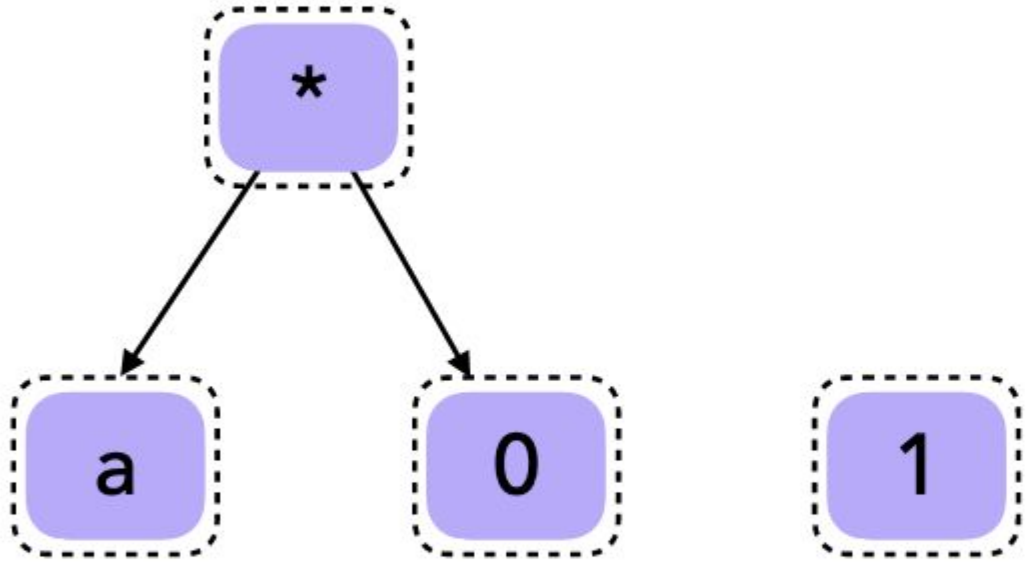

Run equality saturation on term E-graph

Equality saturation "*soundiness*"

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current ruleset

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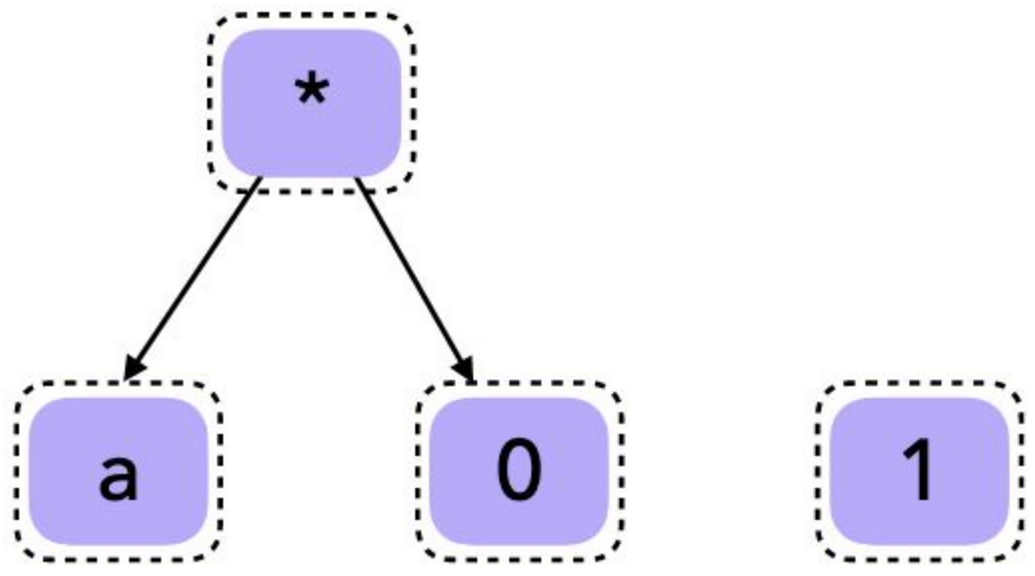

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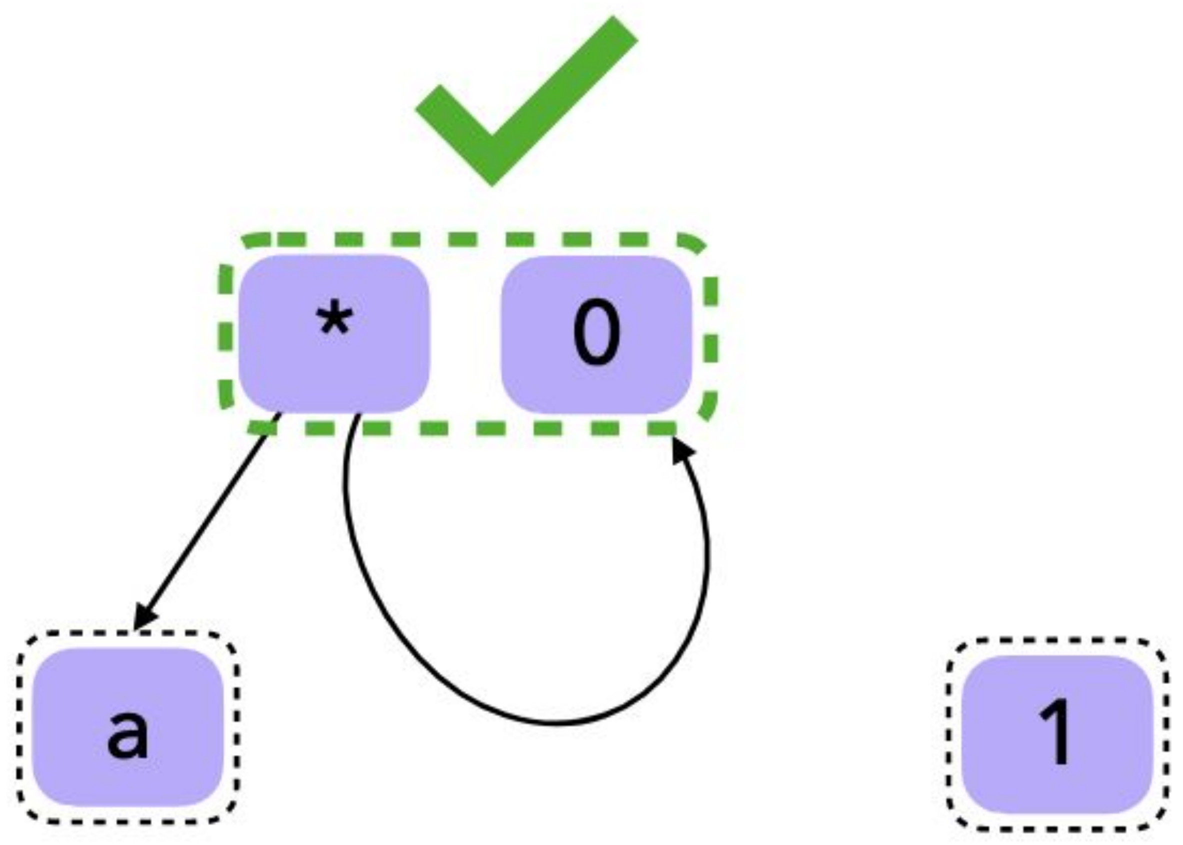

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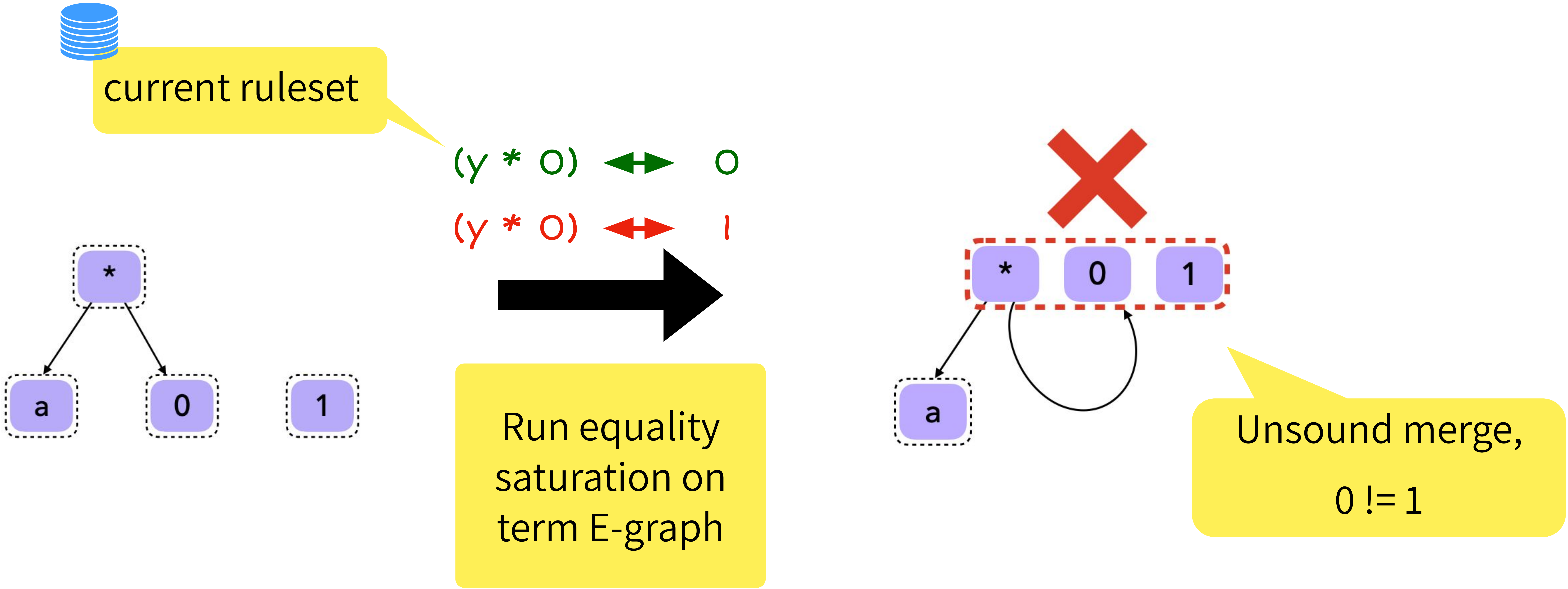
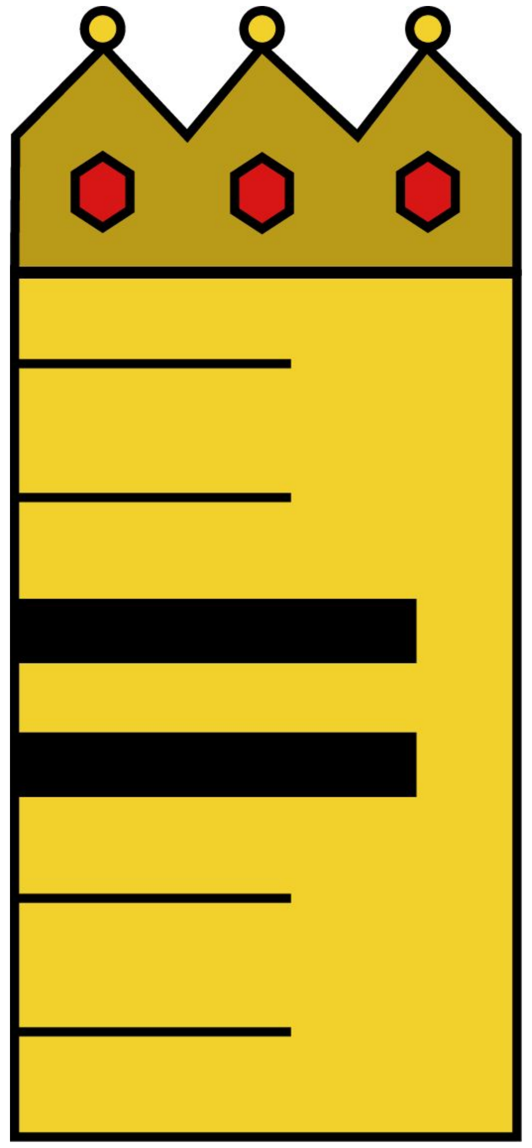


Implementation



https://github.com/uwplse/ruler

Implemented in Rust

Uses egg for equality saturation

Evaluation

Ruler vs Other tools (CVC4)

How do the rulesets compare?

Comparison with CVC4

Parameters		Ruler			CVC4			Ruler / CVC4	
Domain	# Conn	Time (s)	# Rules	Drv	Time (s)	# Rules	Drv	Time	Rules
bool	2	0.01	20	1	0.13	53	1	0.06	0.38
bool	3	0.06	28	1	0.82	293	1	0.07	0.10
bv4	2	0.14	49	1	4.47	135	0.98	0.03	0.36
bv4	3	4.30	272	1	372.26	1978	1	0.01	0.14
bv32	2	13.00	46	0.97	18.53	126	0.93	0.70	0.37
bv32	3	630.09	188	0.98	1199.53	1782	0.91	0.53	0.11

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Fraction of the 1782 rules from CVC4 that the 188 rules from Ruler can derive via equality saturation

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Ruler infers a smaller, useful ruleset faster

Evaluation

Ruler vs Other tools (CVC4)

How do the rulesets compare?

Ruler vs Humans (Herbie)

Can Ruler compete with experts?

Comparison with human-written rules

```
sqrt(x+1) - sqrt(x)  →  1/(sqrt(x+1) + sqrt(x))
```

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x.

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HERBIE

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52 *rational* rules, designed by the developers over 6 years

55 / 155 benchmarks are purely over rational arithmetic

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Herbie can generate more-complex expressions that aren't more precise #261 Edit New issue

Closed nbraud opened this issue on Aug 31, 2019 · 4 comments

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Closed **nbraud** opened this issue on Aug 31, 2019 · 4 comments

$|x * y| \leftrightarrow |x| * |y|$

$|x * x| \leftrightarrow x * x$

Discovered by Ruler, resolved the GitHub issue!

End-to-end: rational Herbie

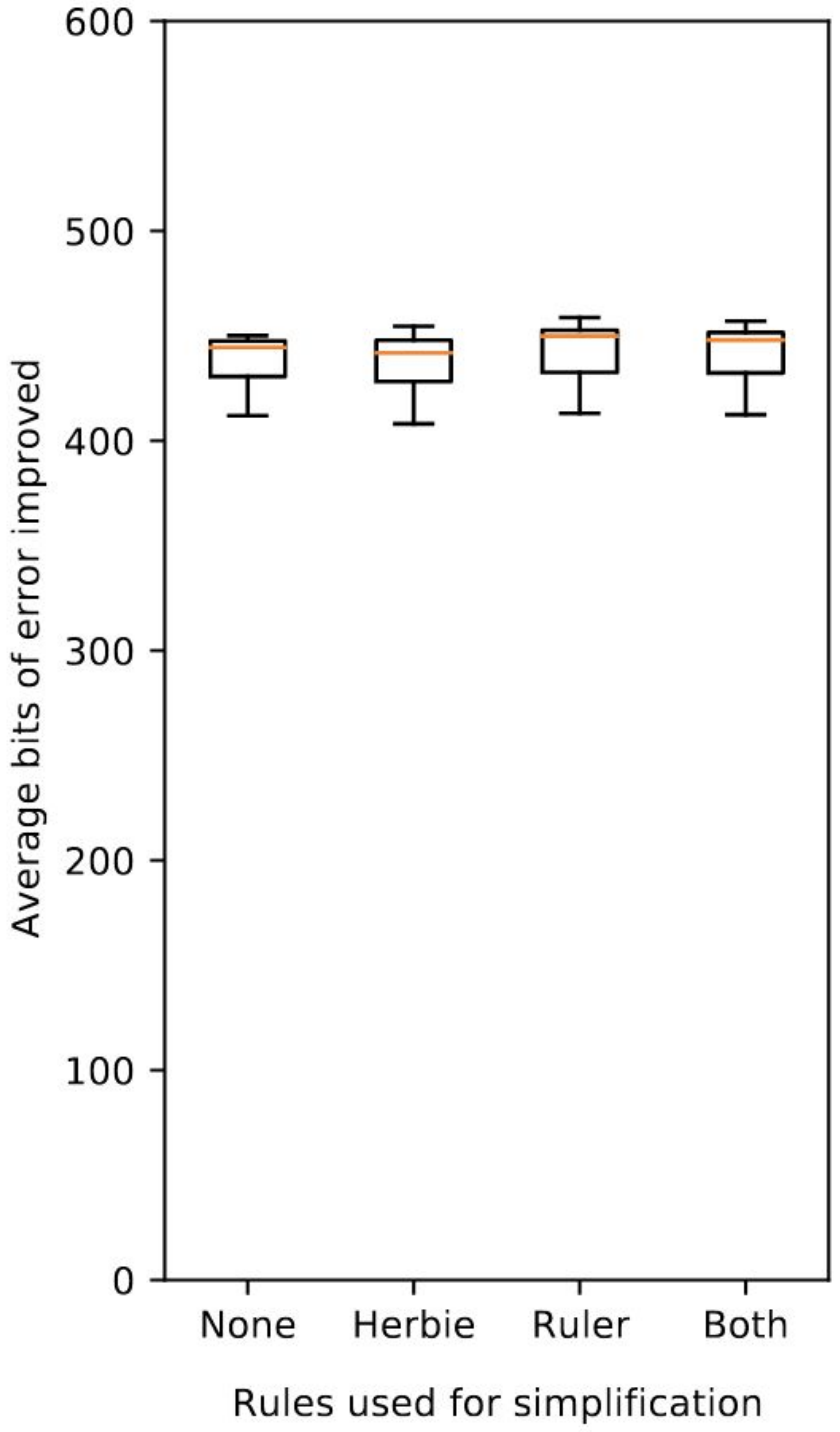
None: Remove all rules

Herbie: Herbie without any changes

Ruler: Herbie with Ruler's rules

Both: Herbie with both original and Ruler's rules

Rational Herbie: comparing accuracy



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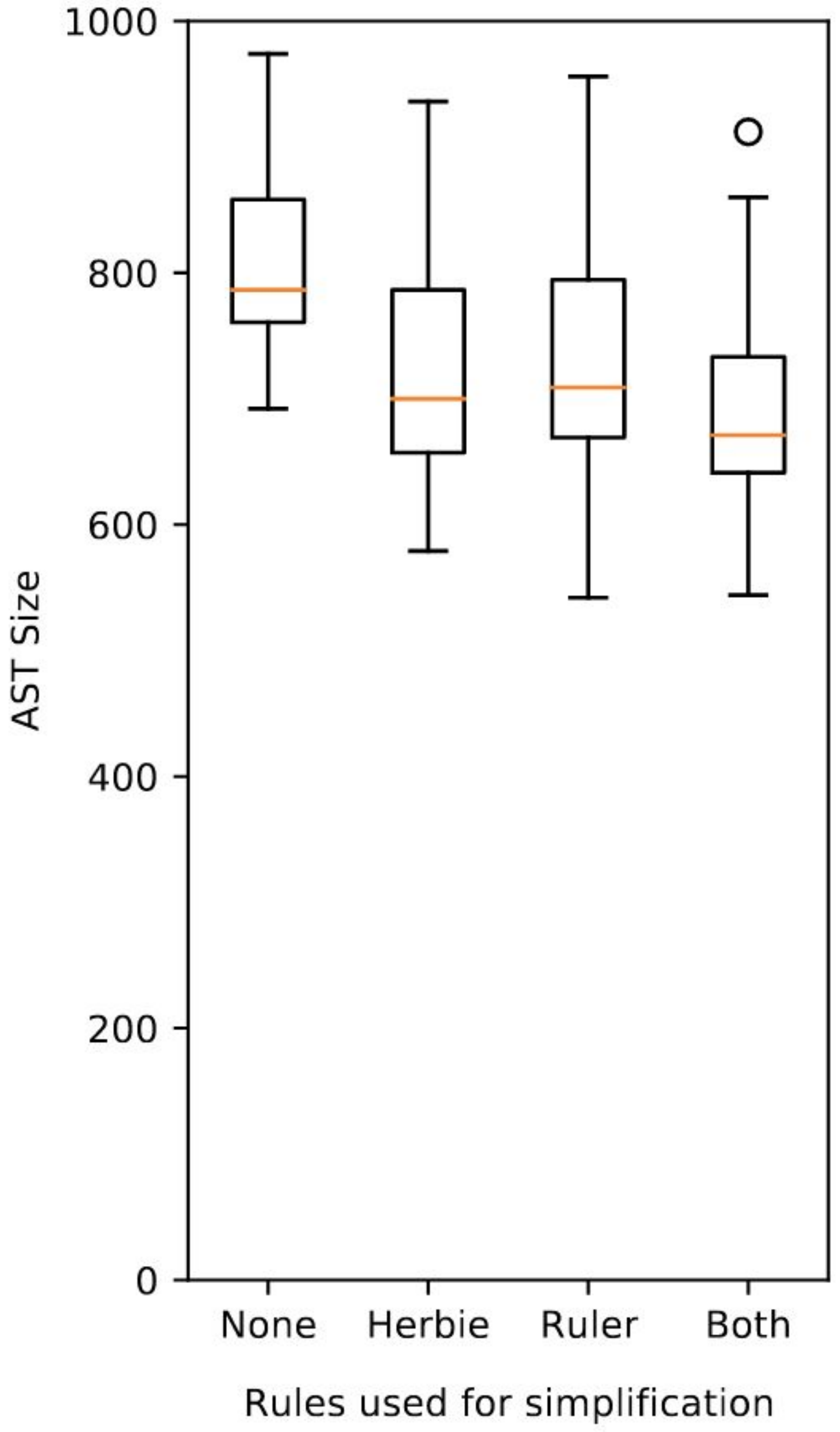
Ruler's rules are at least as good as the original Herbie rules

Rational Herbie: comparing AST size



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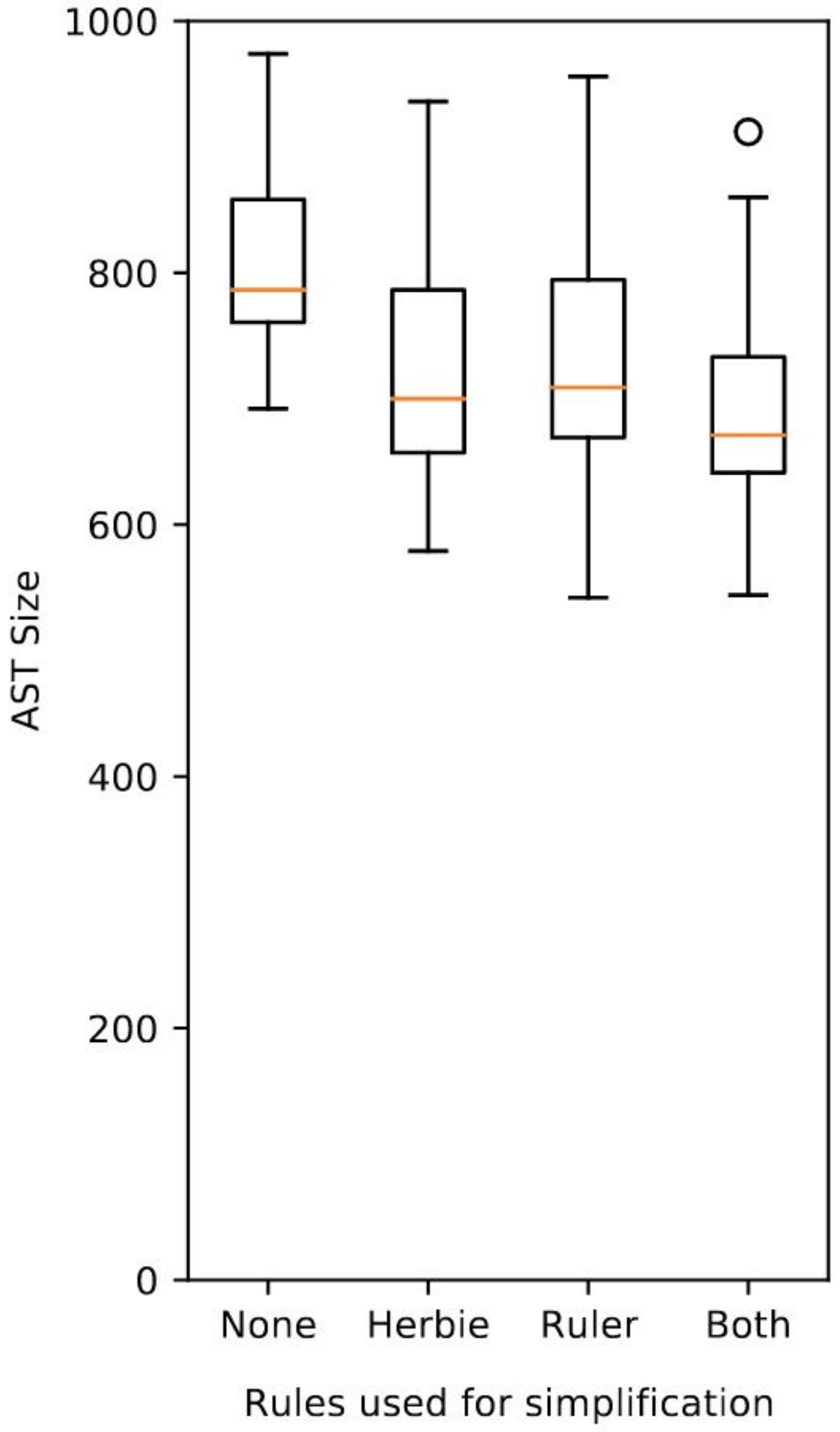

See paper for more results!

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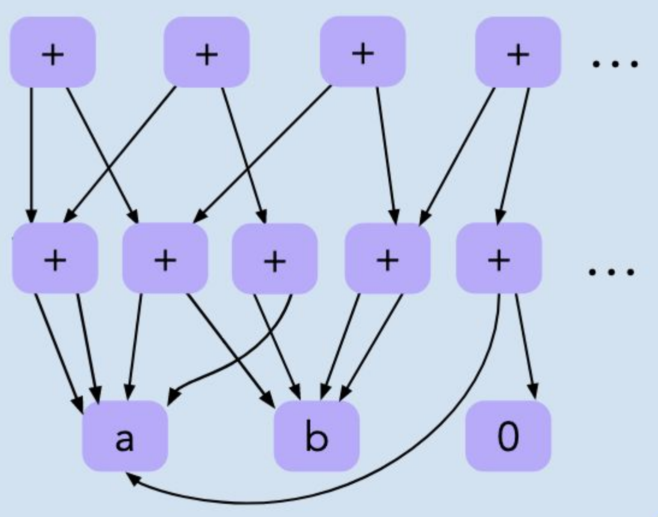
Rewrite Rule Inference Using Equality Saturation

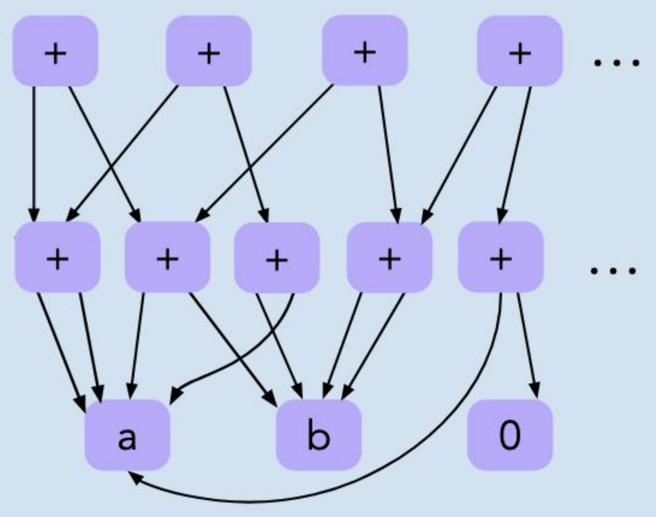
Enumerate terms from a grammar

a, b, 0, +, …



Find candidates: interpret over concrete inputs

“Fingerprints”



Filter candidates to get final ruleset

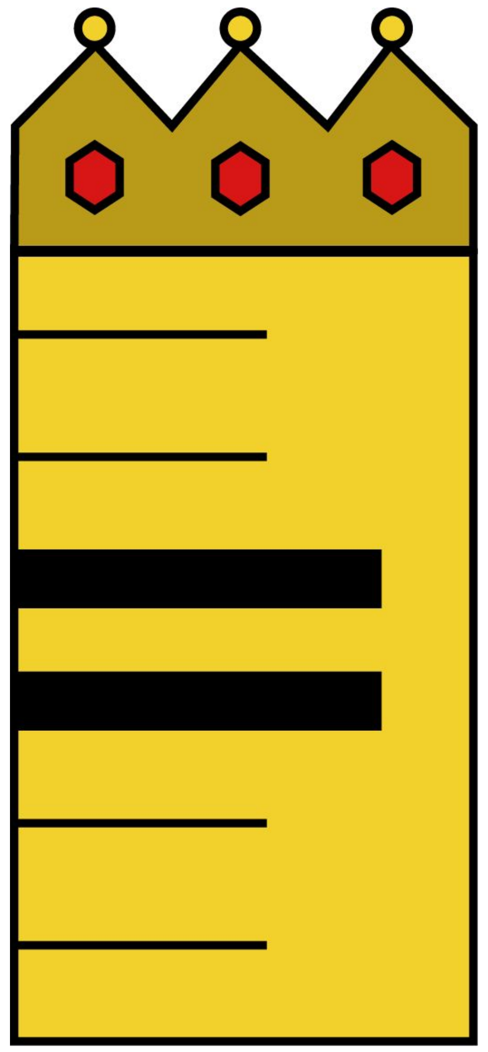
Remove redundant rules

x + 0 ↔ 0 + x

y + 0 ↔ 0 + y

x + y ↔ y + x

Equality Saturation improves all three steps!

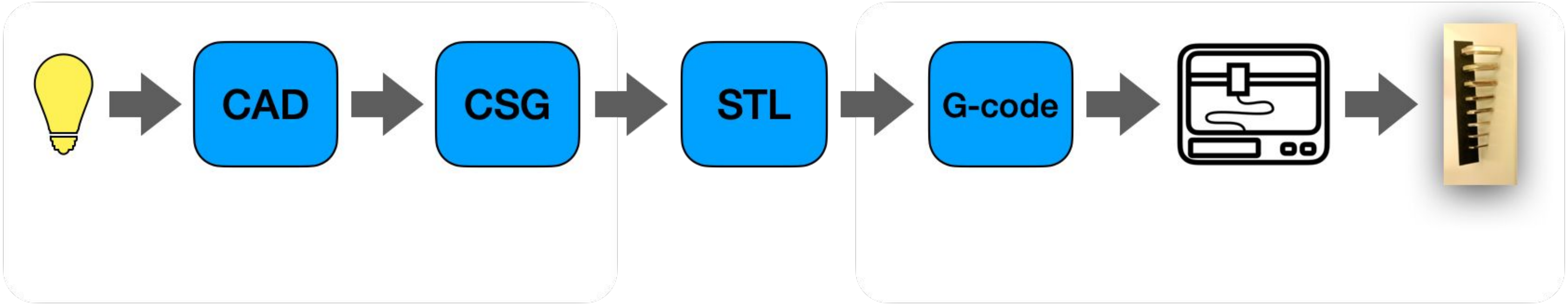


Ruler : https://github.com/uwplse/ruler

egg EqSat Toolkit [POPL 2021, Distinguished Paper]

- ✓ Deferred invariant maintenance & batching
- ✓ Relational e-matching [POPL 2022]
- ✓ E-class analyses
- ✓ Rewrite rule synthesis with Ruler [OOPSLA 2021, Distinguished Paper]
- ❏ Applications
  - ❏ 3D CAD in Szalinski, FP Accuracy in Herbie, Lib Learning in Babble, …
  - ❏ EVM simplify @ Certora, wasm JIT @ Fastly, datapath optimize @ Intel, …

Manufacturing is compilation!



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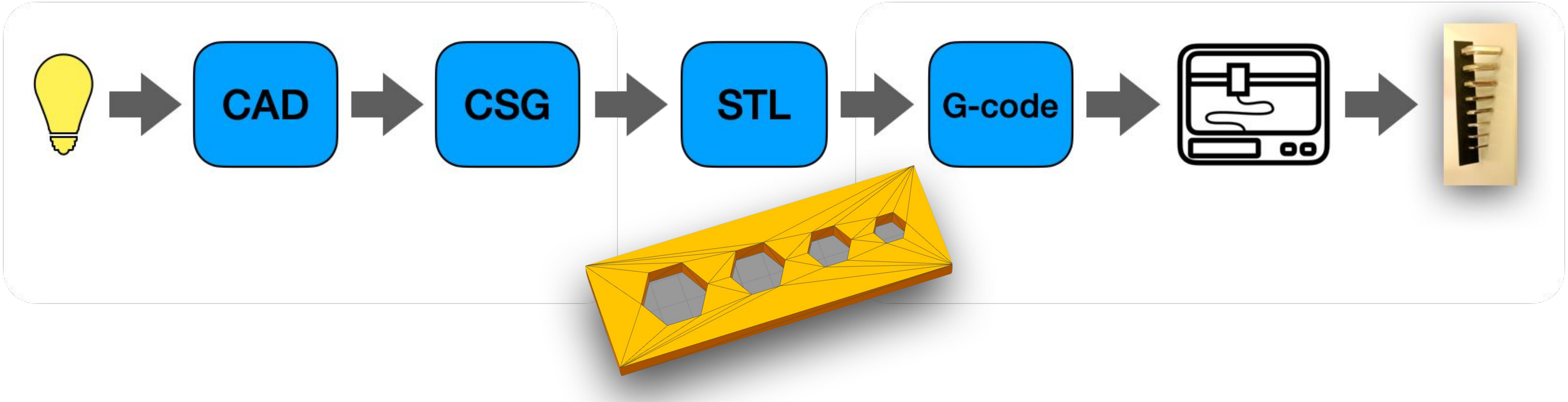
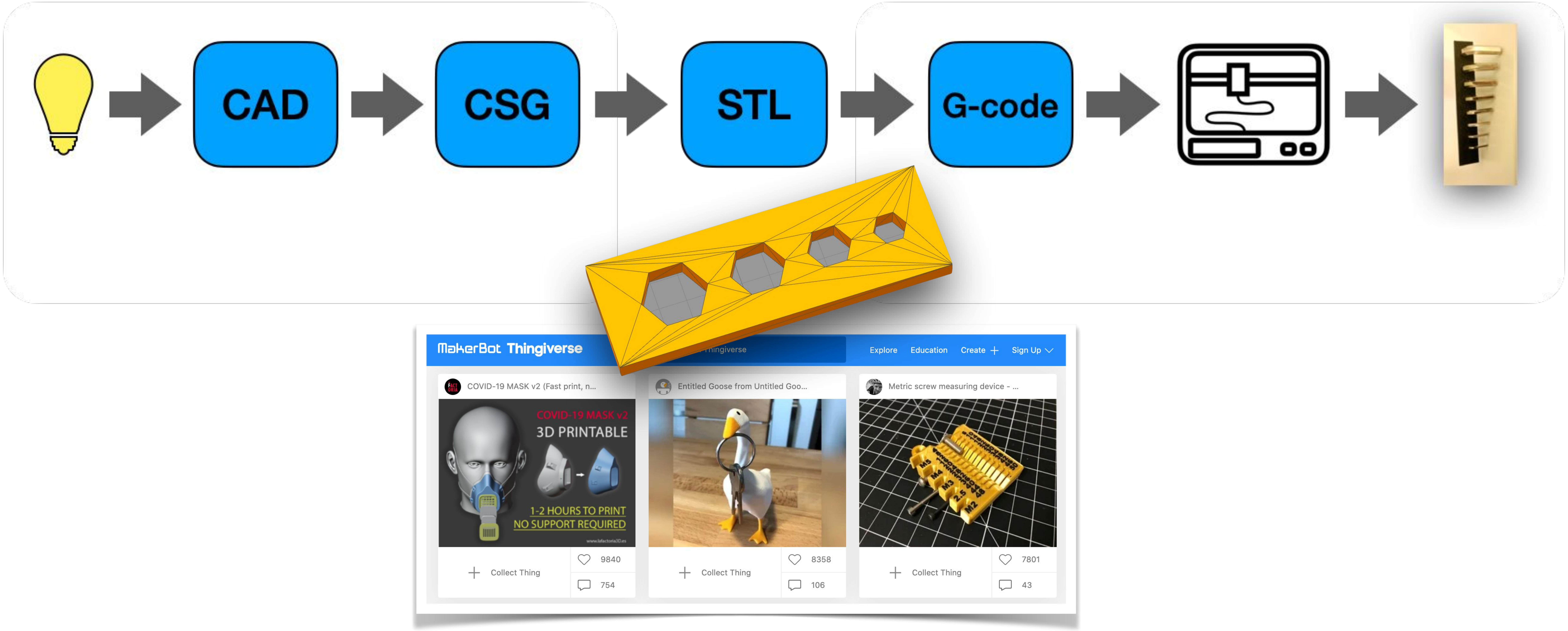


Design is programming!



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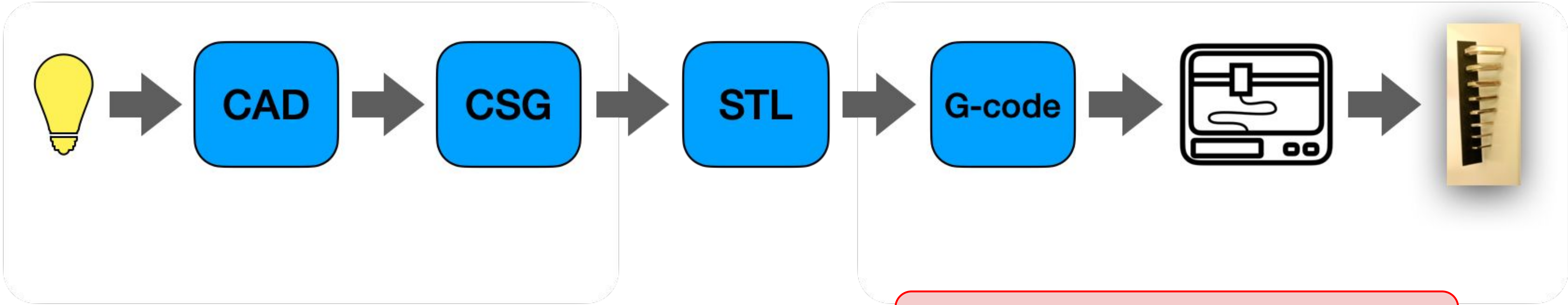

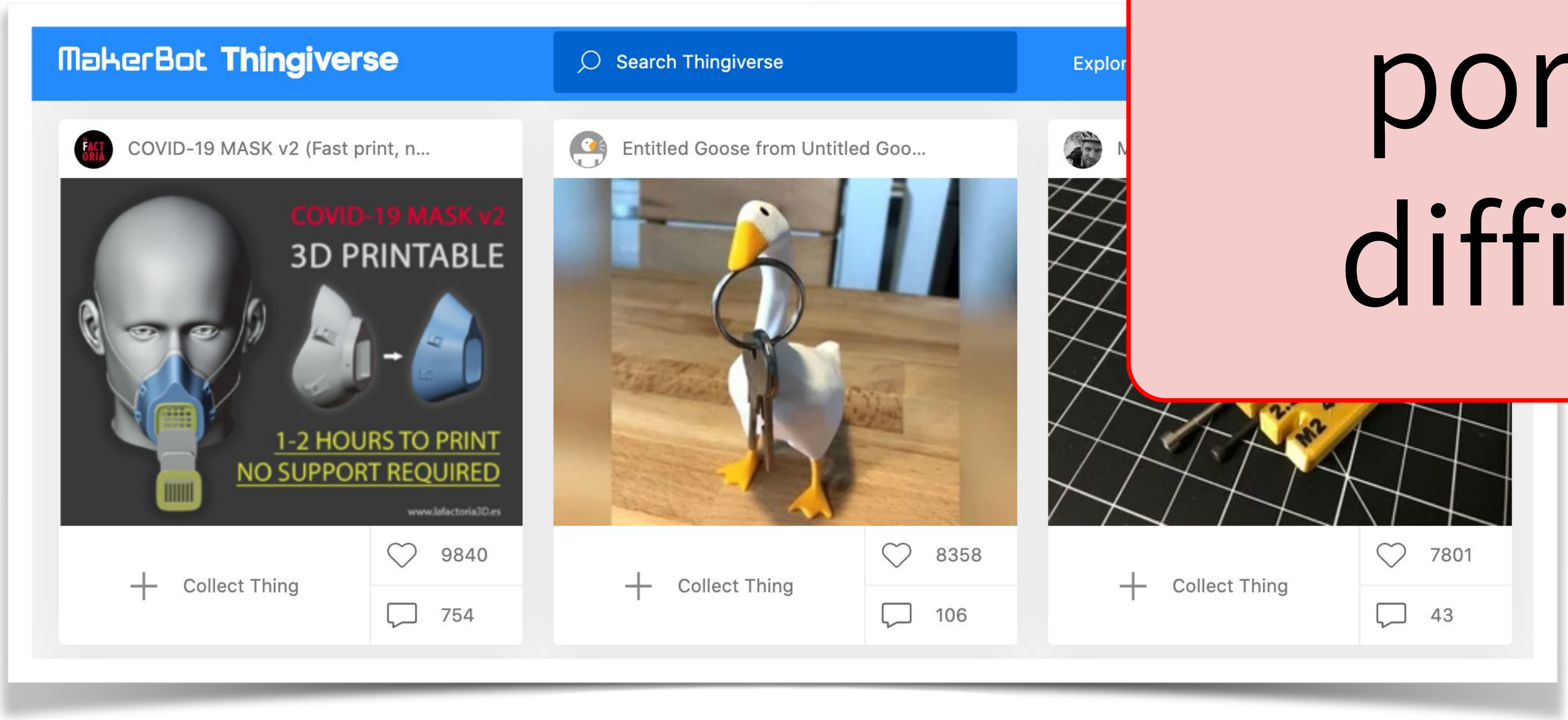

portable, BUT difficult to edit

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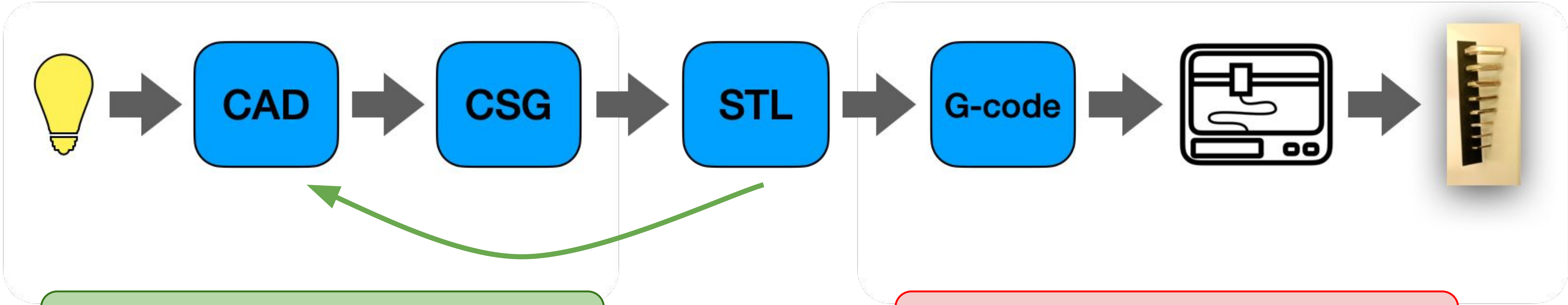
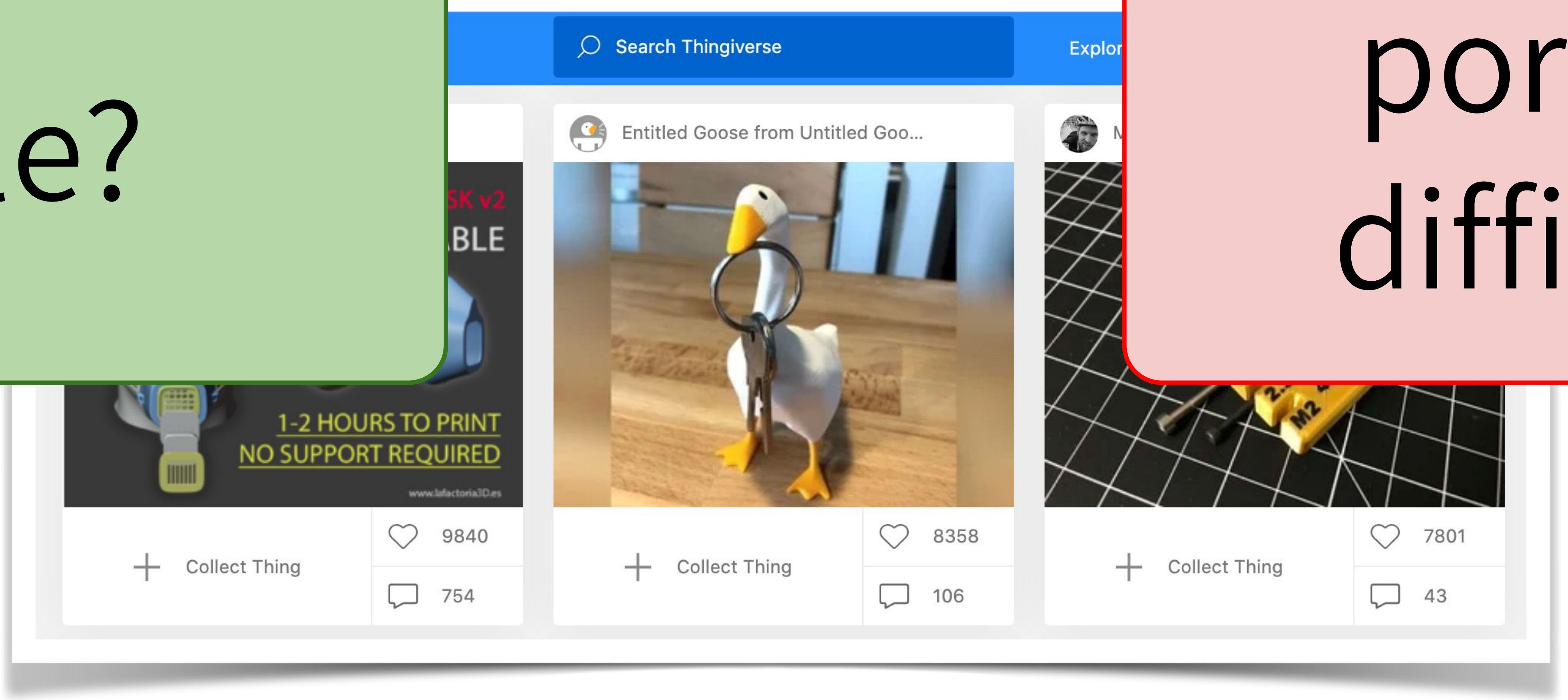


decompile?



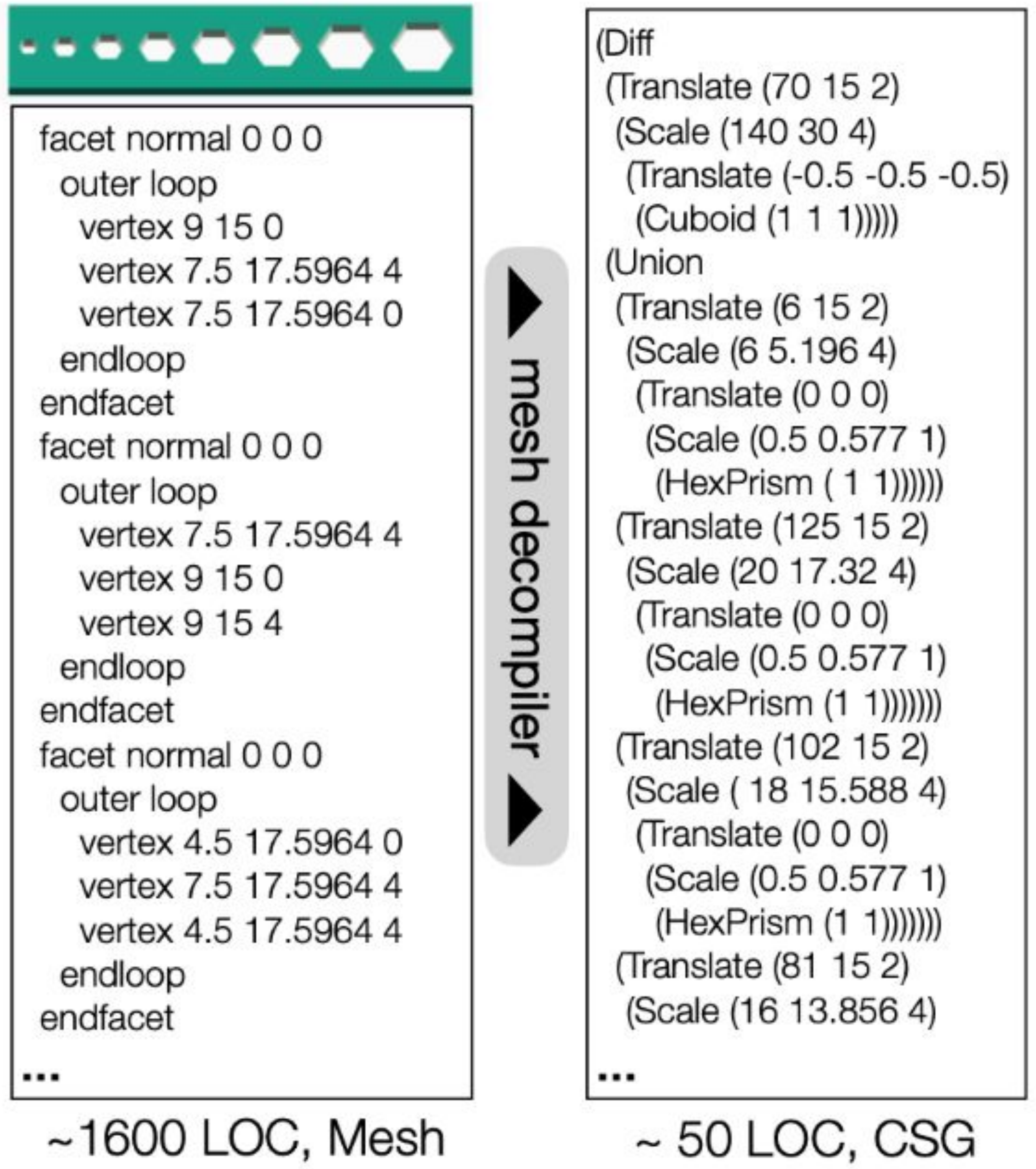
portable, BUT difficult to edit

Szalinski

```
facet normal 0 0 0
  outer loop
    vertex 9 15 0
    vertex 7.5 17.5964 4
    vertex 7.5 17.5964 0
  endloop
endfacet
facet normal 0 0 0
  outer loop
    vertex 7.5 17.5964 4
    vertex 9 15 0
    vertex 9 15 4
  endloop
endfacet
facet normal 0 0 0
  outer loop
    vertex 4.5 17.5964 0
    vertex 7.5 17.5964 4
    vertex 4.5 17.5964 4
  endloop
endfacet
...
```

~1600 LOC, Mesh

Szalinski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...
~1600 LOC, Mesh
mesh decompiler
(Diff
(Translate (70 15 2)
(Scale (140 30 4)
(Translate (-0.5 -0.5 -0.5)
(Cuboid (1 1 1)))))
(Union
(Translate (6 15 2)
(Scale (6 5.196 4)
(Translate (0 0 0)
(Scale (0.5 0.577 1)
(HexPrism (1 1))))))
(Translate (125 15 2)
(Scale (20 17.32 4)
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(Translate (102 15 2)
(Scale (18 15.588 4)
(Translate (0 0 0)
(Scale (0.5 0.577 1)
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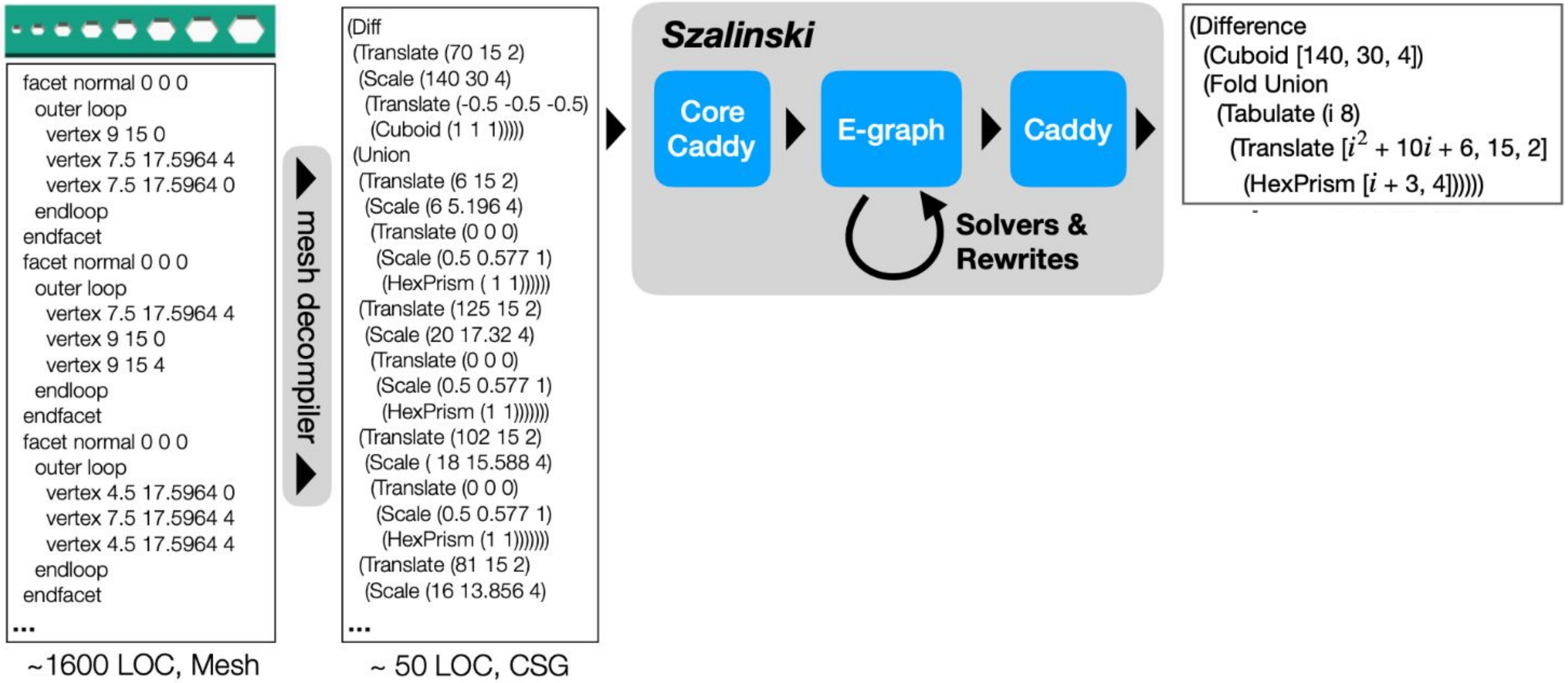
(Translate (81 15 2)
(Scale (16 13.856 4)
...
~ 50 LOC, CSG

Szalinski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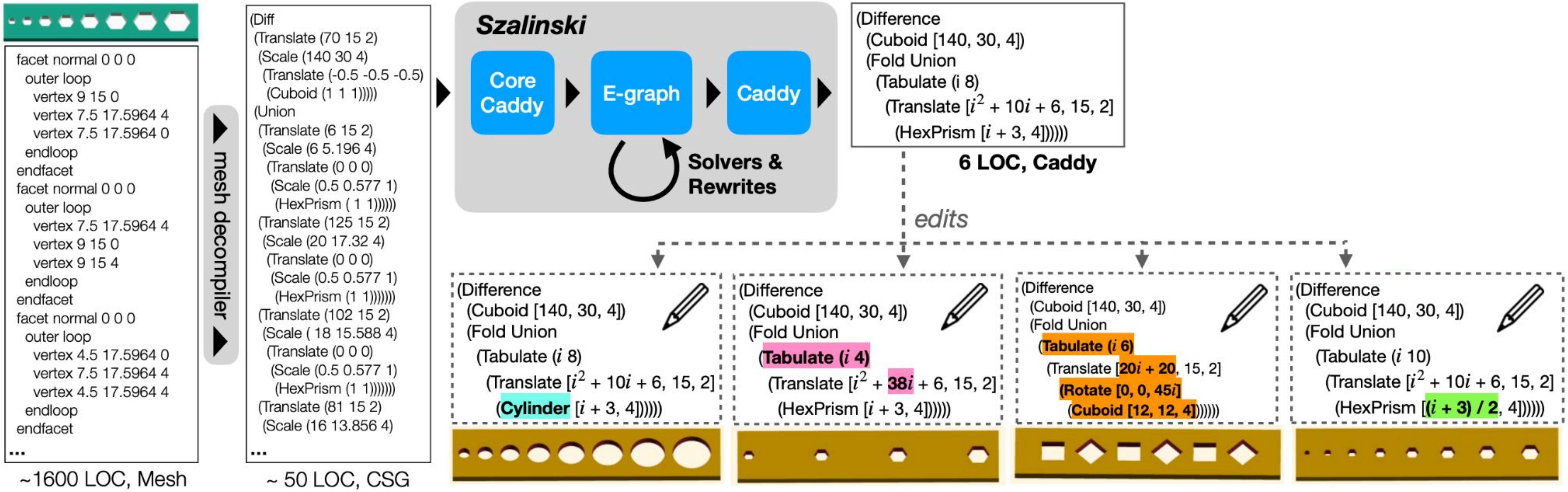
...
~ 50 LOC, CSG
Szalinski
Core Caddy
E-graph
Caddy
Solvers & Rewrites
(Difference
(Cuboid [140, 30, 4])
(Fold Union
(Tabulate (i 8)
(Translate [$i^2 + 10i + 6$, 15, 2]
(HexPrism [$i + 3$, 4])))))

Szalinski



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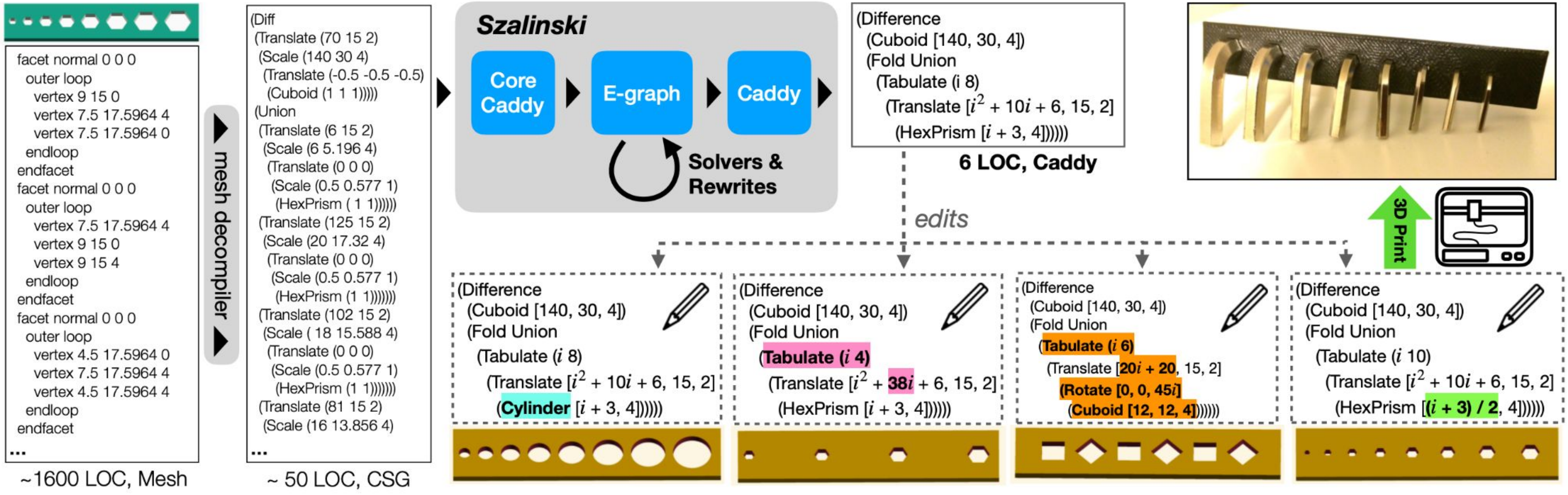
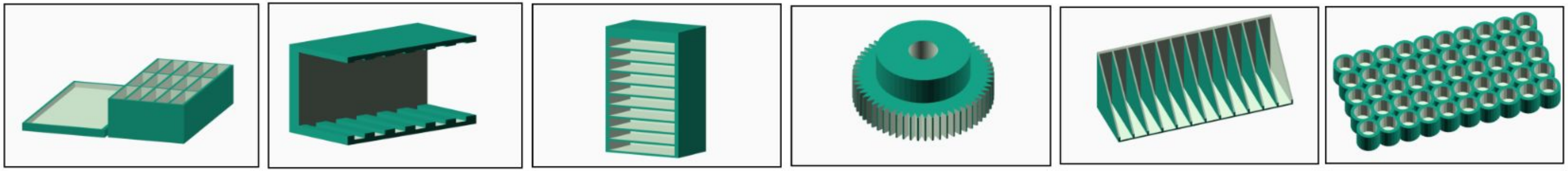
(Translate (0 0 0)
(Scale (0.5 0.577 1)
(HexPrism (1 1)))))))
(Translate (81 15 2)
(Scale (16 13.856 4)
...
~ 50 LOC, CSG
Szalinski
Core Caddy
E-graph
Caddy
Solvers & Rewrites
(Difference
(Cuboid [140, 30, 4])
(Fold Union
(Tabulate (i 8)
(Translate [i^2 + 10i + 6, 15, 2]
(HexPrism [i + 3, 4])))))
6 LOC, Caddy
edits
(Difference
(Cuboid [140, 30, 4])
(Fold Union
(Tabulate (i 8)
(Translate [i^2 + 10i + 6, 15, 2]
(Cylinder [i + 3, 4])))))
(Difference
(Cuboid [140, 30, 4])
(Fold Union
(Tabulate (i 4)
(Translate [i^2 + 38i + 6, 15, 2]
(HexPrism [i + 3, 4])))))
(Difference
(Cuboid [140, 30, 4])
(Fold Union
(Tabulate (i 6)
(Translate [20i + 20, 15, 2]
(Rotate [0, 0, 45i]
(Cuboid [12, 12, 4])))))
(Difference
(Cuboid [140, 30, 4])
(Fold Union
(Tabulate (i 10)
(Translate [i^2 + 10i + 6, 15, 2]
(HexPrism [(i + 3) / 2, 4])))))

Szalinski [PLDI 2020]



Szalinski [PLDI 2020]

- thousands of models decompiled w/ egg, all < 1 second



```
(Union (Difference
  (Cuboid [60, 120, 30])
  (Fold Union
    (Tabulate (i 5) (j 3)
      (Translate [12 * i + 2, 39.3 * j + 2, 2]
        (Cuboid [9.6 37.3 28])))))
 (Fold Difference
  (Map2 Translate
    (List [-67, -2, 0] [-65, 0, 2])
    (List (Cuboid [65, 125, 6])
          (Cuboid [60, 120, 4])))))
```

```
(Difference
 (Cuboid [57, 30, 30])
 (Difference
  (Translate [0, 5, 1.5] (Cuboid [57, 25, 27]))
  (Fold Union
   (Map2 Translate
    (Tabulate (i 7) (j 2) [9 * i, 5, 28 - 26.5 * j])
    (Concat
      (List (Tabulate (i 6) (j 2)
            (Cuboid [4.5, 25, j + 0.5]))
      (List (Cuboid [3, 25, 0.5])
            (Cuboid [3, 25, 1.5])))))))
```

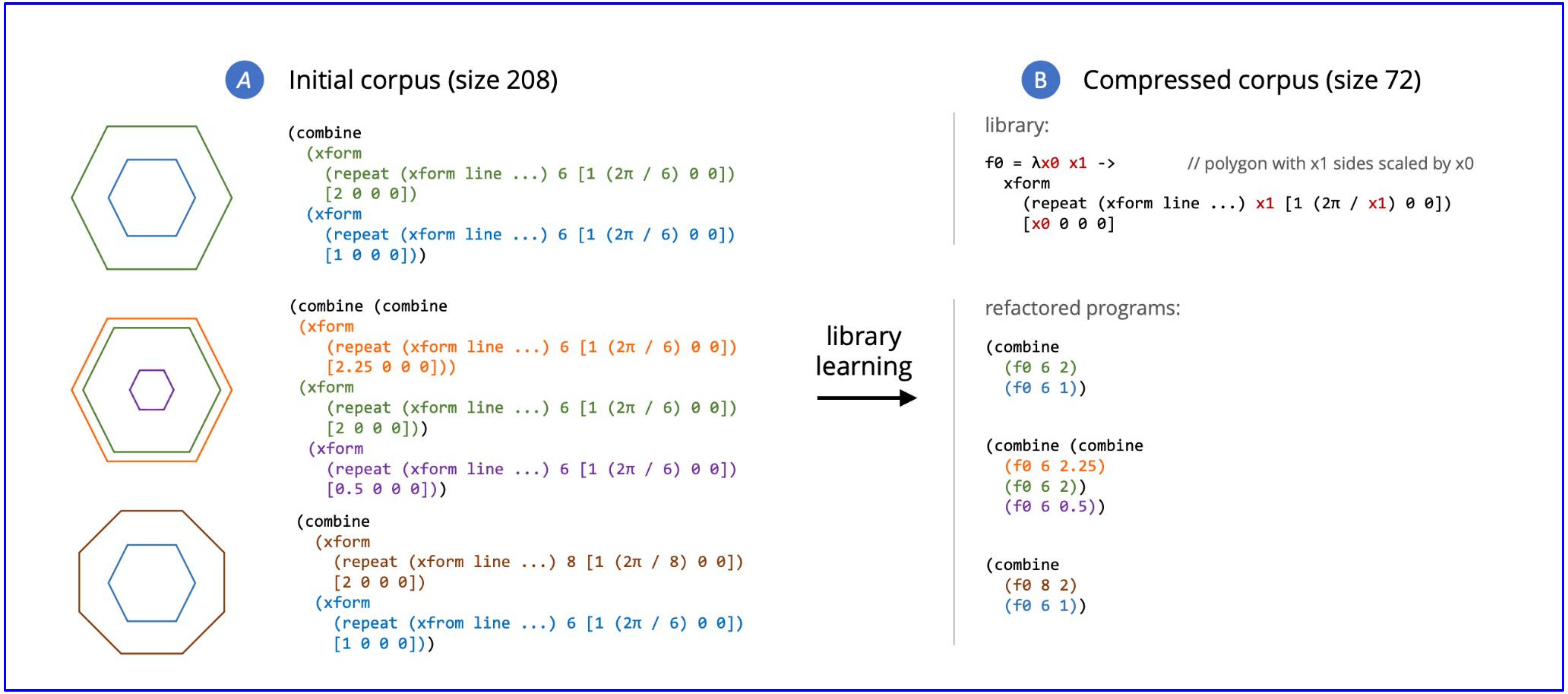
```
(Difference
 (Translate [-57, -3, 3]
  (Cuboid [117, 75, 175]))
 (Fold Union
  (Tabulate (i 10)
   (Translate [-51, -3, 16 * i + 6]
    (Cuboid [105, 58, 13])))))
```

```
(Fold Difference
 (List (Union
   (Cylinder [100, 80, 80])
   (Cylinder [50, 120, 120]))
  (Translate [0, 0, -1] (Cylinder [102, 25, 25]))
  (Fold Union (Tabulate (i 60)
    (Rotate [0, 0, 6 * i]
     (Translate [125, 0, 0]
      (Scale [2.5, 1, 1]
       (Rotate [0, 0, 45]
        (Translate [0, 0, 25]
         (Cuboid [10, 10, 52]))))))))))
```

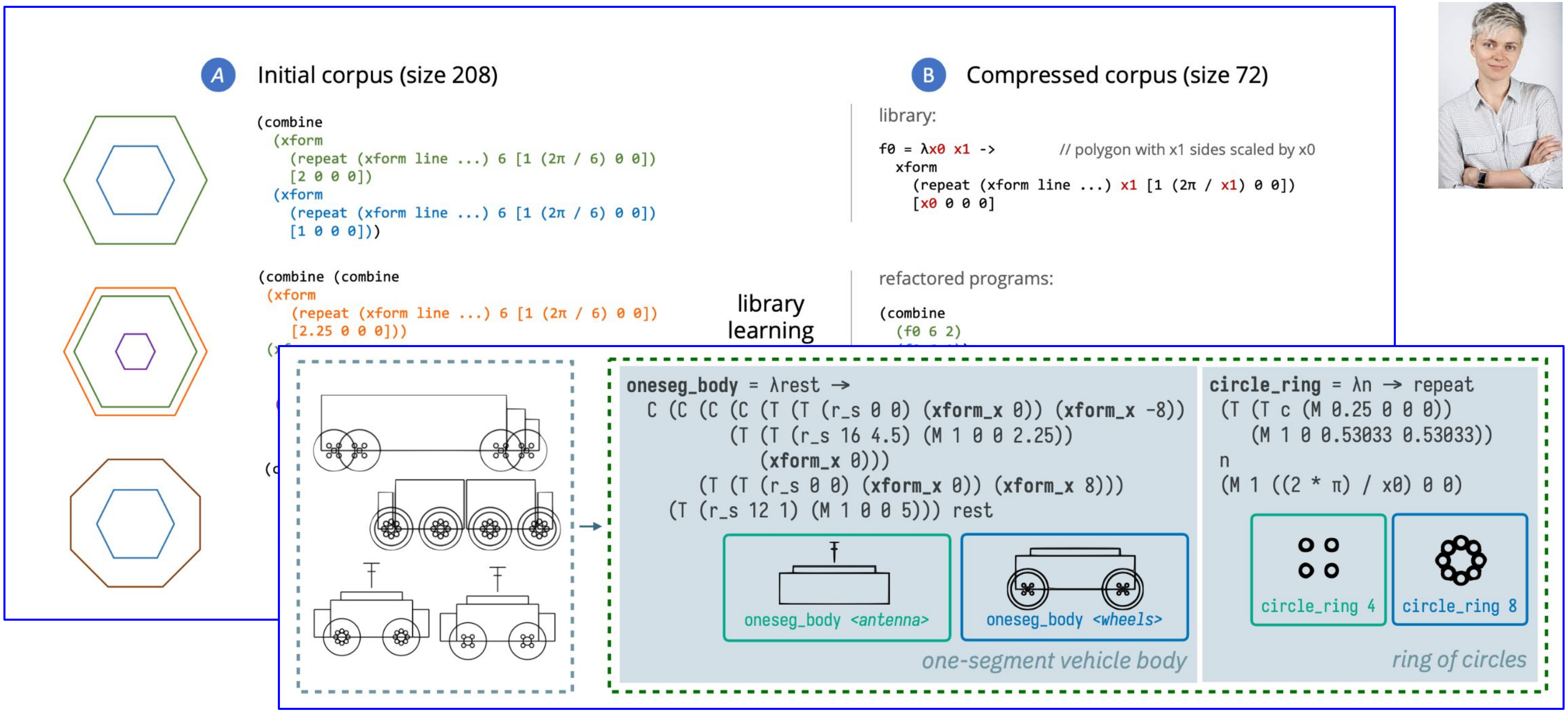
```
(Fold Union
 (Tabulate (i 12)
  (Translate [0, 13* i, 0]
   (Fold Difference
    (List
     (Cuboid [53.1 14.5 58])
     (Translate [1.5, 1.5, 1.5]
       (Cuboid [51.6, 11.5, 56.6]))
     (Translate [0 0 58]
      (Rotate [0, 45, 0]
       (Cuboid [101.5, 14.5, 100])))))))
```

```
(Fold Union
 (Tabulate (i 10) (j 5)
  (Translate
    [12.2 * i + 12.2, 12.2 * j + 12.2, 0]
   (Difference
    (Cylinder [13, 7.1, 7.1])
    (Translate [0, 0, 3]
     (Cylinder [11, 5.1, 5.1]))))))
```

Library learning with Babble [POPL 2023]



Library learning with Babble [POPL 2023]



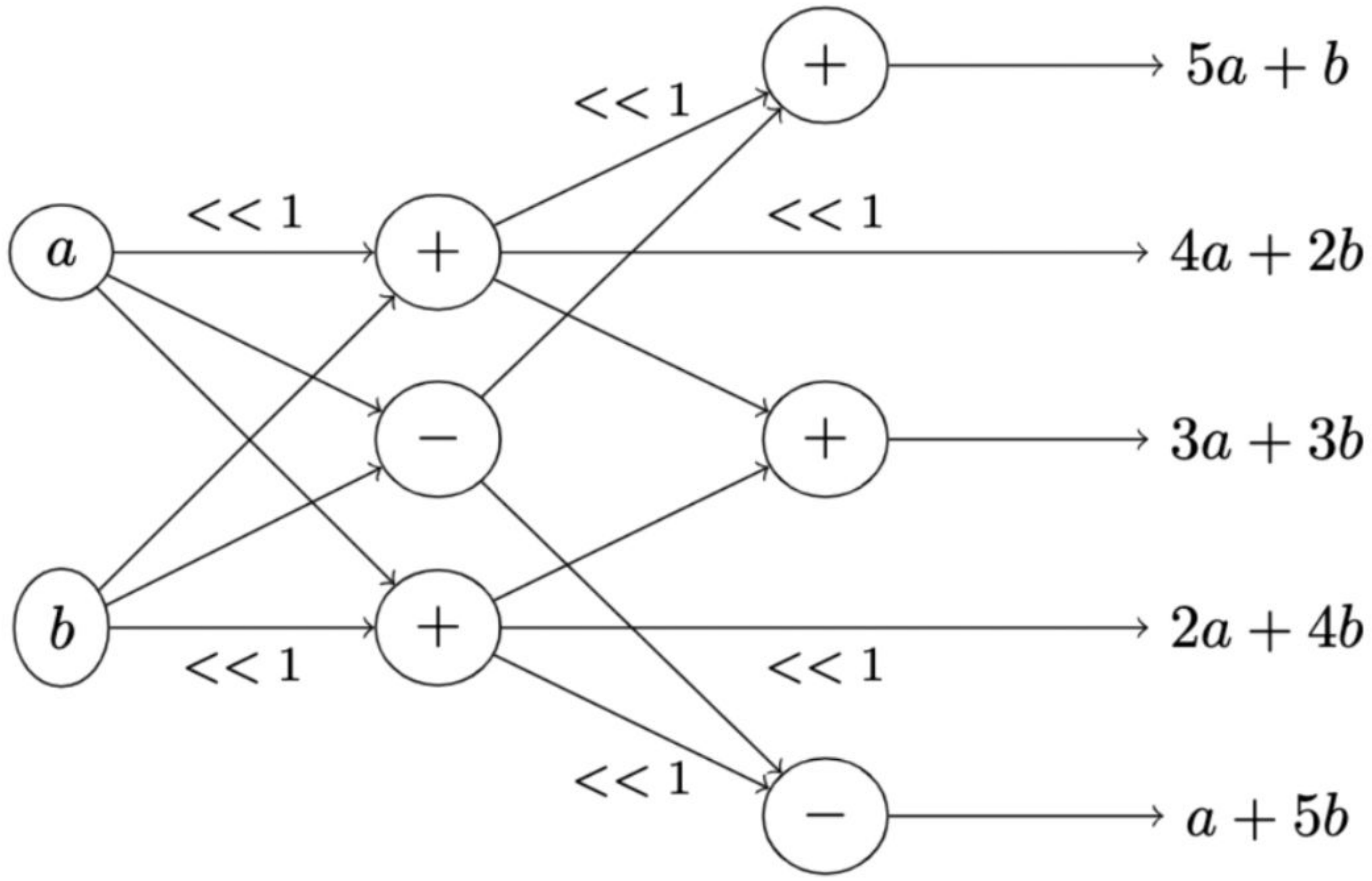
Short Proofs for TV + debugging [FMCAD 2022]

Intel Case Study

Multi-operation circuit optimization and translation validation with egg

4.7 hours -> 2.3 hours

2 inputs



5 outputs

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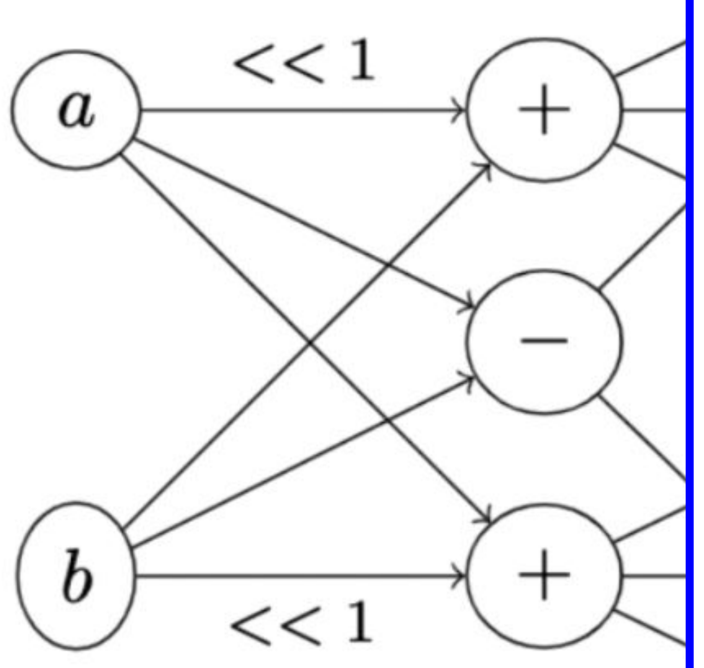
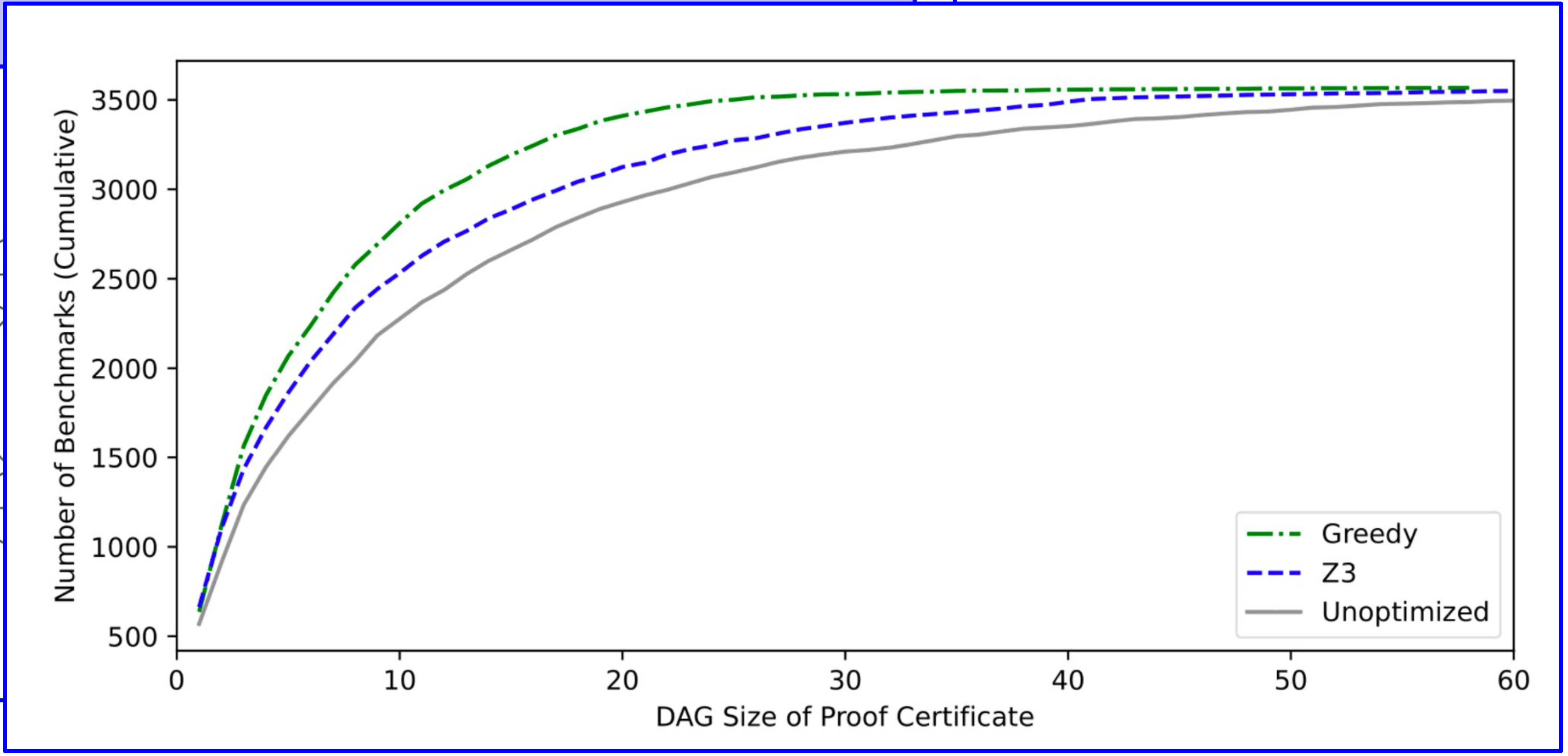
2 inputs





egg case studies

- Herbie: floating point — batching — *3000x faster*
- SPORES: linear algebra kernels — shape analysis — *1.2-5x better*
- Tensat: ML compute graphs — generic library — *23% better, 48x faster*
- Szalinski: CAD synthesis — dynamic rewrites — *12,000 part eval*
  < 1s synthesis
- …, TVM, Java testing, vectorization,
  hw/sw co-design, educational problems, ...

egg EqSat Toolkit [POPL 2021, Distinguished Paper]

- ✓ Deferred invariant maintenance & batching
- ✓ Relational e-matching [POPL 2022]
- ✓ E-class analyses
- ✓ Rewrite rule synthesis with Ruler [OOPSLA 2021, Distinguished Paper]
- ✓ Applications
  - ✓ 3D CAD in Szalinski, FP Accuracy in Herbie, Lib Learning in Babble, …
  - ✓ EVM simplify @ Certora, wasm JIT @ Fastly, datapath optimize @ Intel, …