Learning to Solve SMT Formulas

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Abstract

We present a new approach for learning to solve SMT formulas. We phrase the challenge of solving SMT formulas as a tree search problem where at each step a transformation is applied to the input formula until the formula is solved. Our approach works in two phases: first, given a dataset of unsolved formulas we learn a policy that for each formula selects a suitable transformation to apply at each step in order to solve the formula, and second, we synthesize a *strategy* in the form of a loop-free program with branches. This strategy is an interpretable representation of the policy decisions and is used to guide the SMT solver to decide formulas more efficiently, without requiring any modification to the solver itself and without needing to evaluate the learned policy at inference time. We show that our approach is effective in practice – it solves 17% more formulas over a range of benchmarks and achieves up to $100\times$ runtime improvement over a state-of-the-art SMT solver.

1 Introduction

Satisfiability Modulo Theories (SMT) solvers are a powerful class of automated theorem provers which can deduce satisfiability and validity of first-order formulas in particular logical theories (e.g., real numbers, arrays, bit vectors). SMT solvers are more general than SAT solvers and have been used in a variety of application domains including verification (e.g., neural networks [28]), program synthesis, static analysis, scheduling, and others [16].

To efficiently solve complex real world problems, state-of-the-art SMT solvers (e.g., `Z3` [15], `Yices` [18], `CVC4` [4], `MathSAT5` [13], `Boolector` [35]) contain hand-crafted heuristics combining algorithmic proof methods and satisfiability search techniques. Indeed, crafting the right heuristic is critical and can be the difference between solving a complex formula in seconds or not at all. To enable end users to create suitable heuristics, several modern SMT solvers such as `Z3` [15] provide a so called tactic language for expressing heuristics. Typically, such tactics are combined and performed in a sequence (specified by the user), forming an interpretable program called a *strategy*. However, the resulting strategies can often end up being quite complex (e.g., containing loops and conditionals). Combined with the vast search space, this means that manually finding a well-performing strategy can be very difficult, even for experts.

Our Contributions We present a new approach, based on a combination of learning and synthesis techniques, which addresses the problem of automatically finding the right SMT strategy. Given a dataset of formulas whose solution is initially unknown, we first train a policy that searches for strategies that are fast at solving the formulas in this dataset. Then, we synthesize a single strategy (a program with branches) that captures the policy decision making in an interpretable manner. The resulting strategy is then passed on to the SMT solver which uses it to make more effective decisions when solving a given formula. We note that our approach does not require any changes to the solver's internals and thus can work with any decision procedure which cleanly exports its tactics.

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We performed an extensive experimental evaluation of our approach on formulas of varying complexity across 3 different theories (QF_NRA, QF_BV and QF_NIA). We show that our synthesized strategies solve 17% more formulas and are up to $100\times$ faster when compared to the default strategy used in the state-of-the-art Z3 solver. Further, our learned strategies generalize well and can solve formulas which are much more challenging than those seen during training. Finally, we make our tool, called fastSMT, datasets and experiments available at http://fastsmt.ethz.ch/.

2 Related work

Given the importance and wide range of SMT solver applications, a number of approaches have been developed to improve both, their runtime as well as the range of formulas they can solve.

Portfolio based approaches The most common approach of tools such as SATzilla [44], CPhydra [12], SUNNY [2], Proteus [22], ISAC [27] is a portfolio based method. The key idea is that different SMT solvers use different heuristics and hence work well for different types of formulas. Then, given several SMT solvers and a dataset of formulas, the goal is to learn a classifier which uses features extracted from the given formula to select the right SMT solver (or alternatively defines order in which to run the solvers). In comparison, we address a harder problem - learn how to instantiate an SMT solver with a strategy that efficiently solves the given dataset of formulas.

Evolutionary search The work most closely related to ours is StratEVO [39] which also studies the task of generating SMT strategies. However, StratEVO has several limitations – it performs search using an evolutionary algorithm which does not incorporate any form of learning, the search does not depend on the actual formula, and local mutations tend to get stuck in local minima (as we show in our experiments in Section 5). As a result, for many tasks where the suitable strategy cannot be trivially found (e.g., is very short) their approach reduces to random search. Instead, we leverage models that learn from previously explored strategies as well as the current formula. As we show, this enables discovery of complex strategies that are out of reach for approaches not based on learning.

Learning branching heuristics and premise selection Recently, learning techniques have been applied to improving performance of SAT solvers [32, 42], constraint programming solvers [34], solving quantified boolean formulas [41], solving mixed integer programming problems [29] as well as premise selection in interactive theorem provers [1, 43, 33]. At a high level, these are complementary to us – we learn to search across many tactics and combine them into high level strategies while they optimize a single tactic (e.g., by learning which variable to branch on in SAT). Our work also supports formulas from a wide range of theories (as long as there is a corresponding tactic language) where selecting a suitable high level strategy leads to higher speedups compared to optimizing a single tactic in isolation. However, there are also common challenges such as defining a suitable formula representation. This representation can range from a set of hand-crafted features [32, 36], to recursive and convolutional neural networks [33, 1], to graph embeddings [43]. We extend this line of work by considering fast to compute representations based on bag of words, syntactic features, and features extracted from a graph representation of the formula.

Parameter tuning Finally, a number of approaches exist for finding good parameter configurations from a vast space of both discrete and continuous parameters, including ParamILS [25], GGA [3], TB − SPO [24] or SMAC [23]. An interesting application of such approaches to the domain of SAT/SMT solvers is proposed by SATenstein [30] which first designs a highly parameterized solver framework to be subsequently optimized by off-the shelf tools. Although such tools are not applicable for the task of searching for strategy programs (that include loops and conditionals) considered in our work, they can be used to fine-tune the strategy parameters once a candidate strategy is found.

3 SMT strategies: preliminaries

At a high level, an SMT solver takes as input a first-order logical formula and then tries to decide if the formula is satisfiable. In the process, solvers employ various heuristics that first transform the input formula into a suitable representation and then use search procedures to check for satisfiability. Existing heuristics include: simplify which applies basic simplification rules such as constant

Table 1: Formalization of the Strategy language used to express SMT strategies in Z3 [15].

(Strategy)	q	::=	t | q; q | if p then q else q | q else q | repeat q, c | try q for c | using t with params
(Tactics)	t	∈	Tactics = { bit_blast, solve_eqs, elim_uncnstr ... }
(Predicates)	p	::=	p ∧ p | p ∨ p | expr ⋈ expr
(Expressions)	expr	::=	c | probe | expr ⊕ expr
(Constants)	c	∈	Consts = ℚ
(Probes)	probe	::=	Probe → ℚ, Probe = { num_consts, is_pb, ... }
(AOperators)	⊕	::=	+ | − | ∗ | /
(BOperators)	⋈	::=	> | < | ≥ | ≤ | = | ≠
(Parameter)	param	::=	(Param, ℚ), Param = { hoist_mul, flat, som, ... }
(Parameters)	params	::=	ϵ | param; params

folding ($x + 0 \rightarrow x$), removal of redundant expressions ($x - x \rightarrow 0$), gaussian_elim which eliminates variables ($x = 1 \wedge y \geq x + z \rightarrow y \geq 1 + z$) using Gaussian elimination, elim_term_ite which replaces the term if-then-else with fresh auxiliary declarations ($\texttt{if}\ x > y\ \texttt{then}\ x\ \texttt{else}\ y) > z \rightarrow k > z \wedge (x > y \Rightarrow k = x) \wedge (x \leq y \Rightarrow k = y)$, or bit_blast which reduces bit-vector expressions by introducing fresh variables, one for each bit of the original bit-vector (e.g., a bit vector of size 4 is expanded into four fresh variables). In total, the Z3 SMT solver defines more than 100 such heuristic transformations (called tactics) that can be combined together to define a custom strategy. For example, a strategy for integer arithmetic can be defined as[1]:

using simplify with (som: true); normalize_bounds; lia2pb; pb2bv; bit_blast; sat

Although the above sequence of transformations (tactics) works well for some types of input formulas (e.g., in case every variable has a lower and an upper bound), for other formulas a different set of tactics is more suited. In some cases, the suitable set of tactics can be obtained by a small modification of the original tactic while in others, a completely different set of tactics needs to be defined. As a concrete example, consider the following strategy implemented in the Yices SMT solver [18, 17]:

$$\texttt{if}\ (\neg\texttt{diff} \wedge \frac{\texttt{num\_atoms}}{\texttt{dim}} < k)\ \texttt{then simplex else floyd\_warshall}$$

Here, two high level tactics can be applied to solve an input formula – the Simplex algorithm or the algorithm based on Floyd-Warshall all-pairs shortest distance. The Simplex algorithm is used if the input formula is not in the difference logic fragment (denoted using ¬diff) and the ratio of inequalities divided by the number of uninterpreted constants is smaller than a threshold k.

Language for expressing SMT solver strategies In Fig. 1 we formalize the language used by Z3 to enable control over the core solver heuristics. The basic building blocks of the language are called tactics and represent various heuristic transformations that might be applied during the search. Optionally, each tactic can define a set of parameters that affect its behavior. For example, the simplify tactic contains > 50 different parameters that control which simplifications are performed (e.g., if som : true then simplify puts polynomials in som-of-monomials form). The tactics are combined into larger programs either sequentially or using one of the following control structures:

- if p then q_1 else q_2: If the predicate p evaluates to true apply q_1, otherwise apply q_2. The predicate can contain arithmetic expressions as well as Probes which collect statistics of the current formula (e.g., num_consts returns the number of non-boolean constants).
- q_1 else q_2: First apply q_1 and if it fails then apply q_2 on the original input.
- repeat q, c: Keep applying q until it does not modify the input formula any more or the number of iterations is greater than the constant c.
- try q for c: Apply q to the input and if it does not return in c millisenconds then fail.
- using t with params: Apply the given tactic t with the given parameters params.

[1]For more information and examples we refer the reader to the online tutorial available at: https://rise4fun.com/z3/tutorial/strategies

4 Learning SMT strategies

We now present our approach for synthesizing SMT strategies. Formally, the task we are interested in solving is defined as follows:

Problem statement Given a dataset of SMT formulas $\mathcal{F} = \{f_i\}_{i=1}^{N}$, our goal is to find a strategy:

$$q_{best} = \underset{q \in \texttt{Strategy}}{\arg\min} \sum_{i=1}^{N} cost(f_i, q) \quad \text{where } cost(f_i, q) \stackrel{\text{def}}{=} \begin{cases} \texttt{runtime}(q(f_i)) & \text{if } q \text{ solves } f_i \\ \texttt{timeout\_penalty} & \text{otherwise} \end{cases} \tag{1}$$

Here, $\texttt{runtime}(q(f_i)) \in \mathbb{Q}$ denotes the runtime required for strategy q to solve formula f_i and $\texttt{timeout\_penalty} \in \mathbb{Q}$ is a constant denoting the penalty for not solving f_i (either due to timeout or because the strategy q is not powerful enough). Our goal is to find a strategy that minimizes the time required to solve the formulas in the dataset $\mathcal{F}$. Note that our $cost$ function reflects the fact that we aim to synthesize a strategy that solves as many formulas as possible yet one that is also the fastest. Generally, optimizing for Equation 1 directly is problematic as using runtime makes the optimization problem inherently noisy and non-deterministic. It also makes learning hard to parallelize due to significant impact of hardware and environment on overall runtime. Thus, instead of runtime, we use the amount of work measured as the number of basic operations performed by the solver required to solve a formula (e.g., implemented via the `rlimit` counter in Z3).

Challenges To solve the problem of learning SMT strategies, one has to address two challenges:

- *Interpretability*. We are interested in learning a model that is not only efficient at solving a given set of formulas but also expressible as programs in the `Strategy` language. This is important as the learned strategies can then be directly used an input to existing solvers.
- *No prior domain knowledge*. Our learning does not assume any prior knowledge about the dataset $\mathcal{F}$ and which strategies work best in general – initially the solution to all the formulas in the dataset is unknown, no existing strategies are used to boostrap the learning (not even the default strategies already written by the SMT solver developers) and we do not rely on any heuristics (and their combination) that may be useful in solving formulas from $\mathcal{F}$. Indeed, this represents the most challenging setting for learning.

Our approach The key idea behind learning a program $q_{best} \in \texttt{Strategy}$ efficiently is to take advantage of the fact that each program q can be decomposed into a set of smaller branch-free programs $q_{1,\ldots,k}$, each q_i corresponding to one execution path of q. This is possible because programs in the `Stategy` language do not contain state (all state is implicitly captured in the formula being solved). As a result, in our approach we learn a program $q_{best} \in \texttt{Strategy}$ via a two step process:

- *Learn candidate strategies*. First, we learn a policy which finds a set of candidate strategies consisting of only sequences of tactics where each strategy performs well on a different subset of the dataset $\mathcal{F}$. This allows us to phrase the learning problem as a tree search over tactics for which state-of-the-art models can be used. This step is described in Section 4.1.
- *Synthesize a combined strategy*. Then, given a set of candidate strategies, we combine these into a single best strategy q_{best} by synthesizing control structures such as branches and loops as supported by the `Strategy` language. This step is described in Section 4.2.

4.1 Learning candidate strategies

We phrase the problem of learning candidate strategies as a tree search problem. We start from an initial state s_0 and at each timestep t we choose an action a_t that expands the tree by exploring a single edge. In our setting, a state corresponds to a tuple consisting of an SMT formula and a strategy used to compute the formula. An action is described by a tactic and its parameters (if any) $a \in \texttt{Tactics} \times \texttt{Params}$. Applying an action transforms the formula into another one. Terminal states are those that decide the initial formula f, that is, f was reduced to a form that is either trivially satisfiable or unsatisfiable. Further, for practical reasons, terminal states also include those to which applying an action (tactic) leads to a timeout.

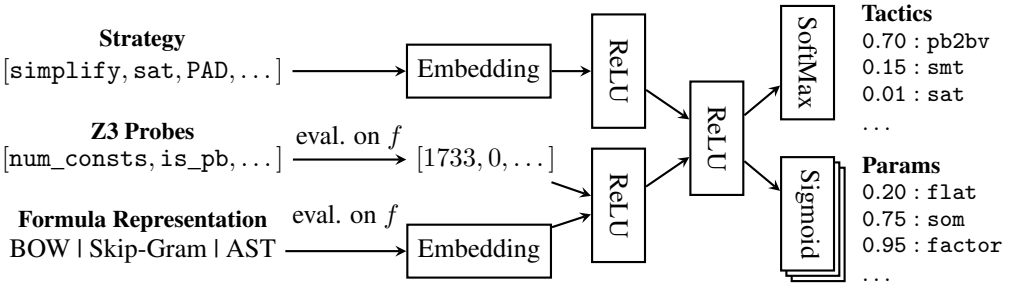


Figure 1: Architecture of the *neural network* model which predicts next tactic and arguments.

A terminal state defines a strategy q (i.e., a sequence of tactics starting from the tree root) with an associated $cost(f, q)$ (as defined in Equation 1) that we aim to minimize. During the search we keep a priority queue of tuples (s, a, p) consisting of a state, possible action and its associated probability, initialized with $(s_0, \epsilon, 1)$. At each step, we remove the tuple with highest probability from the queue, apply its action to obtain a new state s_i and update the priority queue with new tuples $(s_i, a, p \cdot \pi(a \mid s_i))$ for all actions $a \in \texttt{Tactics} \times \texttt{Params}$ capturing the possible transitions from s_i. Our goal is to learn a policy, denoted as $\pi(a \mid s)$, that represents a probability distribution of actions conditioned on a state s. Ideally, the learned distribution should be such that selecting the most likely action at that state minimizes the expected $cost$. In what follows we first describe the models used to represent the policy π considered in our work and then describe how the models are trained, including how to construct a suitable training dataset.

Bilinear model Based on matrix factor models used in unsupervised learning of word embeddings [11, 38] as well as supervised text classification [26] we define a bilinear model as follows:

$$\pi(a \mid s) = \sigma(\boldsymbol{U}\boldsymbol{V}\phi(s, a))$$

where $\phi(s, a)$ is a bag of features computed for action a taken in state s, $\boldsymbol{U} \in \mathbb{R}^{k \times h}$ and $\boldsymbol{V} \in \mathbb{R}^{h \times |V|}$ are the learnable weight matrices, V is the input vocabulary and σ is softmax computing the probability distribution over output classes (in our case actions to be taken in a given state s). As the set of all possible actions is too large to consider, we randomly generate a subset of parameters for each tactic before training starts, thus obtaining the overall set of actions $S \subseteq \texttt{Tactics} \times \texttt{Params}$. We define $\phi(s, a)$ to be all n-grams constructed from the strategy of state s. For example, for strategy $a_1; a_2; a_3$, the extracted n-grams (features) are described by the vector: $\langle a_1, a_2, a_3, a_1a_2, a_2a_3, a_1a_2a_3 \rangle$. Then, the vocabulary set V is simply the collection of all possible n-grams formed over S.

Neural network Our second model is based on a neural network and improves over the bilinear model in two key aspects – it considers a richer state when making the predictions and predicts tactics as well as their corresponding parameters. The architecture of the neural network model is illustrated in Fig. 1 and uses two inputs: (i) the strategy in the current state (as in the bilinear model), and (ii) the formula f to be solved in the current state. The strategy is encoded by first padding it to be of fixed size, concatenating the embedding of each action in the strategy into a single vector and finally feeding the resulting vector into a fully-connected layer. We encode the input formula in two ways: first, by computing a set of formula measures such as the number of expressions and constants, and second, by learning a representation of the formula itself. The formula measures are computed using probes supported by Z3 and are a subset of the possibilities one could define. For the learned representation of the formula we experimented with three different approaches:

- **Bag-of-words (BOW)**: The formula is treated as a sequence of tokens from the SMT-LIB language. We obtain its bag-of-words and use it as the formula embedding.
- **Abstract Syntax Tree (AST)**: From the formulas's AST, we extract all subtrees of depth at most two. The bag-of-words over such subtrees is used as the formula embedding. Note that subtrees which appear in less than 5% of formulas in $\mathcal{F}$ are discarded.
- **Skip-gram**: Each formula in the dataset is treated as a sequence of tokens over which we learn a skip-gram model. We define embedding of the formula as the average of all embeddings of its tokens.

Algorithm 1: Iterative algorithm used to train policy π

Data: Formulas $\mathcal{F}$, Number of iterations N, Number of formulas to sample K, Exploration rates $\boldsymbol{\beta}$, Exploration policy $\pi_{explore}$ (e.g., random policy)
Result: Trained policy π, Explored strategies $\mathcal{Q}$

1 $\mathcal{D} \leftarrow \emptyset$; $\mathcal{Q} \leftarrow \emptyset$; $\pi \leftarrow$ policy initialization
2 **for** $i = 1$ **to** N **do**
3 $\quad \hat{\pi} \leftarrow \boldsymbol{\beta}_i \pi + (1 - \boldsymbol{\beta}_i)\pi_{explore}$ ▷ policy $\hat{\pi}$ explores with probability $(1 - \boldsymbol{\beta}_i)$
4 $\quad \mathcal{Q} \leftarrow \mathcal{Q} \cup$ Top K most likely strategies for each formula in $\mathcal{F}$ according to $\hat{\pi}$
5 $\quad \mathcal{D} \leftarrow \mathcal{D} \cup$ Extract training dataset from strategies $\mathcal{Q}$
6 $\quad \pi \leftarrow$ Retrain model π on $\mathcal{D}$

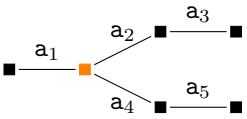
The network output consists of two parts – a probability distribution over tactics to apply next and an assignment to each tactic parameter. The possible tactic parameters are captured by the set `Param` from Fig. 1. We provide a full list of tactics and their parameters in the extended version of our paper. To compute arguments for the tactic parameters, the network introduces a separate output layer for each parameter type. The layer performs regression and outputs normalized values in the range $[0, 1]$. For boolean parameters, values 1 and 0 correspond to `true` and `false`, respectively. For integer parameters, the output of the network is mapped and discretized into the range of allowed values.

Model training At a high level, our training is based on the `DAgger` method [40] and is shown in Algorithm 1. The training starts with a randomly initialized policy used to sample the top K most likely strategies for each formula in the training dataset $\mathcal{F}$ (line 4). Selected strategies are evaluated and used to create a training dataset (line 5, described below) on which the policy π is retrained (line 6). As the model is initially untrained the strategies are sampled at random and only as the training progresses the policy will learn to select strategies that perform well on formulas from $\mathcal{F}$. As an alternative, one could pre-train the initial policy using strategies supplied by an expert in which case the algorithm would correspond to imitation learning. However, in our work we assume such expert strategies are not available and therefore we start with a model that is initialized randomly.

Building the training dataset Each sample in our training dataset $\mathcal{D} = \{\langle(\boldsymbol{t}_i, \boldsymbol{p}_i), s_i\rangle\}_{i=1}^{M}$ for the neural network consists of a state and its associated training label. Here, the label is a tuple consisting of a probability distribution over tactics $\boldsymbol{t} \in \mathbb{R}^{|\texttt{Tactics}|}$ and the values of all tactic parameters $\boldsymbol{p} \in \mathbb{R}^{|\texttt{Param}|}$. The intuition behind this choice is that for a state s, the vector $\boldsymbol{t}$ encodes the likelihood that a given tactic is fast at solving the formula whereas $\boldsymbol{p}$ contains the best parameter values found so far during training. Generating the dataset in this way encodes the preference for strategies that are most efficient in contrast to finding any strategy that solves the input formula. To train the neural network model using such a dataset, the loss is constructed as a weighted average of the cross-entropy loss for tactic prediction and mean-squared-error for parameter prediction.

We build the dataset as follows. First, we evaluate each strategy in $\mathcal{Q}$ on the formula for which it was generated and keep only those that succeeded in deciding the formula. Second, let us denote with $r(t, s_i)$ the best runtime (or `timeout`) achieved from state s_i by first applying tactic t, with $r_{best}(s_i)$ the best runtime achieved from state s_i, and with $v(p, s_i)$ the value assigned to parameter p in tactics with the best runtime from state s_i. We obtain r, r_{best} and v by considering all the states and actions performed when evaluating the strategies in $\mathcal{Q}$. Then, for each non-terminal state s_i that eventually succeeded we create one training sample $\langle(\sigma(\tilde{\boldsymbol{t}}_i), \boldsymbol{p}_i), s_i\rangle$ where $\tilde{\boldsymbol{t}}_i = [1/r(t, s_i)]_{t \in \texttt{Tactics}}$, σ normalizes $\tilde{\boldsymbol{t}}_i$ to a valid probability distribution and $\boldsymbol{p}_i = [v(p, s_i)]_{p \in \texttt{Param}}$. To generate the training dataset for bilinear model we follow a similar procedure with the exception that we use $\tilde{\boldsymbol{t}}_i = \mathbb{1}[r_{best}(s_i) = r(t, s_i)]_{t \in \texttt{Tactics}}$ which assigns probability 1 to the best tactic and 0 to others.

Pruning via equivalence classes A challenge in training the models presented above is that whether a strategy solves the formula is known only at terminal states. This is especially problematic for datasets where majority of the effort is spent on finding any successful strategy. To address this issue we take advantage of the fact that some information can be learned also from partial strategies – namely their current runtime and their transformed formula. This allows us to check if multiple transformed formulas are equivalent (we consider two formulas equivalent if their abstract syntax tress are identical) and keep only the one which was fastest to reach (and prune the others).



	f_1	f_2	f_3	f_1	f_2	f_3
$q_1 = \texttt{a}_1;\texttt{a}_2;\texttt{a}_3$	10	100	100	200	250	30
$q_2 = \texttt{a}_1;\texttt{a}_4;\texttt{a}_5$	30	TIMEOUT	20	num_expr		

tree representation of q_1 and q_2 **formulas f_1, f_2, f_3 and their associated costs** **formula measure**

`if true then a2 else _` **branch with cost** $0.276 = (-1/3 \cdot \log(1/3) - 2/3 \cdot \log(2/3)) + 0$

`if num_expr > 100 then a2 else a4` **branch with cost** $0.2 = -2/3 \cdot \log(1/2) + 1/3 \cdot 0$

Figure 2: Illustration of decision tree representation of strategies, scored formulas and branches.

4.2 Synthesizing a combined, interpretable strategy

The policy π learned in Section 4.1 can be used to speed-up SMT solving as follows: invoke the solver with the current formula f and the action a_0 selected by π, obtain a new intermediate formula f_1 from the solver, then again invoke the solver from scratch with f_1 and a new action a_1, obtaining another intermediate formula f_2, and so on. Unfortunately, even if the final formula f_k is solved (e.g., determined to be SAT), we would have lost the connection with the original formula f as at each step we are making a new, fresh invocation of the SMT solver. This is problematic for tasks (e.g., planning) that require more information from SMT solvers, beyond satisfiability of f, for instance, the model itself. To address this challenge without changing the internals of the SMT solver, we propose to synthesize an interpretable policy q_{best} that follows π and can be directly expressed in the `Strategy` language from Fig. 1 (and thus be given as input to the SMT solver).

Recall from Fig. 1 that the `Strategy` language defines two types of statements that can be used to combine programs: `if-then-else` and `or-else`. However, notice that the `or-else` statement is in fact a special version of the `if` statement with a condition that checks if a given tactic terminates within `c` milliseconds. As a result, we can reduce the problem of synthesizing programs in the `Strategy` language to the problem of synthesizing branches with predicates over a set of candidate strategies. We can obtain the set of candidate strategies by either evaluating π over the training formulas or by extracting successful strategies from the set $\mathcal{Q}$ explored during policy learning. Next, we discuss how to synthesize q_{best}.

Decision tree learning To synthesize branches with predicates, we adapt techniques based on decision tree learning. Consider the tree illustrated in Fig. 2 (top left) with the same structure as used during search (i.e., edges denoting actions and nodes denoting states) but formed from the two candidate strategies q_1 and q_2. Each SMT formula is evaluated by taking a path in the tree and applying all the corresponding actions. Intuitively, at each node with more than one outgoing edge a decision needs to be taken to determine which path to take. To encode such decisions, our goal is to introduce one or more branches at each such node (denoted as orange square in Fig. 2).

More formally, let $\mathcal{Q} = \{q_1, \ldots, q_n\}$ be a set of candidate strategies, $\mathcal{F}$ be our dataset of formulas and $b ::= \texttt{if p then } \boldsymbol{q}_{\texttt{true}} \texttt{ else } \boldsymbol{q}_{\texttt{false}}$ a branch that partitions $\mathcal{F}$ into two parts – $\mathcal{F}_{\texttt{true}}$ are formulas on which predicate `p` evaluates to true and $\mathcal{F}_{\texttt{false}}$ for the rest. We define a notion of multi-label entropy of a dataset of formulas, denoted as $H(\mathcal{F})$ [14]:

$$H(\mathcal{F}) = -\sum\nolimits_{q \in \mathcal{Q}} p(q)\log(p(q)) + (1 - p(q))\log(1 - p(q))$$

where $p(q)$ denotes the ratio of formulas solved by strategy q in $\mathcal{F}$. The goal of synthesis is then to discover branches that partition $\mathcal{F}$ into smaller sets, each of which has small entropy (i.e. either none or all formulas are solved). Using the entropy, we define a cost associated with a branch b as:

$$cost(b, \mathcal{F}_{\texttt{true}}, \mathcal{F}_{\texttt{false}}) = (|\mathcal{F}_{\texttt{true}}|/|\mathcal{F}|)H(\mathcal{F}_{\texttt{true}}) + (|\mathcal{F}_{\texttt{false}}|/|\mathcal{F}|)H(\mathcal{F}_{\texttt{false}})$$

That is, branch cost is a weighted sum of entropies in each of the resulting branches. With this scoring function we build the decision tree in a top-down fashion – for each node with multiple outgoing edges we recursively synthesize predicates that minimize the *cost*. If dataset size for some node is below a certain threshold we greedily select the strategy which can solve the most formulas, breaking ties using runtimes. To express branches, we consider the following types of predicates: (i) `true` which allows expressing a default choice, (ii) `Probes` with arithmetic expression as defined in Fig. 1, and (iii) `try s for c` which allows checking whether tactics terminate within `c` ms.

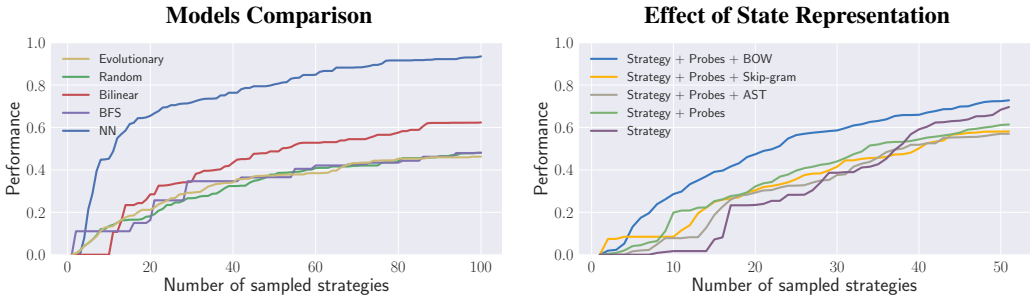


Figure 3: Comparison of proposed models and baselines for learning SMT solver strategies. Each line (higher is better) denotes the quality of the best learned strategy among top x most likely strategies found by a given model (x axis) proportional to the best known strategy overall (y axis).

5 Experiments

We implemented our method in a system called `fastSMT` and evaluated its effectiveness on 5 different datasets – `AProVE` [19, 5], `hycomp` [7], `core` [6], `leipzig` [8] and `Sage2` [9, 21]. These contain formulas of varying complexity across 3 logics `QF_NRA`, `QF_BV` and `QF_NIA`. The formulas have on average 336, 35, 228, 153, 345 assertions, 929, 10690, 1672, 886, 1887 expressions and 118, 49, 46, 20, 79 variables for `leipzig`, `core`, `hycomp`, `AProVE` and `Sage2` benchmarks, respectively and require up to 60 MB for each formula to be stored in the SMT2-lib format. All datasets are part of the official `SMT-LIB` benchmarks [10] and encode formulas from both academic and industrial tools (i.e. `Sage` and `AProVE`). For all datasets we split formulas in training, validation and test set in predetermined ratios. To train our models we use 10 iterations of Algorithm 1 with exponentially decaying exploration rate to choose between policy and random action. We train the bilinear model using `FastText` [26] for 5 epochs with learning rate 0.1 and 20 hidden units. We implemented the neural network model in `PyTorch` [37] and trained using a learning rate 10^{-4}, mini batch size 32, Adam optimizer [31] with Xavier initialization [20] and early stopping. All our datasets, experiments, implementation and extended version of this paper are available at `http://fastsmt.ethz.ch/`.

Search models comparison To compare different learned models we used them to obtain the 100 most likely strategies for each formula in our test dataset. The results are shown in Fig. 3 (left) and additionally include three baseline models that perform random search, breadth-first search and search using an evolutionary algorithm. The x-axis shows the number of most likely strategies sampled from each model and y-axis their runtime proportional to runtime of the best known strategy (i.e., best strategy from top 100 strategies across all models). Here, score one denotes that the best known strategy was found whereas score zero denotes that no strategy was found that solves the formula. Even though baselines perform poorly, they are already able to find simpler strategies that can solve some of the formulas. Note that in our experiment the evolutionary algorithm performed similarly to a random model as it got easily stuck in local minima without enough exploration. Overall, the best model is the neural network which is also most complex and considers the richest set of features.

Effect of state representation In Fig. 3 (right) we evaluate the effect of instantiating the neural network model with a different set of input features capturing the current state. For our task, the representation at the right level of complexity is bag-of-words – if the formula representation is flattened using pre-trained embeddings it loses the relevant information and with more complex AST features the model suffers due to data sparsity. Further, we note that for our task the most important features are those capturing sequences of tactics applied so far, which is illustrated by the strong performance of `Strategy` only model in Fig. 3 (right).

Comparison to state-of-the-art SMT solvers We evaluate the effectiveness of the learned strategies compared to the hand-crafted strategies in Z3 4.6.2 on two metrics – number of solved formulas and runtime. Unless stated otherwise, we use time limit of 10 seconds allocated for solving each formula and instead of runtime, we use the amount of executed basic operations (using `rlimit` counter in Z3) as a deterministic and machine independent measure of the work required to solve the formula. We include additional experiments with runtime in the extended version of the paper.

Table 2: Comparison of best strategy found by any of our models (Section 4.1) against Z3.

	Formulas solved				Speedup percentile		
	Both	Only Z3	Only Learned	None	90th	50th	10th
`leipzig`	57	3	0	8	5.8×	60.7×	191.5×
`Sage2`	332	2	246	220	1.2×	2.7×	35.5×
`core`	270	0	0	0	1.2×	1.3×	1.9×
`hycomp`	273	0	34	18	1.0×	1.8×	4.0×
`AProVE`	283	0	18	14	3.9×	87.8×	1314.0×
Total	68.3%	0.3%	16.8%	14.6%	2.6×	30.9×	309.4×

Table 3: Comparison of the combined strategy synthesized by our approach (Section 4.2) against Z3.

	Formulas solved				Speedup percentile		
	Both	Only Z3	Only Learned	None	90th	50th	10th
`leipzig`	55	5	1	7	1.4×	9.9×	21.7×
`Sage2`	2488	200	705	3051	0.8×	1.2×	3.1×
`core`	270	0	0	0	1.2×	1.3×	1.8×
`hycomp`	1547	93	112	230	0.4×	1.1×	2.3×
`AProVE`	1365	76	112	159	3.2×	65.1×	860.8×
Total	54.6%	3.6%	8.9%	32.9%	1.4×	15.7×	178.0×

Table 2 shows the number of formulas solved by Z3 compared to the best strategy found by any of our methods. We also measure speedups of our strategies over Z3 on all formulas which were solved by both methods. For example, the 90th percentile in the AProVE benchmark denotes that for 90% of formulas the speedup is at least $3.9\times$. The learned strategies significantly outperform Z3 across all benchmarks – solving 17% more formulas, often with up to 3 orders of magnitude speedups and with only 5 formulas not solved by any of the learned strategies even though they can be solved by Z3. This shows that, for all our benchmarks, the strategies found during training generalize well to other formulas from the same dataset.

Table 3 shows performance of the single combined strategy synthesized as described in Section 4.2. Here, the result of synthesis is a program in the `Strategy` language that is used as input to Z3 together with the formula to solve. Naturally, as the `Strategy` language has limited expressiveness (i.e., restricting the kind of expressions that can be used as branch predicates) the performance improvement is smaller compared to best strategy found by any of our methods for each formula as shown in Table 2. However, more importantly, the improvement over the default Z3 strategy is still significant allowing us to solve 8.9% more formulas compared to Z3.

Generalization to harder to solve formulas So far, in all our experiments the time limit for both training and evaluation was set to 10 seconds. To evaluate how our learned strategies generalize to harder to solve formulas we kept the 10 seconds time limit for training but used 600 seconds time limit for the evaluation. Then, our learned strategies can solve 97.7% formulas (up from 85.1%) across all the benchmarks with even slightly better speedups than those shown in Table 2.

6 Conclusion

We presented a new approach for solving SMT formulas based on a combination of training a policy that learns to discover strategies that solve formulas efficiently and a synthesizer that produces interpretable strategies based on this model. The synthesized strategies are represented as programs with branches and are directly usable by state-of-the-art SMT solvers to guide its search. This avoids the need to evaluate the learned policy at inference time and enables close integration with existing SMT solvers. Our technique is practically effective – it solves 17% more formulas over a range of benchmarks and achieves up to $100\times$ runtime improvements over state-of-the-art SMT solver.

Acknowledgments

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Appendix

We provide two appendices. Appendix A includes in depth descriptions of the algorithms used for learning and synthesis. Appendix B provides two additional experiments - evaluating with time limit of 10 minutes and the effect of iterative training used by Algorithm 1.

A Additional details on learning and synthesis

Finding most likely strategies As described in Algorithm 1, a key step of training is a procedure that finds the top K most likely strategies to solve a given formula (line 4 of Algorithm 1). This algorithm, shown in Algorithm 2, takes as input a single formula $f \in \mathcal{F}$, a model π (e.g., a neural network policy, bilinear model, etc.) and integer K (number of strategies to explore).

During the search we keep a priority queue of tuples (s_j, a_j, p_j) consisting of a state, possible action and its associated probability, initialized with $(s_0, \epsilon, 1)$, s_0 denoting initial state. At each step, we remove the tuple with highest probability from the queue, apply its action to obtain a new state s'_j and update the priority queue with new tuples $(s'_j, a, p_j \cdot \pi(a \mid s'_j))$ for all actions $a \in \texttt{Tactics} \times \texttt{Params}$ capturing the possible transitions from s'_j. For practical reasons we approximate the set of all possible actions (denoted as $\text{ACTIONS}(s'_j, \pi)$) as follows:

- If we are using the neural network policy, we consider the most likely parameters for each tactic according to that policy, or
- If we are not using the neural network policy and instead are using models which do not predict parameters, we consider 20 different parameter configurations for each tactic which are selected at random before training starts.

As described in Section 4.1, we additionally perform pruning of states (line 9) which can not possibly lead to an optimal strategy. Finally, we note that in practice we perform the search for batch of formulas at once in order to leverage parallelization capabilities of our system.

Algorithm 2: Procedure for finding top K strategies

Data: Formula f, Model π, Number of strategies to sample K
Result: Top K most likely strategies according to the model

```
1  s_0 = (f, ε)
2  S = ∅
3  queue = PRIORITY_QUEUE()
4  queue.PUSH(s_0, ε, 1)
5  for j = 1 : K do
6      (s_j, a_j, p_j) = queue.POP()
7      s_j --a_j--> s'_j                ▷ State s'_j is obtained by applying action a_j in state s_j
8      S ← S ∪ {s'_j.strategy}          ▷ Strategy is extracted from state s'_j and added to S
9      if ¬ PRUNED(s'_j) then
10         for a ∈ ACTIONS(s'_j, π) do
11             queue.PUSH(s'_j, a, p_j · π(a | s'_j))
12 return S
```

Building the training dataset In Section 4.1 we described how we construct the dataset used to train the neural network policy. Here we illustrate the process on a concrete example. Let us consider an example where the model explored seven different states as shown in Fig. 4 (left). Further, in addition to the visited states we also keep information about the cumulative runtime required to compute the state (starting from the initial state s_0) as well as whether the state successfully solves the formula. Then, according to the procedure described in Section 4.1, the dataset is constructed by generating one training sample $\langle(\sigma(\boldsymbol{t}_i), \boldsymbol{p}_i), s_i\rangle$ for each non-terminal state that eventually succeeded in solving the formula. In our example, this corresponds to generating one training sample for states s_1, s_2 and s_3 as shown in Table 4 where we use ϵ to denote that no tactic was applied so far. Note that

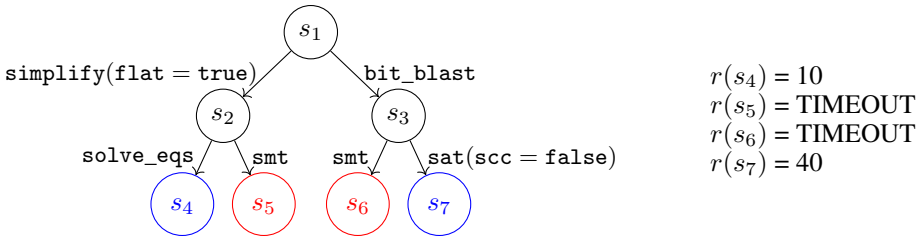

Figure 4: Example with all states visited during solving a formula. Terminal states are colored blue if formula was solved and red otherwise. Runtime for each terminal state is shown on the right.

State	Target tactics	Target parameters
$s_1 = (\varphi_1, \epsilon)$	$\Pr(\texttt{simplify}) = 0.8$	$\texttt{flat} = \texttt{true}$
	$\Pr(\texttt{bit\_blast}) = 0.2$	-
$s_2 = (\varphi_2, \texttt{simplify}(\texttt{flat} = \texttt{true}))$	$\Pr(\texttt{solve\_eqs}) = 1$	-
$s_3 = (\varphi_3, \texttt{bit\_blast})$	$\Pr(\texttt{sat}) = 1$	$\texttt{scc} = \texttt{false}$

Table 4: Dataset constructed from the example shown in Fig. 4.

for state s_1 there are two possible tactics which lead to solving the formula ($\texttt{simplify}(\texttt{flat} = \texttt{true})$ and $\texttt{bit\_blast}$). However, the best strategies in respective subtrees have different runtimes hence probabilities assigned to the corresponding tactics are different as shown in Table 4.

Reducing the set of strategies As the set of synthesized strategies can be large we perform synthesis (described in Section 4.2) only on a subset set of strategies. Intuitively, these strategies should be: strong individually (i.e. each strategy should be able to solve large number of formulas alone) and strong together (i.e. number of formulas solvable by at least one of the strategies should be large). In order to trade-off these two conditions we use greedy procedure shown in Algorithm 3.

We proceed in an iterative manner, choosing one new strategy at every step. In every iteration, strategy receives a score for every formula that it can solve. Score is equal to λ for every formula which was previously unsolved, and 1 - λ if formula was already solved (by another previously selected strategy). One can notice that if $\lambda = 1$ algorithm will try to greedily maximize the number of formulas that strategies can solve in the union. If $\lambda = 1/2$ algorithm will simply select k strategies which can solve the most formulas individually. In our experiments we treat λ as a hyperparameter and optimize it on a validation set of formulas. Concretely, we set λ to 0.5, 0.99, 0.75, 0.95, 0.7 for `Sage2`, `AProVE`, `leipzig`, `core` and `hycomp` respectively.

Algorithm 3: Greedy strategies selection

Data: Set of formulas $\mathcal{F}$, Strategies s_1, ..., s_m, weight $\lambda \in [0, 1]$
Result: Set S consisting of k selected strategies
Initialize $S_{best} = \emptyset$
Initialize $A = \mathcal{F}, B = \emptyset$
for $iter = 1 : k$ **do**
  for $i = 1 : m$ **do**
    if $s_i \notin S_{best}$ **then**
      Calculate A_i = subset of formulas in A which strategy s_i can solve
      Calculate B_i = subset of formulas in B which strategy s_i can solve
      Define $score_i = \lambda|A_i| + (1 - \lambda)|B_i|$
  Add strategy s_i with highest corresponding $score_i$ to S_{best}
  $A = A \setminus A_i$
  $B = B \cup A_i$

Table 5: Comparison of best strategy found by any of our models against Z3 with 10min time limit.

	Formulas solved				Speedup percentile		
	Both	Only Z3	Only Learned	None	10^{th}	50^{th}	90^{th}
leipzig	63	0	1	4	3.5×	43.9×	183.2×
Sage2	630	0	138	32	1.3×	6.5×	199.6×
core	270	0	0	0	1.2×	1.3×	1.9×
hycomp	298	0	10	17	1.0×	2.0×	40.1×
AProVE	306	0	3	6	3.9×	89.3×	1301.5×
Total	88.1%	0.00%	8.6%	3.3%	2.2×	28.6×	345.3×

Table 6: Comparison of best strategy found by any of our models (Section 4.1) against Z3.

	Formulas solved				Speedup percentile		
	Both	Only Z3	Only Learned	None	90^{th}	50^{th}	10^{th}
leipzig	57	0	3	8	5.8×	60.7×	191.5×
Sage2	2531	0	2402	1511	1.2×	2.5×	22.0×
core	270	0	0	0	1.2×	1.3×	1.9×
hycomp	1633	1	210	138	1.0×	2.0×	4.3×
AProVE	1397	3	221	91	4.0×	89.6×	988.7×
Total	56.2%	0.1%	27.0%	16.7%	2.6×	31.2×	241.7×

B Additional experiments

Evaluation with 10 minute time limit The 10 second time limit in our experiments was selected for practical purposes - it is large enough to solve 83.3% of the formulas and to learn strategies in a reasonable time. To check how well our strategies generalize to higher time limits we kept the 10 seconds time limit for training, but used 10 minutes for evaluation. We show results from this experiment in Table 5. With the 10 minute time limit, 88.1% of formulas are solved by both methods. Crucially, our strategies are still able to solve 8.6% more formulas than Z3. In addition, the speedups over Z3 are comparable (and even slightly higher) to speedups achieved with 10 second time limit. For comparison, Table 2 shows the results of evaluating the same strategies on the same set of formulas using a 10 second time limit. Note since the experiments take significantly longer to run we evaluated them only on a subset of the dataset.

Evaluation on larger set of formulas For completeness, we also include the results of the best strategies synthesized by our models on the full set of formulas in our test set using the 10 second time limit. Results are shown in Table 6. The set of formulas is the same as in Table 3 (as opposed to Table 2 and Table 5 which use a subset of the formulas).

Number of basic operations and runtime As stated in Section 4, we use the number of basic operations as a deterministic measure of the amount of work required to solve a formula. In Table 7, we show a comparison between the number of operations and wall clock time for the experiments in Table 5. Note that wall clock time is often imprecise. Especially for formulas which can be solved very fast, wall clock time mostly accounts for initialization of the solver and overhead of the system.

Effect of iterative training In Fig. 5 we show the improvement of our neural network policy as it is continuously retrained using DAgger. We perform a total of 10 stages of DAgger, retraining the model after every stage. In every stage, the current model is used to search for the best strategies, as described in Algorithm 2.

For the purpose of this experiment, we save the models after 2, 4, 6, 8 and 10 stages of DAgger. Then we load each model again and use it to search for the best strategies on the unseen formulas from the

Table 7: Comparison of speedup in number of operations and wall clock time.

	Speedup - number of operations				Speedup - Wall clock time			
	P_{10}	P_{50}	P_{90}	Mean	P_{10}	P_{50}	P_{90}	Mean
`leipzig`	3.5×	43.9×	183.2×	71.6×	0.3×	2.3×	7.3×	3.5×
`Sage2`	1.3×	6.5×	199.6×	62.7×	1.2×	4.8×	72.5×	37.8×
`core`	1.2×	1.3×	1.9×	1.4×	0.5×	0.8×	1.3×	0.9×
`hycomp`	1.0×	2.0×	40.1×	51.9×	0.9×	1.4×	65.7×	25.0×
`AProVE`	3.9×	89.3×	1301.5×	519.9×	0.9×	6.4×	120.8×	45.8×
Total	2.2×	28.6×	345.3×	141.5×	0.8×	3.1×	53.5×	22.6×

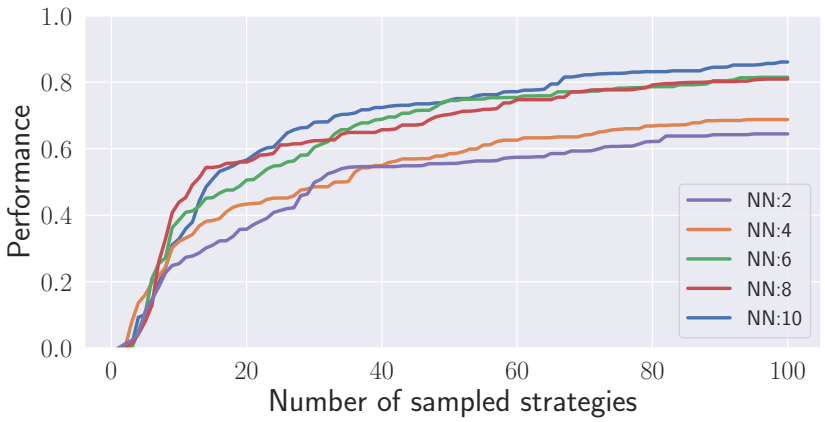

Figure 5: Performance of neural network after 2, 4, 6, 8 and 10 retraining iterations.

test set. We run all of the models for 100 iterations without retraining (which means that each model predicts 100 best strategies for every formula).

One can notice that later models tend to outperform earlier models, thus justifying the increased number of training stages. The only exception in this case are models trained after 6 and 8 stages where an earlier model performs better by a small margin. This can be explained by the stochastic nature of the training procedure.

Tactics For completeness, we also include the set of tactics and parameters used in our experiments. Tactics used for `Sage2` and `core`, both in QF_BV logic are shown in Table 8. Tactics for `hycomp` in QF_NRA logic are in Table 9. Finally, tactics for `leipzig` and `AProVE` in logic QF_NIA are in Table 10. For every tactic we list the parameters that we used (all parameters not listed here are set to the default value) as well as their types.

Table 8: Tactics and parameters used for QF_BV logic (Sage2 and Core benchmarks).

Tactics	Parameters	Type
	elim_and	bool
simplify	blast_distinct	bool
	push_ite_bv	bool
	som	bool
	pull_cheap_ite	bool
	hoist_mul	bool
	local_ctx	bool
	flat	bool
smt	-	-
bit_blast	-	-
bv1_blast	-	-
solve_eqs	-	-
aig	aig_per_assertion	bool
qfnra_nlsat	-	-
sat	-	-
max_bv_sharing	-	-
reduce_bv_size	-	-
purify_arith	-	-
propagate_values	push_ite_bv	bool
elim_uncnstr	-	-
ackermannize_bv	-	-

Table 9: Tactics and parameters used for QF_NRA logic (hycomp benchmark).

Tactics	Parameters	Type
	elim_and	bool
simplify	blast_distinct	bool
	som	bool
	hi_div0	bool
	hoist_mul	bool
	local_ctx	bool
	flat	bool
smt	-	-
bit_blast	-	-
solve_eqs	-	-
	-	-
qfnra_nlsat	inline_vars	bool
	factor	bool
	seed	int
max_bv_sharing	-	-
propagate_values	push_ite_bv	bool
elim_uncnstr	-	-
nla2bv	nla2bv_max_bv_size	int
ctx_simplify	-	-

Table 10: Tactics and parameters used for QF_NIA logic (leipzig and AProVE benchmarks).

Tactics	Parameters	Type
	elim_and	bool
simplify	som	bool
	blast_distinct	bool
	flat	bool
	hi_div0	bool
	local_ctx	bool
	hoist_mul	bool
propagate_values	push_ite_bv	bool
smt	-	-
bit_blast	-	-
solve_eqs	-	-
qfnra_nlsat	-	-
max_bv_sharing	-	-
elim_uncnstr	-	-
nla2bv	nla2bv_max_bv_size	int
ctx_simplify	-	-