

Scalable Certified Segmentation via Randomized Smoothing

Marc Fischer, Maximilian Baader, Martin Vechev

We present a new certification method for image and point cloud segmentation based on randomized smoothing [1]. Key to our approach is the ability to **abstain** from classifying single components (e.g. pixel or point cloud points) and reliance on established **multiple-testing correction** mechanisms.

Robust Classifier For a segmentation model $f: \mathbb{R}^N \rightarrow \mathbb{R}^N$ and threshold $\tau \in [\frac{1}{2}, 1)$ we define

$$\bar{f}_i^\tau(\mathbf{x}) = \begin{cases} c_{A,i} & \text{if } \mathbb{P}_{\epsilon \sim \mathcal{N}(0, \sigma)}(f_i(\mathbf{x} + \epsilon) = c_{A,i}) > \tau \\ \emptyset & \text{else} \end{cases}$$

Then, letting $\mathcal{I}_\mathbf{x} = \{i \mid \bar{f}_i^\tau(\mathbf{x}) \neq \emptyset, i \in 1, \dots, N\}$ denote the set of non-abstain indices for $\bar{f}^\tau(\mathbf{x})$,

$$\bar{f}_i^\tau(\mathbf{x} + \delta) = \bar{f}_i^\tau(\mathbf{x}), \quad \forall i \in \mathcal{I}_\mathbf{x}$$

for $\delta \in \mathbb{R}^{N \times m}$ with $\|\delta\|_2 \leq R := \sigma \Phi^{-1}(\tau)$.

Classification [1]



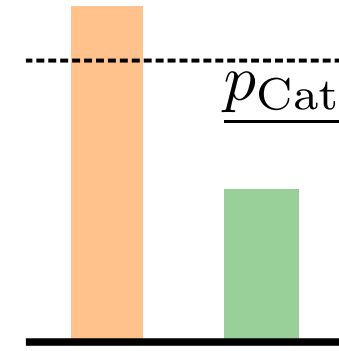
$+\mathcal{N}(0, \sigma I)$

sample perturbations

Segmentation (this work)

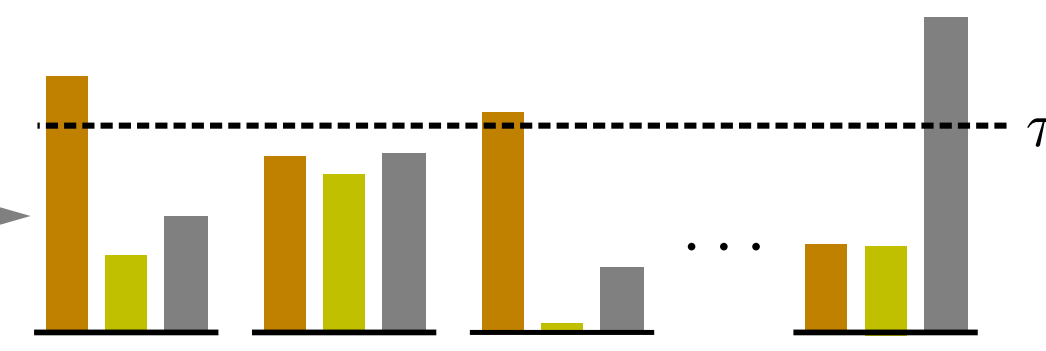


$+\mathcal{N}(0, \sigma I)$



statistical test

left: find lower bound $\underline{p}_{\text{Cat}}$
right: check threshold τ



"Cat" for $\pm \delta$,
 $\|\delta\|_2 \leq \sigma \Phi^{-1}(\underline{p}_{\text{Cat}})$
w.p. $1 - \alpha$

multiple-testing correction

$\text{FWER}(\dots, \alpha)$

$\hat{c}_1 \quad \hat{c}_2 \quad \hat{c}_3 \quad \dots \quad \hat{c}_N$

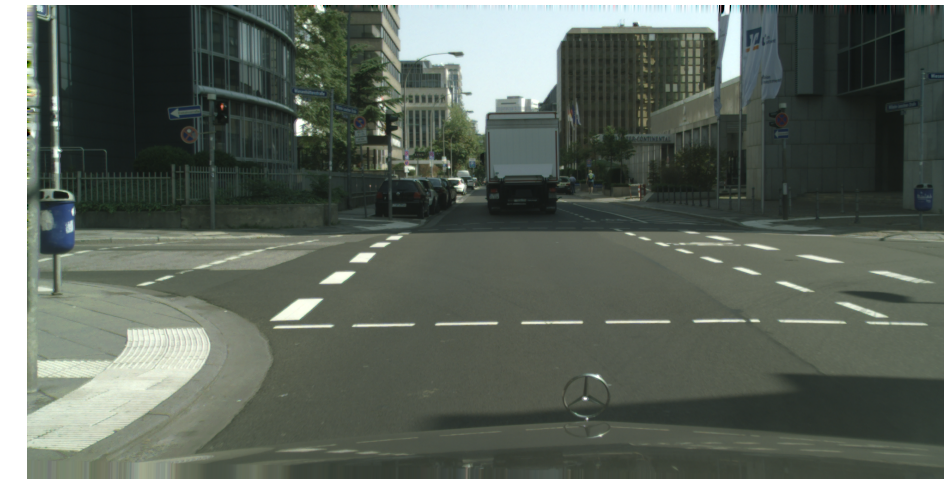
certificate

Segmentation Certificate

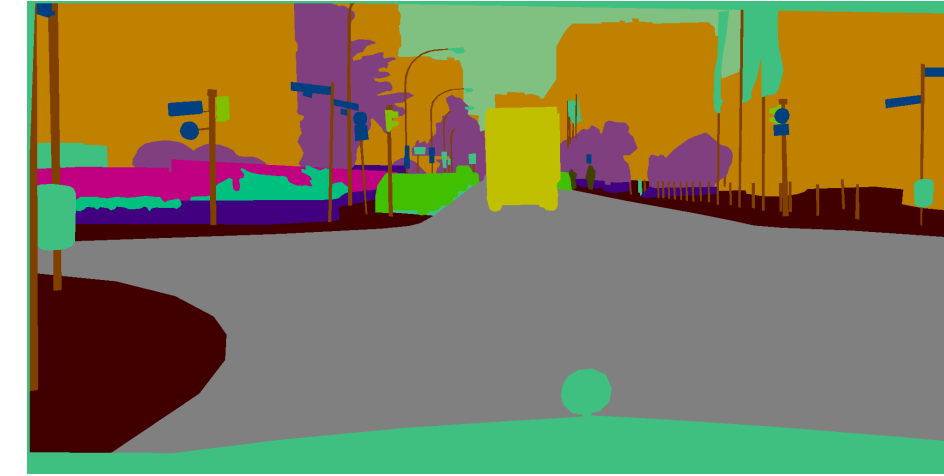
$$\hat{\mathcal{I}} = \{i \mid \hat{c}_i \neq \emptyset\}$$

$$\bar{f}_i^\tau(\mathbf{x}) = \bar{f}_i^\tau(\mathbf{x} + \delta) = \hat{c}_i \quad \forall i \in \hat{\mathcal{I}}, \forall \delta \text{ with } \|\delta\|_2 \leq \sigma \Phi^{-1}(\tau)$$

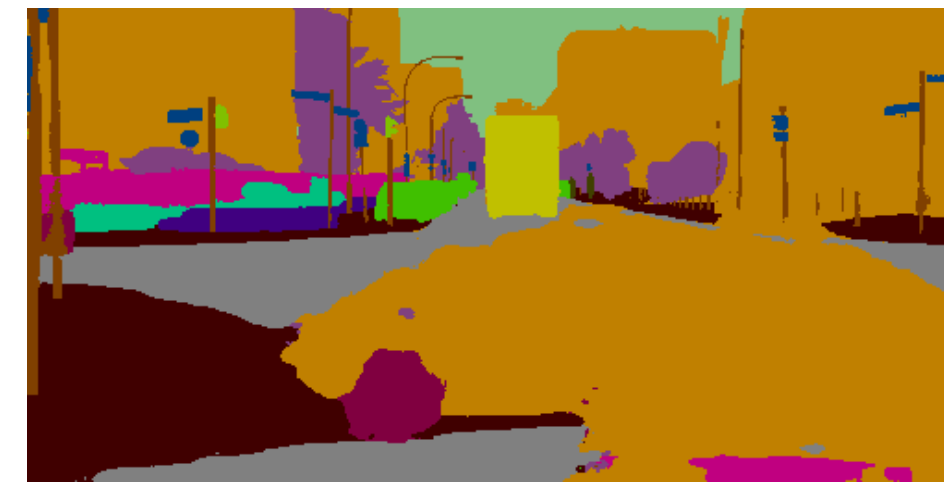
w.p. $1 - \alpha$



Original



Target



Attacked



Certified

Figure 1: Image from [2]. White pixels show abstentions.

Scales to large Semantic Segmentation problems like Cityscapes [2] or Pascal Context [3].

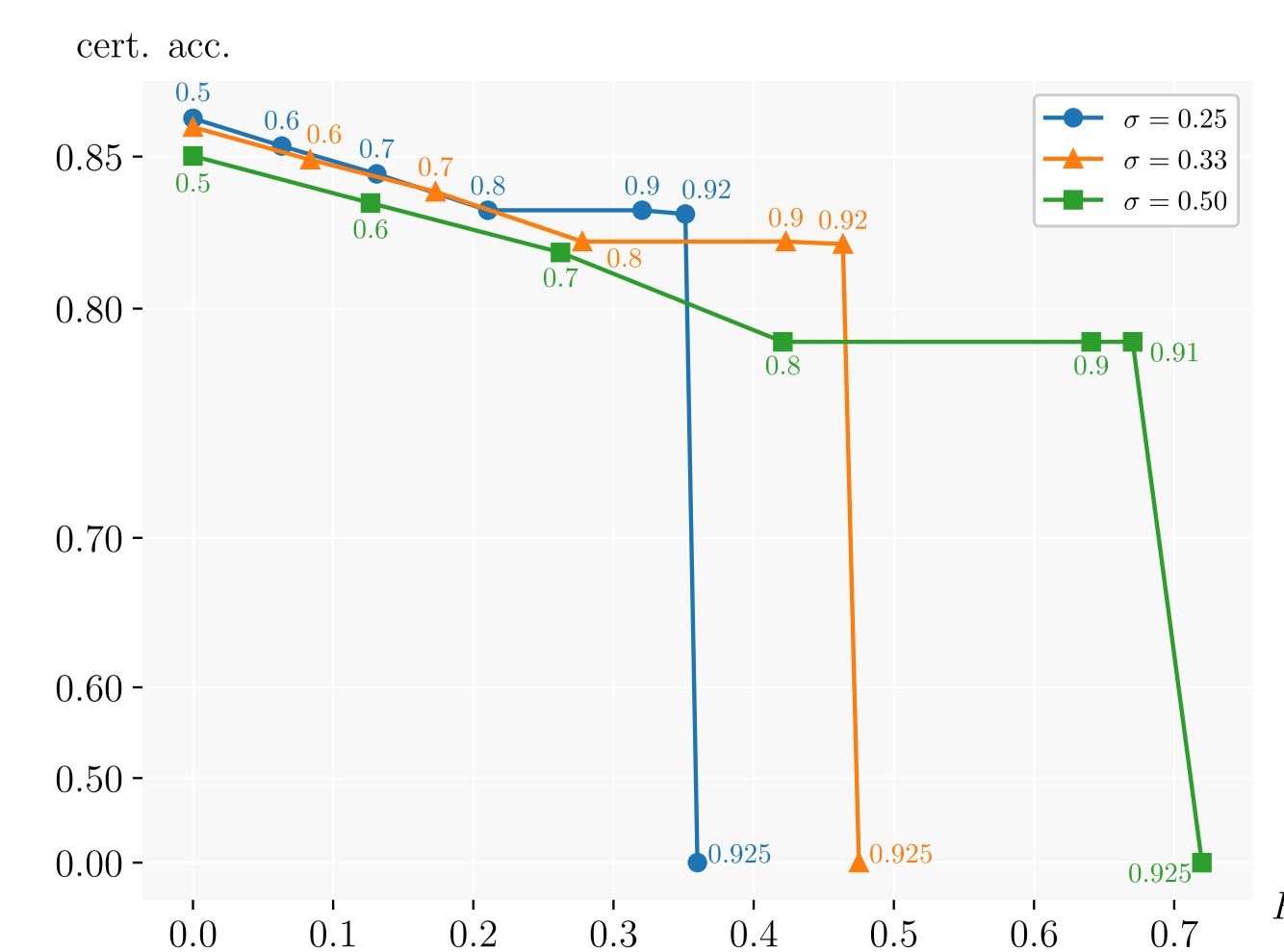


Figure 2: Radius versus certified mean per-pixel accuracy for semantic segmentation on Cityscapes at scale 0.25. Numbers next to dots show τ . The y -axis is scaled to the fourth power for clarity. ($n = 300$)

Table 1: Segmentation results for 100 images. acc. shows mean per-pixel accuracy, mIoU the mean intersection over union, $\% \emptyset$ abstentions and t runtime in seconds. All SEG CERTIFY results are certifiably robust at radius R w.h.p. ($n_0 = 10, \alpha = 0.001$)

		Cityscapes					
scale		σ	R	acc.	mIoU	$\% \emptyset$	t
0.25	non-robust model	-	-	0.93	0.60	0.00	0.38
	base model	-	-	0.87	0.42	0.00	0.37
	SEG CERTIFY	0.25	0.17	0.84	0.43	0.07	70.00
	$n = 100, \tau = 0.75$	0.33	0.22	0.84	0.44	0.09	70.21
		0.50	0.34	0.82	0.43	0.13	71.45
0.5	non-robust model	-	-	0.96	0.76	0.00	0.39
	base model	-	-	0.89	0.51	0.00	0.39
	SEG CERTIFY	0.25	0.17	0.88	0.54	0.06	75.59
	$n = 100, \tau = 0.75$	0.33	0.22	0.87	0.54	0.08	75.99
		0.50	0.34	0.86	0.54	0.10	75.72
1.0	non-robust model	-	-	0.97	0.81	0.00	0.52
	base model	-	-	0.91	0.57	0.00	0.52
	SEG CERTIFY	0.25	0.17	0.88	0.59	0.11	92.75
	$n = 100, \tau = 0.75$	0.33	0.22	0.78	0.43	0.20	92.85
		0.50	0.34	0.34	0.06	0.40	92.48
multi	non-robust model	-	-	0.97	0.82	0.00	8.98
	base model	-	-	0.92	0.60	0.00	9.04
	SEG CERTIFY	0.25	0.17	0.88	0.57	0.09	1040.55
	$n = 100, \tau = 0.75$						

Applicable to Point Cloud Part Segmentation, such as ShapeNet [4].

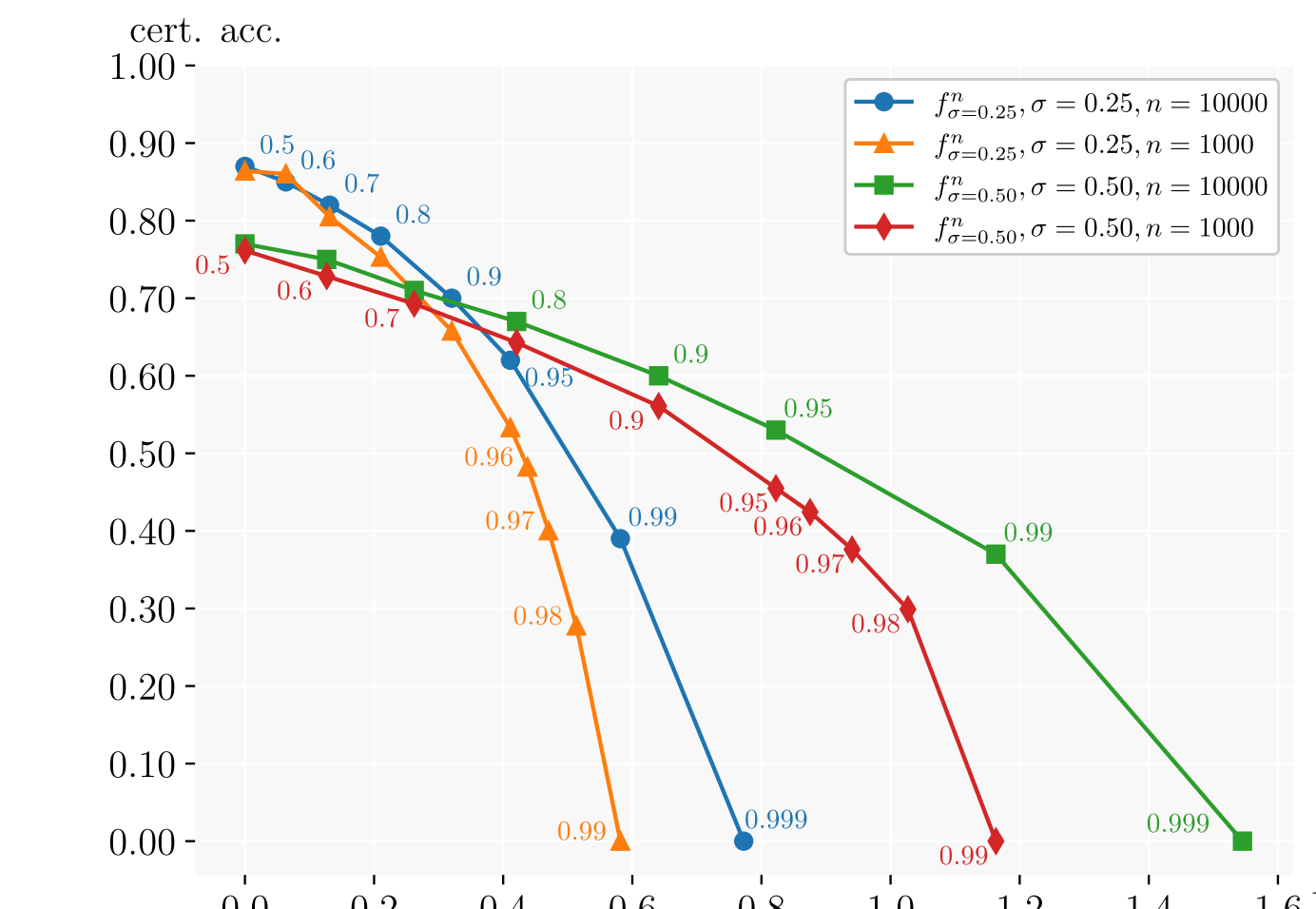


Figure 3: Radius versus certified accuracy at different radii for Point Cloud part segmentation. Numbers next to dots show τ .

Table 3: Results on 100 point clouds part segmentations, showing the mean per-point accuracy (acc), abstentions ($\% \emptyset$) and t runtime in seconds.

	n	τ	σ	acc	$\% \emptyset$	t
non-robust model	-	-	-	0.91	0.00	0.57
base model $f_{\sigma=0.25}^n$	-	-	-	0.86	0.00	0.57
	1000	0.75	0.25	0.78	0.16	54.46
SEG CERTIFY $f_{\sigma=0.25}^n$	1000	0.85	0.25	0.71	0.25	54.41
	10000	0.95	0.25	0.62	0.35	496.65
	10000	0.99	0.25	0.39	0.60	496.68
base model $f_{\sigma=0.50}^n$	-	-	-	0.86	0.00	0.57
	1000	0.75	0.50	0.67	0.21	54.58
SEG CERTIFY $f_{\sigma=0.50}^n$	1000	0.85	0.50	0.61	0.30	54.44
	10000	0.95	0.50	0.53	0.41	497.40
	10000	0.99	0.50	0.37	0.60	497.87

Certification of challenging specifications (such as rotations) beyond ℓ_2 .

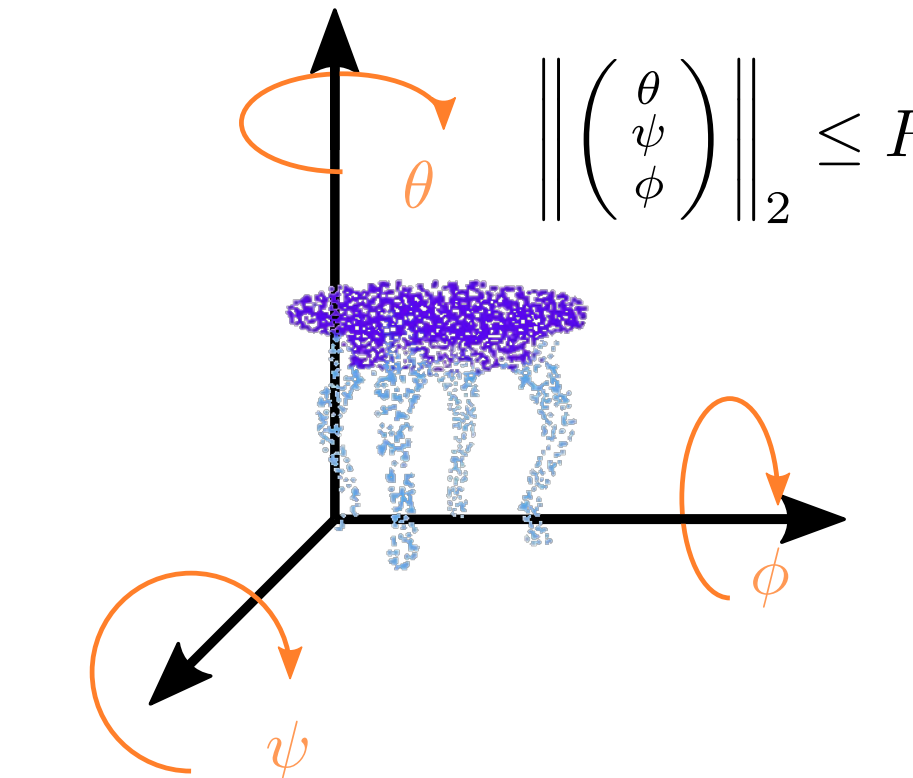


Figure 4: Certification of 3d-rotations via [5].

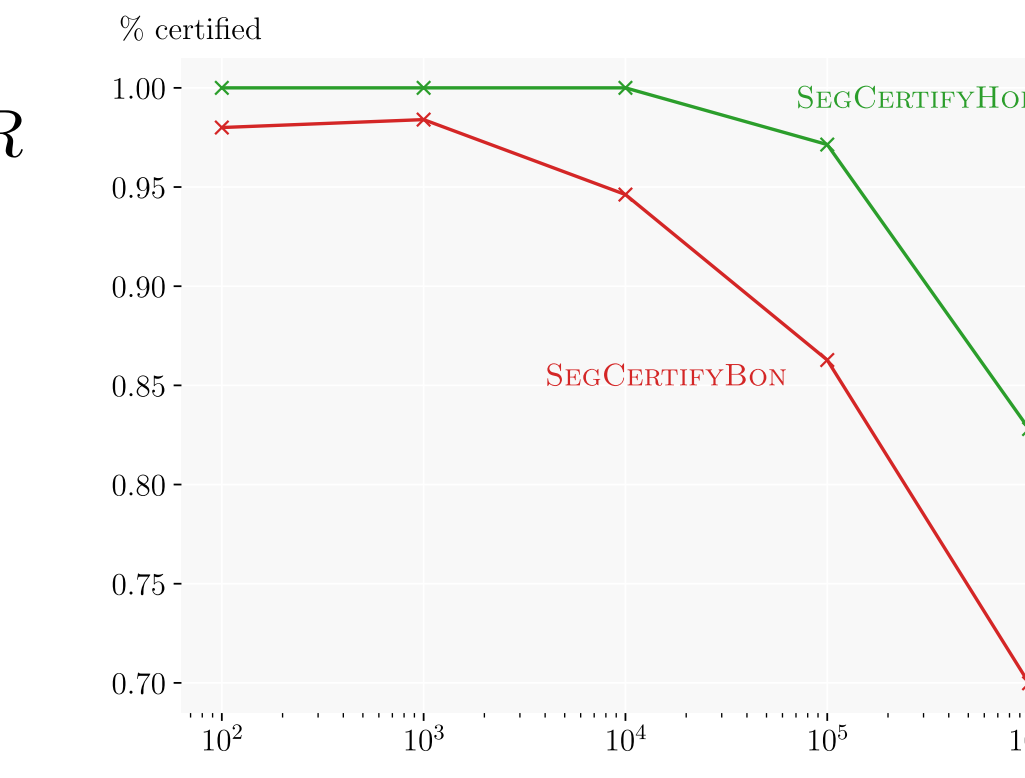


Figure 5: Difference between Bonferroni and Holm FWER-CONTROL for SEG CERTIFY.

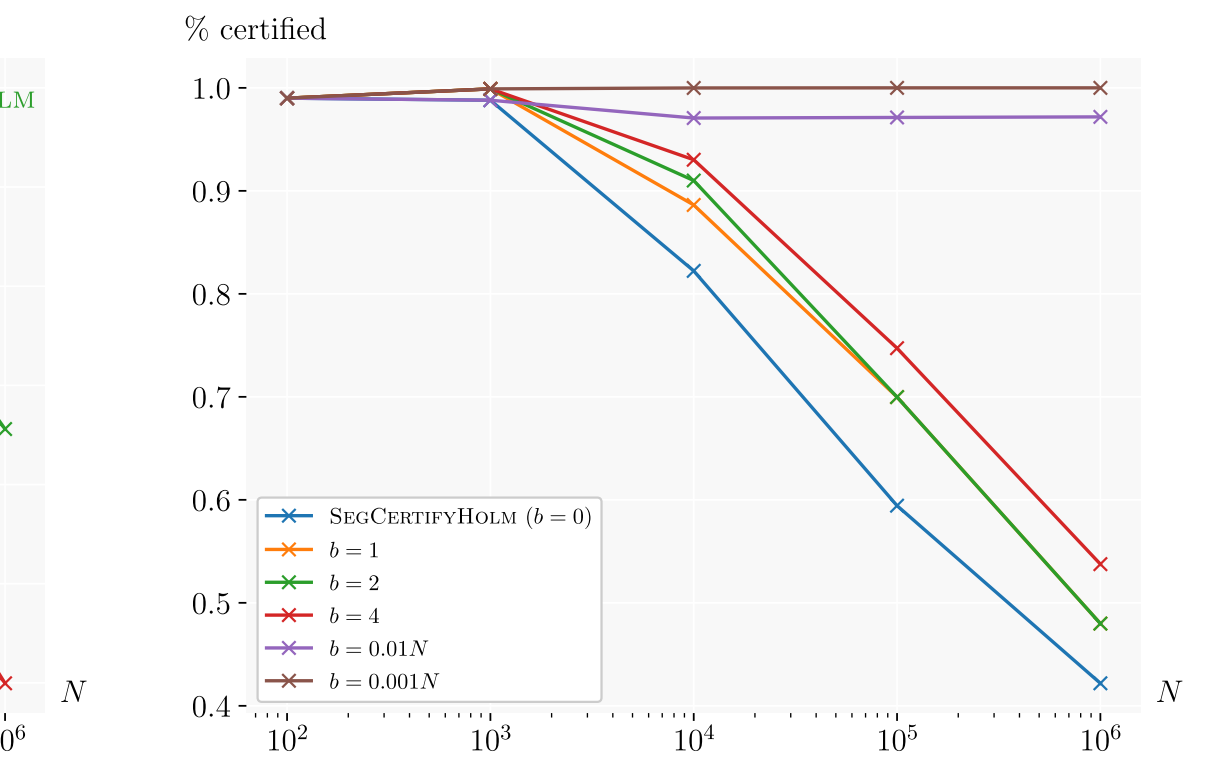
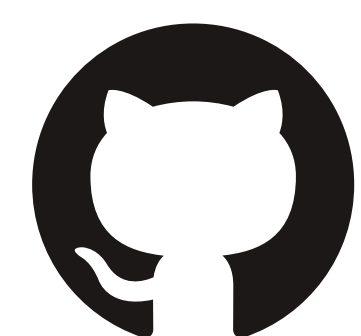


Figure 6: SEG CERTIFY with k -FWERCONTROL that allows for up to b errors.

Error Rate Control and Error Budgets Figure 5, shows the impact of different choices of FWERCONTROL on the number of certified components when f is an oracle with error rate 0.05. FWERCONTROL limits the probability of making any error. k -FWERCONTROL, displayed in Figure 6, allows up to b errors.

Advanced FWERCONTROL allows to improve statistical power, e.g. by allowing errors for a small fraction of components.

- [1] Cohen et al. Certified adversarial robustness via randomized smoothing ICML'19
- [2] Cordts et al. The cityscapes dataset for semantic urban scene under-standing CVPR'16
- [3] Mottaghi et al. The role of context for object detection and semantic segmentation in the wild CVPR'14
- [4] Chang et al. Shapenet: An information-rich 3d model repository. 2015
- [5] Fischer et al. Certified defense to image transformations via randomized smoothing NeurIPS'20



github.com/eth-sri/segmentation-smoothing

ETH zürich

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