Exercise 04

Adversarial Defenses and Certification

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Problem 1 (Coding)**.** In this task, you are going to implement adversarial training with PGD (originally introduced in [1]) and an alternative defense.

1. Complete the provided code skeleton in `train.py` to train the network `model` with PGD defense. That is, for data distribution D (the MNIST dataset in our case) and network parameters θ, optimize the following objective:

$$\min_{\theta} \mathbb{E}_{(x,y)\sim D}\left[\max_{x'\in\mathbb{B}_\epsilon(x)} L(\theta, x', y)\right]. \tag{1}$$

   Here, $\mathbb{B}_\epsilon(x) := \{x' \mid \|x - x'\|_\infty \leq \epsilon\}$ denotes the ϵ-sized ℓ_∞-ball around x. L is the usual classification loss $L(\theta, x', y) := \mathcal{H}(y, f_\theta(x'))$, where $f_\theta = (\texttt{model} \circ \text{softmax})$ denotes the output distribution of the neural network, and $\mathcal{H}$ the cross entropy[1] between distributions (being a discrete value, we treat y as a one-hot distribution). In PyTorch, you can use `nn.CrossEntropyLoss`[2] to implement L.

   Use PGD to solve the inner optimization problem with $\epsilon = 0.1$, $k = 7$ steps, and $\epsilon_{\text{step}} = 2.5\frac{\epsilon}{k}$. You can reuse your implementation of untargeted PGD from the previous exercise, or create a more efficient (batched) version better suited for training.

   Compare the accuracy results with and without PGD training.

2. The TRADES [2] algorithm minimizes the following objective (see [2] for details):

[1] `https://en.wikipedia.org/wiki/Cross_entropy`
[2] `https://pytorch.org/docs/stable/generated/torch.nn.CrossEntropyLoss.html`

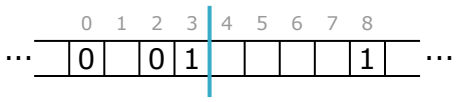
$$\min_{\theta} \mathbb{E}_{(x,y)\sim D}\big[\underbrace{L(\theta,x,y)}_{\text{for accuracy}} + \underbrace{\lambda \max_{x'\in\mathbb{B}_\epsilon(x)} L(\theta,x',f_\theta(x))}_{\text{regularization for robustness}}\big]. \tag{2}$$

Extend your implementation in `train.py` to the TRADES defense. Again, the inner optimization problem is solved using PGD. Use $\lambda = 1.0$ and the same parameters as in the previous task. Compare your results with the previous task.

Note: Here, $L(\theta, x', f_\theta(x))$ still denotes the cross entropy loss. However, PyTorch's `nn.CrossEntropyLoss` cannot be used as it expects one of the distributions to be a one-hot encoding of the label. Instead, you will need to manually implement L. To this end, some existing functions in PyTorch[3] [4] may be useful.

Problem 2 (Alternating optimization)**.** We formalized adversarial defense as the optimization problem in (1). In general, finding the globally optimal parameters θ^* is hard due to the nesting of maximization and minimization. In the lecture, we considered a method that *approximates* θ^* by individually solving the outer and inner optimization problem in alternation. In this task, you will show that this approach can result in a local minimum that is not globally optimal.

Consider the basic case in one dimension where the data is of the form (x, y) with discretized $x \in \mathbb{Z}$ and label $y \in \{0, 1\}$. Assume a very simple classifier ρ in one parameter $\theta \in \mathbb{Z}$, which classifies a point x as $\rho(\theta, x) = [x \geq \theta]$, where $[\cdot]$ is the Iverson bracket. [5] Consider a scenario where $D = \{(0, 0), (2, 0), (3, 1), (8, 1)\}$ as illustrated below:



For a datapoint (x, y), we define the loss $L(\theta, x, y) := [\rho(\theta, x) \neq y]$. Assuming all data points in D are equally likely, the expected loss of a model $\rho(\theta, \cdot)$ is the fraction of misclassified points in D. In our example, the model for $\theta = 4$ (indicated as a blue line) has expected loss $\frac{1}{4}$ as it misclassifies one out of four points. We set $\epsilon = 2$ for the adversarial region $\mathbb{B}_\epsilon(x)$ to denote the set of integers with distance at most 2 to x. For example, point $(8, 1)$ can be perturbed to $(6, 1)$.

Show how in this scenario, alternating optimization of (1) can result in a local optimum that is strictly worse than the global optimum.

[3] `https://pytorch.org/docs/stable/nn.html#loss-functions`
[4] `https://pytorch.org/docs/stable/generated/torch.nn.LogSoftmax.html#torch.nn.LogSoftmax`
[5] `https://en.wikipedia.org/wiki/Iverson_bracket`

Problem 3 (Box Transformers)**.** In the lecture, you have seen the dox domain for numerical analysis. For vectors $a, b \in \mathbb{R}^m$ with $\forall i.\ a_i \leq b_i$, the box $[a, b]$ is a hypercube in $\mathbb{R}^m$. We can use abstract transformers to obtain an over-approximation of the behavior of a function. Given a function $f\colon \mathbb{R}^n \to \mathbb{R}^m$ and an input box $[a, b] \subset \mathbb{R}^n$, a sound abstract transformer $f^\sharp$ finds $[c, d] \subset \mathbb{R}^m$ such that $\forall x \in [a, b].\ f(x) \in [c, d]$.

For example, let $x \in [1, 3]$ and $y \in [2, 4]$, and assume we want to approximate the result of $2x - y$. Using the basic abstract transformers from the lecture, we can compute

$$2 \cdot^\sharp [1, 3] -^\sharp [2, 4] \ = \ [2, 6] +^\sharp [-4, -2] \ = \ [-2, 4]$$

and conclude that $2x - y \in [-2, 4]$.

1. Show that the box transformers lose precision, by approximating the outcome of $x - x$ for $x \in [0, 1]$ using the transformers $+^\sharp$ and $-^\sharp$ from the lecture.

2. Prove or disprove: The alternative transformer $[a, b] +' [c, d] = [a + c,\ b + |d|]$ for addition is sound (i.e., the output box is an over-approximation of all possible result values).

3. Prove or disprove: The alternative transformer $[a, b] +'' [c, d] = [-\infty,\ a + b + d]$ for addition is sound.

4. Derive a sound abstract transformer $f^\sharp$ for the function $f(x) := x^2$. That is, derive expressions for g, h such that $[g, h] = f^\sharp([a, b])$ for $a, b \in \mathbb{R}$.

5. Derive a sound abstract transformer $\cdot^\sharp$ for multiplication. That is, derive expressions for g, h such that $[g, h] = [a, b] \cdot^\sharp [c, d]$, where $a, b, c, d \in \mathbb{R}$.

References

[1] Aleksander Madry et al. "Towards deep learning models resistant to adversarial attacks". ICLR (2018).

[2] Hongyang Zhang et al. "Theoretically Principled Trade-off between Robustness and Accuracy". ICML (2019).